Network Coding Based CQI Reporting for Two-Way Multi-Relay Networks

Quoc-Tuan Vien, *Student Member, IEEE* and Huan X. Nguyen, *Member, IEEE*

Abstract

This paper considers channel quality indicator (CQI) reporting for data exchange in a two-way multi-relay network. We first propose an efficient CQI reporting scheme based on network coding, where two terminals are allowed to simultaneously estimate the CQI of the distant terminal-relay link without suffering from additional overhead. In addition, the transmission time for CQI feedback at the relays is reduced by half while the increase in complexity and the loss of performance are negligible. This results in a significant improvement of system throughput especially when the number of relays is large. Upper and lower bounds of the mean square error (MSE) of the estimated CQI are derived to study performance behaviour of our proposed scheme. It is found that the MSE of the estimated CQI increases proportionally with the square of the cardinality of CQI level sets though an increased number of CQI levels would eventually lead to a higher data-rate transmission. Based on the derived bounds, a low-complexity relay selection (RS) scheme is then proposed. Simulation results show that, in comparison with optimal methods, our suboptimal bound-based RS scheme achieves satisfactory performance while reducing the complexity at least three times in case of large number of relays.

Quoc-Tuan Vien is with School of Engineering and Built Environment, Glasgow Caledonian University, Cowcaddens Road, Glasgow G4 0BA, United Kingdom. Email: quoctuan.vien@gcu.ac.uk.

Huan X. Nguyen is with School of Engineering and Information Sciences, Middlesex University, The Burroughs, London NW4 4BT, United Kingdom. Email: h.nguyen@mdx.ac.uk.

Index Terms

Two-way wireless relay networks, network coding, CQI reporting.

I. Introduction

Relaying techniques have recently attracted significant attention from researchers worldwide in the areas of wireless networks, such as broadcast channels in wireless communication networks [1], cooperative communications in cellular networks [2], ad hoc networks [3], sensor networks [4], and ultra-wideband body area networks [5]. Data transmission in wireless single-relay networks can be classified into two categories: one-way and two-way single-relay network. One-way single-relay network (OWSRN) considers unidirectional communication where a source node sends data to a destination node through a relay node with several proposed cooperation strategies [6], [7]. On the other hand, two-way single-relay network (TWSRN) refers to a scenario where two parties want to exchange information with each other through a relay node. Researchers have also been dedicated to investigating the application of network coding (NC) [8], [9] to improve the network throughput of some particular relay topologies, such as relay-assisted broadcast channels [10], [11], multicast channels [12], [13], unicast channels [14], [15], and bidirectional channels (i.e., TWSRNs) [16], [17]. Here, we investigate the application of NC to TWSRNs, which have interested many studies [18]–[23]. In an NC-based TWSRN, the relay node mixes the signals received from two terminal nodes before broadcasting it. From this combined signal, each terminal node can extract the data sent by the other terminal node using network decoding mechanisms. In real-time applications, e.g., in wireless ad-hoc or sensor networks, strict complexity and time delay constraints are generally required, especially when there are multiple intermediate relay nodes. This stimulates us to consider a nonregenerative two-way multi-relay network (TWMRN) model that includes two terminal nodes and multiple relay nodes. Under nonregenerative protocol, the relay nodes simply combine the received data signal and forward this combined signal to both terminal nodes.

In general, in order to help one terminal node decode the data sent by the other in nonregenerative TWMRNs, the channel state information (CSI) of the terminal-relay links should be feedbacked to both terminal nodes [24]. Common mechanisms for CSI feedback are via channel quality indicator (CQI) reporting [25]. Relaying strategies may require the CQI information at all nodes [17] and this motivates us to investigate the CQI reporting mechanism for TWMRNs where each terminal node is required to know the CQIs of the distant terminal-relay links. However, most of recent work investigated CQI reporting or feedback in OWSRNs only for some applications such as adaptive non-orthogonal cooperation [26], user selection with multiple destination nodes [27], adaptive resource scheduling in multihop OFDMA systems [28], and adaptive utilisation of time-varying channels [29]. Extending these CQI feedback schemes to TWSRNs obviously results in doubling the signaling overhead and requiring two time slots at each relay node to forward these overheads to both terminal nodes.

In this paper, we propose an efficient CQI reporting scheme for TWMRNs so as to reduce the number of transmissions at the relays and to avoid the additional overhead. The idea of our proposed scheme is originated from the NC concept and can be summarized by the following steps: i) each relay node combines the estimated CQIs of the two terminal-relay channels; ii) the relay node then broadcasts the combined signal to both terminal nodes; and iii) based on the received combined signal, two terminal nodes can simultaneously estimate the CQI of the distant links. With our proposed CQI reporting scheme, the CQIs of the terminal-relay channel are conveyed to the other terminal at no additional cost in terms of bandwidth or energy. It can be seen that N signalling overheads and N transmission time slots are reduced for an N-relay network when compared with the conventional schemes. The major contributions of our paper are summarised as follows:

- *The upper and lower bounds of mean square error (MSE) of the estimated CQI over Rayleigh flat fading channel are derived*. To the best of our knowledge, these bound derivations have not been achieved before. The tight bounds reflect well the behavior of the numerical MSE. It is found that while the MSE of the estimated CQI increases proportionally with the square of the number of CQI levels, a higher data rate could be achieved with an increased number of CQI levels by using various adaptive modulation and coding (AMC) schemes. It is also shown that the loss of performance and the increase of complexity of our proposed scheme are negligible compared with conventional CQI reporting schemes.
- *A complexity-reduced relay selection scheme is proposed based on the derived MSE bounds*.

Since the data exchange between two terminals in TWMRNs can be assisted by all available relay nodes, relay selection (RS) should be considered. In particular, opportunistic RS scheme was generally investigated, where the best relay is chosen based on a specific selection criterion, e.g., minimizing the sum of bit error rate (BER) or maximizing the sum-rate [22]. We observe that the RS can also be simply realized by maximizing the sum of channel gains of both terminal users. However, in our work, we consider a system where CQI is required at the transmitter and therefore CQI reporting is an important performance metric for the system. This motivates us to design an efficient RS scheme based on the previously determined MSE of estimated CQI at the two terminals, where the best relay is chosen such that the sum of MSE (sum-MSE) of the estimated CQI is minimized. The RS is carried out by a scheduler of a coordinator node in a centralized manner [30], [31], i.e., each relay informs the coordinator its sum-MSE through a specific feedback channel and then the coordinator selects the best relay based on these information. The optimal RS scheme requires a full search of available relays, which results in high complexity. This motivates us to propose a suboptimal bound-based RS scheme where the searching process will stop whenever the maximum of MSE (max-MSE) is smaller than the upper bound. The resulting complexity is reduced by at least three times compared with the optimal RS scheme if the number of relays is sufficiently large while its performance is still satisfactory.

The rest of the paper is organized as follows: In Section II, we describe our proposed CQI reporting scheme, related algorithms, and complexity analysis. Section III provides the analysis of MSE of estimated CQI. Then, different opportunistic RS schemes are proposed and analyzed in Section IV. Numerical results are discussed in Section V and Section VI concludes this paper.

II. Proposed CQI Reporting Scheme

The system model of a general TWMRN under investigation is shown in Fig. 1 where the data exchange between two terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ is assisted by a group of N relays $\mathcal{R}^{(N)} = \{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_N\}$. The channel quality reporting at these relays is assumed to be concurrently carried out. For convenience, let us consider the channel quality reporting at $\mathcal{R}_1$ only, i.e., a typical TWSRN including $\mathcal{T}_1$, $\mathcal{T}_2$, and $\mathcal{R}_1$. Let h_{AB} denote the channel coefficient of the $A \to B$

link where $A, B \in \{\mathcal{T}_1, \mathcal{T}_2, \mathcal{R}_1\}$. We make the following assumptions: i) there is no direct link between the terminal nodes due to power limit in each node; ii) time division duplex (TDD) signalling is employed in our considered system; iii) channel in each link is reciprocal, i.e., $h_{\mathcal{T}_i\mathcal{R}_1} = h_{\mathcal{R}_1\mathcal{T}_i} = h_i$, $i = 1, 2$; iv) the channel in each link is Rayleigh flat fading; and v) pilot signals are used to initially estimate the link quality of all channels (i.e., instantaneous signal to noise ratio (SNR) at the receiver).

It is noteworthy that for various signal processing mechanisms in TWSRNs such as data detection or adaptive modulation [17], each terminal node $\mathcal{T}_i$ requires the channel quality information of not only its associated link $\mathcal{T}_i - \mathcal{R}_1$ but also that of the distant link $\mathcal{T}_j - \mathcal{R}_1, j \neq i$. In order to reduce the amount of feedback information, the value of channel quality, SNR, should be quantized into a finite bit sequence called CQI with different levels. The CQI reporting in TWSRNs can be divided into two phases as follows: In the first phase, $\mathcal{T}_i, i = 1, 2$, and $\mathcal{R}_1$ transmit pilot signals to each other to estimate the CQI of the associated link $\mathcal{T}_i - \mathcal{R}_1$. In the second phase, $\mathcal{R}_1$ helps $\mathcal{T}_i$ estimate the CQI of the distant link $\mathcal{T}_j - \mathcal{R}_1, j = 1, 2, j \neq i$, which cannot be directly obtained at $\mathcal{T}_i$ since there is no direct link available between $\mathcal{T}_i$ and $\mathcal{T}_j$. We observe that the CQI estimation in the first phase can straightforwardly follow conventional pilot-based approaches. We therefore focus on the CQI reporting in the second phase. Conventionally, a double amount of signaling overhead should be required at $\mathcal{R}_1$ to consecutively forward the CQIs of the links $\mathcal{T}_1 - \mathcal{R}_1$ and $\mathcal{T}_2 - \mathcal{R}_1$ to $\mathcal{T}_2$ and $\mathcal{T}_1$, respectively, in two time slots. This considerably reduces the network throughput. Therefore, we propose a new efficient CQI reporting scheme based on NC to eliminate the additional overhead as well as reduce the number of time slots by half. By using NC, $\mathcal{R}_1$ can combine the estimated CQIs of two links $\mathcal{T}_1 - \mathcal{R}_1$ and $\mathcal{T}_2 - \mathcal{R}_1$ before broadcasting it to allow each terminal $\mathcal{T}_i$ to simultaneously estimate the CQI of the distant link $\mathcal{T}_j - \mathcal{R}_1$ $(j \neq i)$.

Let γ_i and ρ_i denote the SNR and CQI, respectively, of link h_i $(i = 1, 2)$. Assume that $\rho_i \in \mathcal{C}_i$ where $\mathcal{C}_i$ is the set of all possible CQI levels of link h_i. Let Q_i denote the cardinality of $\mathcal{C}_i$. Thus, it requires $L_i = \lceil \log_2 Q_i \rceil$ bits to represent a ρ_i level, where $\lceil . \rceil$ denotes the ceiling function of a real number. The lists of ρ_1 and ρ_2 levels are assumed to be available at $\mathcal{R}_1$, $\mathcal{T}_1$, and $\mathcal{T}_2$. Practically, there are multiple ways to map SNR to CQI [32], [33]. One of the common ways is that CQI can be approximated by a linear function of SNR as follows

$$\rho_i = \lceil a\gamma_i[\text{dB}] + b \rceil, \tag{1}$$

where a and b are the constants and γ_i is calculated in dB. Assume that the range of SNR for CQI mapping is from 0 to γ_{mdB} [dB], where γ_{mdB} is positive and measured in dB. Following the above approach, we divide the range $[0 : \gamma_{mdB}]$ into Q_i levels $(1, 2, \ldots, Q_i)$ by setting $a = Q_i/\gamma_{mdB}$ and $b = 0$. As a result, we can obtain ρ_i as

$$\rho_i = \left\lceil \frac{Q_i}{\gamma_{mdB}} \gamma_i[\text{dB}] \right\rceil = \left\lceil \frac{10 Q_i \log_{10} \gamma_i}{\gamma_{mdB}} \right\rceil. \tag{2}$$

Let $\rho_{i,\mathcal{T}}$ and $\rho_{i,\mathcal{R}}$ denote the estimated values of ρ_i at $\mathcal{T}_i$ and $\mathcal{R}_1$, respectively, in the first phase. It can be seen that $\rho_{i,\mathcal{T}}, \rho_{i,\mathcal{R}} \in \mathcal{C}_i$. Our proposed CQI reporting scheme is carried out in the second phase. $\mathcal{R}_1$ combines two estimated CQIs, i.e., $\rho_{1,\mathcal{R}}$ and $\rho_{2,\mathcal{R}}$, to create

$$\mathbf{b} \triangleq \mathbf{b}_{\rho_{1,\mathcal{R}}} \oplus \mathbf{b}_{\rho_{2,\mathcal{R}}}, \tag{3}$$

where $\oplus$ denotes the bitwise XOR operator and $\mathbf{b}_{\rho_{i,\mathcal{R}}}, i = 1, 2$, denotes the bit-level format of $\rho_{i,\mathcal{R}}$. We notice that the terms in XOR operations in (3) must have the same length. Thus, zero-padding is used throughout the paper to match the length of CQIs, i.e., the length of $\mathbf{b}$ is $\max\{L_1, L_2\} \triangleq L_m$. For the CQI estimation at $\mathcal{T}_1$ and $\mathcal{T}_2$, $\mathcal{R}_1$ broadcasts $\mathbf{b}$ to $\mathcal{T}_1$ and $\mathcal{T}_2$. The received signal at $\mathcal{T}_i$, $i = 1, 2$, can be written by:

$$\mathbf{y}_i = \sqrt{P} h_i \mathbf{x} + \mathbf{n}_i, \tag{4}$$

where P is the transmission power of $\mathcal{R}_1$, $\mathbf{x}$ is the modulated version of $\mathbf{b}$, and $\mathbf{n}_i$ is the white Gaussian noise vector with each entry having zero mean and variance of σ_i^2.

At $\mathcal{T}_i$, $i = 1, 2$, the question is how to estimate $\rho_{j,\mathcal{R}}$, $j \neq i$ of the distant link $\mathcal{T}_j - \mathcal{R}_1$. Based on the estimated CQI of the link $\mathcal{T}_i - \mathcal{R}_1$ at $\mathcal{T}_i$ (i.e., $\rho_{i,\mathcal{T}}$) in the first phase, $\mathcal{T}_i$ can create a list of all possible NC-based combinations of $\rho_{i,\mathcal{T}}$ and ρ_j as follows:

$$\mathbf{b}'_{\rho_j} = \mathbf{b}_{\rho_{i,\mathcal{T}}} \oplus \mathbf{b}_{\rho_j}, \tag{5}$$

where $\mathbf{b}_{\rho_{i,\mathcal{T}}}$, $\mathbf{b}_{\rho_j}$, and $\mathbf{b}'_{\rho_j}$ denote the bit-level formats of $\rho_{i,\mathcal{T}}$, ρ_j, and the NC-based combination of $\rho_{i,\mathcal{T}}$ and ρ_j, respectively. Note that $\rho_j \in \mathcal{C}_j$ and therefore there are Q_j possible candidates of $\mathbf{b}_{\rho_j}$. $\mathcal{T}_i$ then compares the received signal $\mathbf{y}_i$ given in (4) with all possible $\mathbf{b}'_{\rho_j}$'s in order to choose the matched $\mathbf{b}_{\rho_j}$. Correspondingly, the matched $\rho_j \in \mathcal{C}_j$ can be found. This matched ρ_j is the estimated value of $\rho_{j,\mathcal{R}}$, which is denoted by $\hat{\rho}_{j,\mathcal{R}}$. We observe that finding $\hat{\rho}_{j,R}$ can be carried out by using an exhaustive search method, where the correlation-based decision is based on the received signal $\mathbf{y}_i$ and the NC-based combination sample $\mathbf{b}'_{\rho_j}$. This correlation-based decision is represented by the following correlation value:

$$\vartheta_{\rho_j} = \sum_{l=1}^{L_m} \mathbf{y}_i[l] \frac{\mathbf{x}_{\rho_j}[l]}{|\mathbf{x}_{\rho_j}[l]|^2}, \tag{6}$$

where $\mathbf{x}_{\rho_j}$ denotes the modulated version of $\mathbf{b}'_{\rho_j}$. Here, $\mathbf{y}_i[l]$ and $\mathbf{x}_{\rho_j}[l]$ denote the l-th element of vectors $\mathbf{y}_i$ and $\mathbf{x}_{\rho_j}$, respectively.

Theorem 1. ϑ_{ρ_j} is almost surely upper bounded by $(\sqrt{P}h_i L_m + \sqrt{L_m}\sigma_i N_{\rho_j})$ when $\rho_{i,\mathcal{R}} = \rho_{i,\mathcal{T}}$ and $\rho_j = \rho_{j,\mathcal{R}}$, where N_{ρ_j} is an independent complex-valued random number [34].

Proof: See Appendix A. ■

In Theorem 1, two conditions to maximize ϑ_{ρ_j} mean that the estimated ρ_i and ρ_j at $\mathcal{R}_1$ in the first phase should be equal to the estimated ρ_i in the first phase and the required ρ_j in the second phase at $\mathcal{T}_i$, respectively. Thus, the estimated value of $\rho_{j,\mathcal{R}}$ in the second phase is chosen from $\mathcal{C}_j$ to maximize ϑ_{ρ_j}, i.e.,

$$\hat{\rho}_{j,\mathcal{R}} = \arg\max_{\rho_j \in \mathcal{C}_j} \vartheta_{\rho_j}. \tag{7}$$

Note that the estimation of $\rho_{2,\mathcal{R}}$ at $\mathcal{T}_1$ and the estimation of $\rho_{1,\mathcal{R}}$ at $\mathcal{T}_2$ are carried out simultaneously.

Remark 1 (*Imperfect CQI Estimation*)**.** The required condition $\rho_{i,\mathcal{R}} = \rho_{i,\mathcal{T}}$ in order to maximize ϑ_{ρ_j} causes a loss in the performance of our proposed scheme when compared with the conventional scheme[1] in terms of the MSE of the estimated $\rho_{j,\mathcal{R}}$ at $\mathcal{T}_i$. This condition may not be achieved due to the imperfect estimation of ρ_i at $\mathcal{R}_1$ and $\mathcal{T}_i$. Thus, the overall performance

[1]The conventional scheme is referred to as a scheme where $\mathcal{R}$ sequentially transmits $\rho_{i,\mathcal{R}}$ and $\rho_{j,\mathcal{R}}$ to $\mathcal{T}_j$ and $\mathcal{T}_i$, respectively, in two separate time slots.

of our proposed CQI reporting scheme depends on that of the pilot-based CQI estimation in the first phase.

Remark 2 (*Throughput Improvement*)**.** It can be seen that our proposed CQI reporting scheme for one-relay system need five transmissions of five signalling overheads while the traditional scheme requires six transmissions. This means that one transmission and one overhead are reduced with our proposed scheme for one-relay system. Extending to N-relay networks, N transmission time slots and N signaling overheads are reduced. Thus, the system throughput is significantly improved with our proposed scheme when N is large.

Remark 3 (*Complexity*)**.** For complexity analysis, we compare our scheme with the conventional scheme. First, we discuss the complexity of exhaustive search at $\mathcal{T}_i$, $i = 1, 2$. Second, the computation complexity at $\mathcal{R}_1$ and $\mathcal{T}_i$ is studied. For the first comparison, the complexity is measured by the number of searches to find the desired CQI. In our proposed scheme, at $\mathcal{T}_i$, ρ_j is chosen in $\mathcal{C}_j$ to maximize the correlation value ϑ_{ρ_j} given by (6). Thus, Q_j searches are required. In the conventional scheme where exhaustive search is also used, the same number of searches is required at $\mathcal{T}_i$ to find the desired ρ_j. However, our proposed scheme has a slightly higher computation complexity compared to conventional scheme. The XOR operation is required for the generation of $\mathbf{b}$ in (3). Thus, the complexity at $\mathcal{R}_1$ in our proposed scheme increases by L_m XOR operations. At $\mathcal{T}_i$, the difference between our proposed scheme and the conventional scheme is the generation of $\mathbf{b}'_{\rho_j}$ defined in (5). The XOR operation in (5) results in L_m more computations at $\mathcal{T}_i$. Similarly, L_m more computations are required at $\mathcal{T}_j$. If we let L_T denote the total number of computations in the whole system using the conventional scheme, then our scheme would require $(L_T + 3L_m)$ computations. However, it can be seen that $L_T \gg L_m$, and thus this increase of complexity in our proposed scheme is insignificant.

III. Analysis of MSE of estimated CQI

In this section, we derive the MSE expression of the estimated CQI of our proposed scheme. For simplicity, we study the CQI estimation performed at $\mathcal{T}_2$ only. The analysis of the CQI estimation at $\mathcal{T}_1$ can be similarly obtained. The estimation error occurs if the estimated $\rho_{1,\mathcal{R}}$ at

$\mathcal{T}_2$ in the second phase (i.e., $\hat{\rho}_{1,R}$) is different from the value of ρ_1 estimated at $\mathcal{R}_1$ in the first phase (i.e., $\rho_{1,\mathcal{R}}$). Thus, the MSE of estimated CQI can be computed by

$$\text{MSE} = E\left\{[\rho_{1,\mathcal{R}} - \hat{\rho}_{1,\mathcal{R}}]^2\right\}, \tag{8}$$

where $E\{.\}$ denotes the expectation.

As it is difficult to derive $\hat{\rho}_{1,\mathcal{R}}$ and $\rho_{1,\mathcal{R}}$ for any arbitrary characteristics of two links $\mathcal{T}_1 \to \mathcal{R}_1$ and $\mathcal{R}_1 \to \mathcal{T}_2$ simultaneously, we observe that it is still useful to understand the behaviour of the MSE in (8) in an asymptotic case and gain some insights from it. Thus, for simple analysis, let us assume that the link $\mathcal{T}_1 \to \mathcal{R}_1$ is at a high SNR[2] , i.e., $\gamma_1[dB] = \gamma_{mdB}$, and thus from (2), $\rho_{1,\mathcal{R}}$ can be approximated by Q_1.

We now derive $\hat{\rho}_{1,\mathcal{R}}$. From (4), the SNR γ_2 of $\mathcal{R}_1 \to \mathcal{T}_2$ link can be expressed as

$$\gamma_2 = \frac{P|h_2|^2}{\sigma_2^2}. \tag{9}$$

Note that, in the second phase, $\mathbf{x}$ in (4) is constructed by both $\rho_{1,R}$ and $\rho_{2,R}$. We assume that $\rho_{2,R} \approx \rho_{2,T}$ in the first phase. Since $\rho_{2,T}$ is known at $\mathcal{T}_2$, it can be removed from the received signal. Thus, it can be approximated that γ_2 decides the mapping of $\rho_{1,R}$, i.e.,

$$\hat{\rho}_{1,R} \approx \left\lceil \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} \right\rceil. \tag{10}$$

Substituting (10) into (8) with the assumption that $\mathcal{T}_1 \to \mathcal{R}_1$ link is at a high SNR, (8) can be approximated as

$$\text{MSE} \approx E\left\{\left(Q_1 - \left\lceil \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} \right\rceil\right)^2\right\}. \tag{11}$$

Let $\alpha = e^{-\gamma_m/\bar{\gamma}}$, $\beta = e^{-1/\bar{\gamma}}$, $\gamma_m = 10^{\gamma_{mdB}/10}$, $Q_1' = 10Q_1/(\gamma_{mdB}\ln 10)$ where $\bar{\gamma}$ is average SNR of γ_2 and $\ln x$ is natural logarithm of x. Let $E_i(.)$ denote exponential integral and $\mathfrak{G}_{p,q}^{m,n}\left(\substack{a_1,\ldots,a_p \\ b_1,\ldots,b_q}\middle| z\right)$ denote the Meijer G function [35], we have the following finding:

Theorem 2. The MSE given in (11) is upper-bounded and lower-bounded by MSE_u and MSE_l, respectively, where

$$\text{MSE}_u = \lambda_1 + \lambda_2 A + \lambda_3 B, \tag{12}$$

[2]This assumption of high SNR is for analysis purpose only. Our proposed CQI reporting algorithm is actually for a general case and valid for any SNR value of the uplink.

$$\text{MSE}_l = \lambda_1' + \lambda_2' A + \lambda_3 B, \tag{13}$$

and

$$\lambda_1 = (Q_1 - Q_1' \ln\bar{\gamma})^2(\beta - \alpha), \lambda_2 = -2Q_1'(Q_1 - Q_1' \ln\bar{\gamma}), \lambda_3 = Q_1'^2,$$

$$\lambda_1' = (Q_1 - 1 - Q_1' \ln\bar{\gamma})^2(\beta - \alpha), \lambda_2' = -2Q_1'(Q_1 - 1 - Q_1' \ln\bar{\gamma}),$$

$$A = \beta\ln(-\ln\beta) - \alpha\ln(-\ln\alpha) + E_i(\ln\alpha) - E_i(\ln\beta),$$

$$B = \beta\ln^2(-\ln\beta) - \alpha\ln^2(-\ln\alpha) - 2\ln(-\ln\alpha)\mathfrak{G}_{1,2}^{2,0}\left({}_{0,0}^{1}\middle| -\ln\alpha\right)$$
$$+ 2\ln(-\ln\beta)\mathfrak{G}_{1,2}^{2,0}\left({}_{0,0}^{1}\middle| -\ln\beta\right) - 2\mathfrak{G}_{2,3}^{3,0}\left({}_{0,0,0}^{1,1}\middle| -\ln\alpha\right) + 2\mathfrak{G}_{2,3}^{3,0}\left({}_{0,0,0}^{1,1}\middle| -\ln\beta\right).$$

Proof: See Appendix B. ■

Corollary 1. MSE bounds in (12) and (13) increase as a function of Q_1^2.

Proof: See Appendix C. ■

Remark 4 (*Impact of* Q_1)**.** Although increasing the number of CQI levels (i.e., Q_1) would provide a more precise representation of channel quality and add more flexibility to the implementation of various adaptive schemes, from Corollary 1 we can observe that the performance of the CQI reporting scheme is significantly reduced with Q_1. Thus, a trade-off between performance of CQI reporting and performance of data transmission should be considered when choosing Q_1.

IV. Opportunistic CQI Based Relay Selection

In Section II, the proposed CQI reporting scheme is considered for each relay node. In this Section, we will consider all the relays and therefore requires an efficient RS mechanism. Particularly, based on the estimated CQIs at the relays and two terminals, we propose different RS schemes for TWMRNs where only one best relay is opportunistically selected by a coordinator in the network to exchange data between two terminal nodes. Specifically, an optimal RS (ORS) scheme is proposed where the relay is chosen to minimize the sum-MSE given by

$$\text{SMSE}(n) = \text{MSE}_1(n) + \text{MSE}_2(n), \tag{14}$$

where $\text{MSE}_i(n)$ denotes the MSE of the estimated ρ_i at $\mathcal{T}_j$, $i, j \in \{1, 2\}$, $i \neq j$, in a TWSRN using $\mathcal{R}_n$, $n \in \{1, \ldots, N\}$. From the MSE analysis for TWSRNs in Section III, $\text{MSE}_i(n)$ is computed by

$$\mathrm{MSE}_i(n) \approx E\left\{\left(Q_i - \left\lceil \frac{10\log_{10}(\gamma_j(n))}{\gamma_{mdB}/Q_i} \right\rceil \right)^2\right\}. \tag{15}$$

Thus, the ORS based on the sum-MSE is represented by

$$n^* = \arg\min_n \mathrm{SMSE}(n). \tag{16}$$

However, the computation complexity of this scheme is high. Let us consider a suboptimal RS (SRS) scheme based on the max-MSE. In fact, it is well-known that minimizing the sum can be approximated to minimizing the maximum. Therefore, the relay chosen by SRS scheme is determined by

$$n^*_{sub} = \arg\min_n \max\{\mathrm{MSE}_1(n), \mathrm{MSE}_2(n)\}. \tag{17}$$

Due to the quantization carried out in the mapping process as explained for TWSRNs, we can derive the upper bound and lower bound of $\mathrm{MSE}(n^*_{sub})$. For simple analysis, we assume that Q_1 and Q_2 are equal, and, $\gamma_1(n)$ and $\gamma_2(n)$ have the same probability density function. Let $\alpha_N = e^{-2\gamma_m/\bar{\gamma}}$, $\beta_N = e^{-2/\bar{\gamma}}$, $Q = Q_1 = Q_2$, and $Q' = 10Q/(\gamma_{mdB}\ln 10)$, we have the following finding:

Theorem 3. $\mathrm{MSE}(n^*_{sub})$ is upper-bounded and lower-bounded by $\mathrm{MSE}_u(n^*_{sub})$ and $\mathrm{MSE}_l(n^*_{sub})$, respectively, where

$$\mathrm{MSE}_u(n^*_{sub}) = \lambda_{1N} + \lambda_{2N}A_N + \lambda_{3N}B_N, \tag{18}$$

$$\mathrm{MSE}_l(n^*_{sub}) = \lambda'_{1N} + \lambda'_{2N}A_N + \lambda_{3N}B_N, \tag{19}$$

and

$$\lambda_{1N} = \left(Q - Q'\ln\frac{\bar{\gamma}}{2}\right)^2 \left[(1-\alpha_N)^N - (1-\beta_N)^N\right], \lambda_{2N} = -2Q'\left(Q - Q'\ln\frac{\bar{\gamma}}{2}\right), \lambda_{3N} = Q'^2,$$

$$\lambda'_{1N} = \left(Q - 1 - Q'\ln\frac{\bar{\gamma}}{2}\right)^2 \left[(1-\alpha_N)^N - (1-\beta_N)^N\right], \lambda'_{2N} = -2Q'\left(Q - 1 - Q'\ln\frac{\bar{\gamma}}{2}\right),$$

$$A_N = (-1)^{N-1}\sum_{m=1}^{N}(-1)^{m-1}\frac{\prod_{j=1}^{m-1}(N-j+1)}{(m-1)!}\left\{E_i\left[(N-m+1)\ln\alpha_N\right]\right.$$

$$\left. - E_i\left[(N-m+1)\ln\beta_N\right] - \alpha_N^{N-m+1}\ln(-\ln\alpha_N) + \beta_N^{N-m+1}\ln(-\ln\beta_N)\right\},$$

$$B_N = (-1)^{N}\sum_{m=1}^{N}(-1)^{m-1}\frac{\prod_{j=1}^{m-1}(N-j+1)}{(m-1)!}\left\{\alpha_N^{N-m+1}\ln^2(-\ln\alpha_N) - \beta_N^{N-m+1}\ln^2(-\ln\beta_N)\right.$$

$$+ 2\ln(-\ln\alpha_N)\mathfrak{G}_{1,2}^{2,0}\left({}^{1}_{0,0}\middle| -(N-m+1)\ln\beta_N\right) - 2\ln(-\ln\beta_N)\mathfrak{G}_{1,2}^{2,0}\left({}^{1}_{0,0}\middle| -(N-m+1)\ln\beta_N\right)$$

$$\left. +2\mathfrak{G}_{2,3}^{3,0}\left({}^{1,1}_{0,0,0}\middle| -(N-m+1)\ln\alpha_N\right) - 2\mathfrak{G}_{2,3}^{3,0}\left({}^{1,1}_{0,0,0}\middle| -(N-m+1)\ln\beta_N\right)\right\}.$$

Proof: See Appendix D. ■

Remark 5 (*Tighter Bounds with Larger N*)**.** The MSE performance of the SRS scheme converges to zero when the number of relays is large. It can be seen that $\lambda_{1N} \to 0$, $\lambda'_{1N} \to 0$, $A_N \to 0$, and $B_N \to 0$ as $N \to \infty$. Thus, $\text{MSE}_u(n^*_{sub}) \to 0$ and $\text{MSE}_l(n^*_{sub}) \to 0$. Since $\text{MSE}_u(n^*_{sub}) \geqslant \text{MSE}(n^*_{sub}) \geqslant \text{MSE}_l(n^*_{sub})$, we can deduce that $\text{MSE}(n^*_{sub}) \to 0$ as $N \to \infty$. We can also deduce that the bounds are tighter as N increases.

Based on the upper and lower bounds of $\text{MSE}(n^*_{sub})$ given in Theorem 3 and their characteristics discussed in Remark 5, we propose a so-called suboptimal bound-based RS (SBBRS) scheme to reduce further the complexity of the searching method in (17). Note that if the previously mentioned SRS scheme (i.e., (17)) is used, N relays would be verified to choose the best one to minimize the max-MSE. Instead, the proposed SBBRS scheme will stop the searching when finding out a relay with max-MSE being smaller than $\text{MSE}_u(n^*_{sub})$. As the result, the number of searches is significantly reduced, especially with larger N (i.e., when $\text{MSE}_u(n^*_{sub})$ decreases). The complexity reduction will be shown and further discussed in the simulation results. The algorithm corresponding to SBBRS scheme is summarized in Table I.

V. Numerical Results

In this section, we compare the MSE of estimated CQI of different schemes considered in our work using computer simulation. Let us first consider the TWSRNs where the CQI estimation is carried out at $\mathcal{T}_2$. The estimation error occurs if the estimated $\rho_{1,\mathcal{R}}$ at $\mathcal{T}_2$ (i.e., $\hat{\rho}_{1,\mathcal{R}}$) is different from ρ_1 estimated at $\mathcal{R}_1$ (i.e., $\rho_{1,\mathcal{R}}$). For comparison, the conventional scheme is considered. Using the conventional scheme, ρ_1 of the link $\mathcal{T}_1 - \mathcal{R}_1$ is fed back to $\mathcal{T}_2$ through one feedback link, and ρ_2 is separately fed back to $\mathcal{T}_1$ through another link, which results in double overhead and two time slots. Using our proposed scheme, combined data broadcast from relay $\mathcal{R}_1$ enables each terminal to estimate the required CQI. This process utilizes only one time slot and requires no additional overhead.

As shown in Fig. 2, the MSE of estimated $\rho_{1,\mathcal{R}}$ of various schemes is drawn against the SNR of the $\mathcal{R}_1 \to \mathcal{T}_2$ link with the assumption that 8 different CQI levels are used, i.e., $Q_1 = Q_2 = 8$. We also assume that the length of pilot sequences used for the CQI estimation in the first phase is 8 bits. The range of SNR in CQI mapping is from 0 to 20 dB, i.e., $\gamma_{mdB} = 20$ dB. The SNRs of the $\mathcal{T}_1 \to \mathcal{R}_1$ link and the $\mathcal{T}_2 \to \mathcal{R}$ link in the first time slot are assumed to be 20 dB. First, the upper and lower bounds given by (12) and (13) are shown to be quite tight and reflect well the behavior of the numerical MSEs. Secondly, we can observe that the performance of our proposed scheme is close to that of the conventional scheme, especially at a high SNR. The expected small loss, as explained in Remark 1, occurs because the perfect condition $\rho_{2,\mathcal{R}} = \rho_{2,\mathcal{T}}$ in Theorem 1 could not be satisfied in the first phase of the CQI estimation process. This is further illustrated in this figure where we additionally carry out the simulation for the case of perfect CQI estimation of ρ_2 at $\mathcal{R}_1$ (i.e., $\rho_{2,\mathcal{R}} = \rho_{2,\mathcal{T}}$). It shows that this perfect condition allows a closer performance to that of the conventional scheme even though there still exists a small loss due to the NC process. Moreover, in order to compare our proposed scheme with the conventional scheme for various SNR values of the transmission link in the first time slot, let us investigate the case $\mathcal{T}_1 \to \mathcal{R}_1$ transmission is of the same SNR as that of the $\mathcal{R}_1 \to \mathcal{T}_2$ transmission (see Fig. 3). It can be observed that the performance of our proposed scheme is still close to that of the conventional scheme, especially when both links are at high SNRs. This observation confirms the validity of our proposed CQI reporting scheme for any SNR value of the uplink transmission.

The effects of the cardinality of CQI sets are shown in Figs. 4, 5, and 6, where the MSE of estimated CQI, sum-rate, and sum-BER are plotted against the number of CQI levels (i.e., Q_1), respectively. The SNRs of the links in the second phase, i.e., $\mathcal{R}_1 \rightarrow \mathcal{T}_1$ and $\mathcal{R}_1 \rightarrow \mathcal{T}_2$ links, are fixed at 10 dB. The data transmission over TWSRNs in Fig. 5 and Fig. 6 is carried out using various adaptive modulation and coding schemes (MCSs) shown in Table II which are empirically selected to map the CQI level to MCS level. The SNRs in the first phase are arbitrarily chosen to be 10 dB or 20 dB. It can be seen in Fig. 4 that the MSE of estimated CQI increases as a function of Q_1^2, e.g., MSE increases by 4 times as Q_1 increases twice. This is further shown in Fig. 6 where data transmission is taken into account. The sum-BER performance is reduced as Q_1 increases. However, the increase of CQI levels is helpful in adding more flexibility in selecting a precise MCS level to achieve a higher sum-rate (see Fig. 5). These observations confirm our discussion in Remark 4 about the trade-off between the performance of CQI reporting and the performance of data transmission.

Next, we consider the TWMRNs where multiple relays are taken into account. For RS, the ORS scheme in (16), the SRS scheme in (17) and the proposed SBBRS scheme are used. For CQI estimation, the conventional scheme is also considered. We assume that the number of CQI levels is 16, the length of pilot sequence is 8, the SNRs of two links from the relay to both terminals are 4 dB, and the SNRs of uplink transmissions from both terminals to the relay are 20 dB. The assumption of the SNRs in the uplink is to assure that they are at high SNRs, and thus we can confirm the consistency of our simulation results with analytical results for high-SNR scenario. As shown in Fig. 7, the performances of different RS schemes are close and converge to zero if the number of relays is large. This observation confirms our discussion in Remark 5. Furthermore, let us investigate the effects of different SNR levels in the uplink on the performance of our proposed RS schemes. Specifically, in Fig. 8, the performances of various RS schemes are plotted versus the number of relays with respect to various SNR levels in the uplink, including low SNR (0 dB) and medium SNR (10 dB). It can be seen that the performance gap between the proposed schemes and conventional schemes is larger when N is small but still converges to zero if N is large. This observation confirms that the proposed RS schemes work reasonably with different SNR levels in the uplink.

Finally, Fig. 9 shows the complexity advantage of the proposed SBBRS scheme. The number of iterations is significantly reduced compared to that of the searching algorithm in (17), especially when the number of relays in TWMRNs is large. For example, the complexity is reduced by at least three times when the number of relays is larger than five.

VI. Conclusion

In this paper, we have proposed an efficient NC-based CQI reporting scheme for two terminals in nonregenerative TWMRNs to reduce the transmission time at the relays by half and eliminate the additional overhead when compared with the conventional scheme. Significantly, these throughput enhancements are obtained at the expense of insignificant increase of complexity and negligible performance loss. In addition, the upper and lower bounds of the MSE of estimated CQI are derived. The bounds are shown to be quite tight and reflect well the behavior of the numerical MSEs. Furthermore, a suboptimal CQI-based relay selection scheme has been proposed to reduce the searching complexity of the optimal schemes. The complexity is significantly reduced while the performance is close to that of the optimal ones. For future work, one can investigate the impact of the CQI estimation errors at the relays.

Appendix A
Proof of Theorem 1

From (4), we can rewrite (6) as

$$\vartheta_{\rho_j} = \sqrt{P} h_i \sum_{l=1}^{L_m} \mathbf{x}[l] \frac{\mathbf{x}_{\rho_j}[l]}{|\mathbf{x}_{\rho_j}[l]|^2} + \sum_{l=1}^{L_m} \mathbf{n}_i[l] \frac{\mathbf{x}_{\rho_j}[l]}{|\mathbf{x}_{\rho_j}[l]|^2}. \tag{20}$$

The first term in (20) can be given by [34]

$$\sqrt{P} h_i \sum_{l=1}^{L_m} \mathbf{x}[l] \frac{\mathbf{x}_{\rho_j}[l]}{|\mathbf{x}_{\rho_j}[l]|^2} = \begin{cases} \sqrt{P} h_i L_m, & \text{if } \rho_{i,\mathcal{R}} = \rho_{i,\mathcal{T}} \text{ and } \rho_j = \rho_{j,\mathcal{R}}, \\ \sqrt{P} h_i \left(\sqrt{\frac{L_m}{2}} \omega_1 + \sqrt{-\frac{L_m}{2}} \omega_2 \right), & \text{otherwise}, \end{cases} \tag{21}$$

where ω_1 and ω_2 are independent Gaussian random numbers with zero mean and unit variance. Additionally, the second term in (20) can be expressed as [34]

$$\sum_{l=1}^{L_m} \mathbf{n}_i[l] \frac{\mathbf{x}_{\rho_j}[l]}{|\mathbf{x}_{\rho_j}[l]|^2} = \sqrt{L_m} \sigma_i N_{\rho_j}, \tag{22}$$

where N_{ρ_j} is an independent complex-valued random number. Thus, (20) can be written as

$$\vartheta_{\rho_j} = \begin{cases} \sqrt{P} h_i L_m + \sqrt{L_m}\sigma_i N_{\rho_j}, & \text{if } \rho_{i,\mathcal{R}} = \rho_{i,\mathcal{T}} \text{ and } \rho_j = \rho_{j,\mathcal{R}}, \\ \sqrt{P} h_i \left(\sqrt{\frac{L_m}{2}}\omega_1 + \sqrt{-\frac{L_m}{2}}\omega_2 \right) + \sqrt{L_m}\sigma_i N_{\rho_j}, & \text{otherwise.} \end{cases} \tag{23}$$

It can be seen that $\left(\sqrt{\frac{L_m}{2}}\omega_1 + \sqrt{-\frac{L_m}{2}}\omega_2 \right)$ is almost surely smaller than L_m when $L_m \geqslant 2$. Therefore, we can conclude that ϑ_{ρ_j} is almost surely upper bounded by $(\sqrt{P} h_i L_m + \sqrt{L_m}\sigma_i N_{\rho_j})$ when $\rho_{i,\mathcal{R}} = \rho_{i,\mathcal{T}}$ and $\rho_j = \rho_{j,\mathcal{R}}$.

APPENDIX B

DERIVATION OF MSE_u AND MSE_l IN THEOREM 2

We notice that $\lceil x \rceil \geqslant x \ \forall x$. Thus,

$$Q_1 \geqslant \left\lceil \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} \right\rceil \geqslant \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} \geqslant 0. \tag{24}$$

Applying the inequality in (24) to (11), MSE has an upper bound given by

$$\text{MSE}_u = E\left\{ \left(Q_1 - \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} \right)^2 \right\}. \tag{25}$$

Let us denote $Q_1' = 10Q_1/(\gamma_{mdB}\ln 10)$, where $\ln x$ is natural logarithm of x, and denote $\gamma = \gamma_2$. (25) can be rewritten by

$$\text{MSE}_u = E\left\{ (Q_1 - Q_1' \ln\gamma)^2 \right\}. \tag{26}$$

This expectation can be computed by

$$\text{MSE}_u = \int_1^{\gamma_m} (Q_1 - Q_1' \ln\gamma)^2 f_\gamma(\gamma) d\gamma, \tag{27}$$

where $\gamma_m = 10^{\gamma_{mdB}/10}$ and $f(.)$ is the probability density function (pdf) of a random variable. Since the fading channel $\mathcal{R}_1 \to \mathcal{T}_2$ is Rayleigh flat fading, $f_\gamma(\gamma)$ is given by [36]

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}} \right), \tag{28}$$

where $\bar{\gamma}$ is the average SNR. Thus, we have

$$\text{MSE}_u = \int_1^{\gamma_m} (Q_1 - Q_1' \ln\gamma)^2 \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}} \right) d\gamma. \tag{29}$$

Let $t = \exp\left(-\gamma/\bar{\gamma}\right)$, $\alpha = e^{-\gamma_m/\bar{\gamma}}$, and $\beta = e^{-1/\bar{\gamma}}$, (29) can be given by

$$\begin{aligned} \mathrm{MSE}_u &= \int_{\alpha}^{\beta} \left[(Q_1 - Q_1' \ln\bar{\gamma})^2 - 2Q_1'(Q_1 - Q_1'\ln\bar{\gamma})\ln(-\ln t) + Q_1'^2 \ln^2(-\ln t) \right] dt \\ &= \lambda_1 + \lambda_2 A + \lambda_3 B, \end{aligned} \tag{30}$$

where

$$\lambda_1 = (Q_1 - Q_1'\ln\bar{\gamma})^2(\beta - \alpha), \tag{31}$$

$$\lambda_2 = -2Q_1'(Q_1 - Q_1'\ln\bar{\gamma}), \tag{32}$$

$$\lambda_3 = Q_1'^2, \tag{33}$$

$$A = \int_{\alpha}^{\beta} \ln(-\ln t) dt, \tag{34}$$

$$B = \int_{\alpha}^{\beta} \ln^2(-\ln t) dt. \tag{35}$$

The derivation of MSE_u is simplified to the integral calculus of A and B. From [35] and some simple algebraic manipulation, the integrals A and B can be calculated by

$$A = \beta \ln(-\ln\beta) - \alpha\ln(-\ln\alpha) + E_i(\ln\alpha) - E_i(\ln\beta), \tag{36}$$

$$\begin{aligned} B = {} & \beta\ln^2(-\ln\beta) - \alpha\ln^2(-\ln\alpha) - 2\ln(-\ln\alpha)\mathfrak{G}_{1,2}^{2,0}\left({}^{1}_{0,0}\middle| -\ln\alpha\right) \\ & + 2\ln(-\ln\beta)\mathfrak{G}_{1,2}^{2,0}\left({}^{1}_{0,0}\middle| -\ln\beta\right) - 2\mathfrak{G}_{2,3}^{3,0}\left({}^{1,1}_{0,0,0}\middle| -\ln\alpha\right) + 2\mathfrak{G}_{2,3}^{3,0}\left({}^{1,1}_{0,0,0}\middle| -\ln\beta\right), \end{aligned} \tag{37}$$

respectively, where $E_i(.)$ is exponential integral and $\mathfrak{G}_{p,q}^{m,n}\left({}^{a_1,\dots,a_p}_{b_1,\dots,b_q}\middle| z\right)$ is Meijer G function.

Another remarkable inequality concerning with ceiling function is that $\lceil x \rceil < x + 1 \; \forall x$. Thus,

$$0 \leqslant \left\lceil \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} \right\rceil < \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} + 1. \tag{38}$$

The lower bound of MSE is then given by

$$\mathrm{MSE}_l = E\left\{ \left(Q_1 - 1 - \frac{10\log_{10}(\gamma_2)}{\gamma_{mdB}/Q_1} \right)^2 \right\}. \tag{39}$$

We observe that the expression of MSE_l has the same form of MSE_u in (25). Thus, MSE_l can be calculated by

$$\mathrm{MSE}_l = \lambda_1' + \lambda_2' A + \lambda_3 B, \tag{40}$$

where

$$\lambda_1' = (Q_1 - 1 - Q_1' \ln\bar{\gamma})^2 (\beta - \alpha), \tag{41}$$

$$\lambda_2' = -2Q_1'(Q_1 - 1 - Q_1' \ln\bar{\gamma}). \tag{42}$$

APPENDIX C
PROOF OF COROLLARY 1

From Theorem 2, λ_1, λ_2, λ_1', λ_2', and λ_3 depend on Q_1, whereas A and B are independent of Q_1. We can rewrite λ_1, λ_2, and λ_3 as

$$\lambda_1 = Q_1^2 \left(1 - \frac{10\ln\bar{\gamma}}{\gamma_{mdB}\ln 10}\right)^2 (\beta - \alpha),$$

$$\lambda_2 = Q_1^2 \frac{-20}{\gamma_{mdB}\ln 10} \left(1 - \frac{10\ln\bar{\gamma}}{\gamma_{mdB}\ln 10}\right),$$

$$\lambda_3 = Q_1^2 \left(\frac{10}{\gamma_{mdB}\ln 10}\right)^2.$$

Thus, MSE_u can be rewritten as

$$\text{MSE}_u = Q_1^2 \left[\left(1 - \frac{10\ln\bar{\gamma}}{\gamma_{mdB}\ln 10}\right)^2 (\beta - \alpha) + \frac{-20}{\gamma_{mdB}\ln 10}\left(1 - \frac{10\ln\bar{\gamma}}{\gamma_{mdB}\ln 10}\right) A + \left(\frac{10}{\gamma_{mdB}\ln 10}\right)^2 B\right]. \tag{43}$$

We observe that if we change Q_1 and fix the other parameters, MSE_u is a function of Q_1^2, i.e.,

$$\text{MSE}_u = \zeta Q_1^2, \tag{44}$$

where ζ is a non-negative constant since $\text{MSE}_u \geqslant 0$.

Considering MSE_l, we have

$$\lambda_1' = Q_1^2 \left(1 - 1/Q_1 - \frac{10\ln\bar{\gamma}}{\gamma_{mdB}\ln 10}\right)^2 (\beta - \alpha),$$

$$\lambda_2' = Q_1^2 \frac{-20}{\gamma_{mdB}\ln 10} \left(1 - 1/Q_1 - \frac{10\ln\bar{\gamma}}{\gamma_{mdB}\ln 10}\right).$$

When Q_1 is large, the term $1/Q_1$ can be omitted. Thus, MSE_l can also be written by

$$\text{MSE}_l = \zeta' Q_1^2, \tag{45}$$

where ζ' is a non-negative constant.

Appendix D

Derivation of $\text{MSE}_u(n^*_{sub})$ and $\text{MSE}_l(n^*_{sub})$ in Theorem 3

From (15), (17), and $Q_1 = Q_2$, n^*_{sub} can be obtained as

$$n^*_{sub} = \arg\max_n \min\{\gamma_1(n), \gamma_2(n)\}. \tag{46}$$

Let us denote $\gamma^* = \max \gamma_{\min}(n)$ where $\gamma_{\min}(n) = \min\{\gamma_1(n), \gamma_2(n)\}$. $\text{MSE}(n^*_{sub})$ can be calculated by

$$\text{MSE}(n^*_{sub}) \approx E\left\{\left(Q - \left\lceil \frac{10\log_{10}\gamma^*}{\gamma_{mdB}/Q}\right\rceil\right)^2\right\}. \tag{47}$$

Similarly, applying the inequalities (24) and (38) to (47), $\text{MSE}(n^*_{sub})$ has an upper bound and a lower bound given by

$$\text{MSE}_u(n^*_{sub}) = E\left\{\left(Q - \frac{10\log_{10}\gamma^*}{\gamma_{mdB}/Q}\right)^2\right\}, \tag{48}$$

$$\text{MSE}_l(n^*_{sub}) = E\left\{\left(Q - 1 - \frac{10\log_{10}\gamma^*}{\gamma_{mdB}/Q}\right)^2\right\}, \tag{49}$$

respectively, where $Q = Q_1 = Q_2$. Observing that (48) and (49) have the same form, we will derive the expression of $\text{MSE}_u(n^*_{sub})$. The derivation for $\text{MSE}_l(n^*_{sub})$ can be carried out similarly.

In order to derive $\text{MSE}_u(n^*_{sub})$, let us calculate the pdf of γ^*. Note that $\gamma_1(n)$ and $\gamma_2(n)$ have the same pdf and cumulative density function (cdf) of Rayleigh fading given by (28) and

$$F_\gamma(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \tag{50}$$

respectively. Applying order statistics [37], the pdf of γ^* can be calculated by

$$f_{\gamma^*}(\gamma) = N f_{\gamma_{\min}}(\gamma) F^{N-1}_{\gamma_{\min}}(\gamma), \tag{51}$$

where

$$f_{\gamma_{\min}}(\gamma) = 2f_\gamma(\gamma)[1 - F_\gamma(\gamma)], \tag{52}$$

$$F_{\gamma_{\min}}(\gamma) = 1 - [1 - F_\gamma(\gamma)]^2, \tag{53}$$

denote the pdf and cdf of $\gamma_{\min}$, respectively. Thus,

$$f_{\gamma^*}(\gamma) = \frac{2N}{\bar{\gamma}}\exp\left(-\frac{2\gamma}{\bar{\gamma}}\right)\left[1 - \exp\left(-\frac{2\gamma}{\bar{\gamma}}\right)\right]^{N-1}. \tag{54}$$

From (48), $\text{MSE}_u(n^*_{sub})$ can be computed by

$$\text{MSE}_u(n^*_{sub}) = \int_1^{\gamma_m} \left(Q - \frac{10\log_{10}\gamma}{\gamma_{mdB}/Q}\right)^2 \frac{2N}{\bar{\gamma}} \exp\left(-\frac{2\gamma}{\bar{\gamma}}\right) \left[1 - \exp\left(-\frac{2\gamma}{\bar{\gamma}}\right)\right]^{N-1} d\gamma. \tag{55}$$

Let $Q' = 10Q/(\gamma_{mdB}\ln 10)$, $t = \exp(-2\gamma/\bar{\gamma})$, $\alpha_N = e^{-2\gamma_m/\bar{\gamma}}$, and $\beta_N = e^{-2/\bar{\gamma}}$, (55) can be rewritten as

$$\begin{aligned}\text{MSE}_u(n^*_{sub}) &= N\int_{\alpha_N}^{\beta_N} \left[\left(Q - Q'\ln\frac{\bar{\gamma}}{2}\right)^2 - 2Q'\left(Q - Q'\ln\frac{\bar{\gamma}}{2}\right)\ln(-\ln t) + Q'^2\ln^2(-\ln t)\right](1-t)^{N-1}dt \\ &= \lambda_{1N} + \lambda_{2N}A_N + \lambda_{3N}B_N,\end{aligned} \tag{56}$$

where

$$\lambda_{1N} = \left(Q - Q'\ln\frac{\bar{\gamma}}{2}\right)^2 \left[(1-\alpha_N)^N - (1-\beta_N)^N\right], \tag{57}$$

$$\lambda_{2N} = -2Q'\left(Q - Q'\ln\frac{\bar{\gamma}}{2}\right), \tag{58}$$

$$\lambda_{3N} = Q'^2, \tag{59}$$

$$A_N = N\int_{\alpha_N}^{\beta_N} \ln(-\ln t)(1-t)^{N-1}dt, \tag{60}$$

$$B_N = N\int_{\alpha_N}^{\beta_N} \ln^2(-\ln t)(1-t)^{N-1}dt. \tag{61}$$

Solving the two integrals A_N and B_N with identities in [35] and some simple algebraic manipulation, we obtain

$$\begin{aligned}A_N = (-1)^{N-1}\sum_{m=1}^{N}(-1)^{m-1}\frac{\prod_{j=1}^{m-1}(N-j+1)}{(m-1)!}&\left\{E_i\left[(N-m+1)\ln\alpha_N\right]\right. \\ &\left.-E_i\left[(N-m+1)\ln\beta_N\right] - \alpha_N^{N-m+1}\ln(-\ln\alpha_N) + \beta_N^{N-m+1}\ln(-\ln\beta_N)\right\},\end{aligned} \tag{62}$$

$$\begin{aligned}B_N = (-1)^N\sum_{m=1}^{N}(-1)^{m-1}\frac{\prod_{j=1}^{m-1}(N-j+1)}{(m-1)!}&\left\{\alpha_N^{N-m+1}\ln^2(-\ln\alpha_N) - \beta_N^{N-m+1}\ln^2(-\ln\beta_N)\right. \\ + 2\ln(-\ln\alpha_N)\mathfrak{G}_{1,2}^{2,0}\left({}^{1}_{0,0}\middle| -(N-m+1)\ln\beta_N\right) &- 2\ln(-\ln\beta_N)\mathfrak{G}_{1,2}^{2,0}\left({}^{1}_{0,0}\middle| -(N-m+1)\ln\beta_N\right) \\ \left.+2\mathfrak{G}_{2,3}^{3,0}\left({}^{1,1}_{0,0,0}\middle| -(N-m+1)\ln\alpha_N\right) - 2\mathfrak{G}_{2,3}^{3,0}\left({}^{1,1}_{0,0,0}\middle| -(N-m+1)\ln\beta_N\right)\right\}.\end{aligned} \tag{63}$$

Similarly, $\text{MSE}_l(n^*_{sub})$ is given by

$$\text{MSE}_l(n^*_{sub}) = \lambda'_{1N} + \lambda'_{2N}A_N + \lambda_{3N}B_N, \tag{64}$$

where

$$\lambda'_{1N} = \left(Q - 1 - Q'\ln\frac{\bar{\gamma}}{2}\right)^2 \left[(1-\alpha_N)^N - (1-\beta_N)^N\right], \tag{65}$$

$$\lambda'_{2N} = -2Q'\left(Q - 1 - Q'\ln\frac{\bar{\gamma}}{2}\right). \tag{66}$$

REFERENCES

[1] Z. Ding, K. K. Leung, D. L. Goeckel, and D. Towsley, "Cooperative transmission protocols for wireless broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3701–3713, Dec. 2010.

[2] I. Garcia, K. Sakaguchi, and K. Araki, "Cell planning for cooperative transmission," in *Proc. IEEE WCNC'08*, Las Vegas, USA, Mar. 2008, pp. 1769–1774.

[3] S. Moh, "Two cooperation models and their optimal routing for cooperative diversity in wireless ad hoc networks," in *Proc. IEEE ISWCS'08*, Reykjavik, Iceland, Oct. 2008, pp. 57–61.

[4] L. Sun, T. Zhang, L. Lu, and H. Niu, "Cooperative communications with relay selection in wireless sensor networks," *IEEE Trans. Consum. Electron.*, vol. 55, no. 2, pp. 513–517, May 2009.

[5] Y. Chen, J. Teo, J. Lai, E. Gunawan, K. S. Low, C. B. Soh, and P. Rapajic, "Cooperative communications in ultra-wideband wireless body area networks: Channel modeling and system diversity analysis," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 1, pp. 5–16, Jan. 2009.

[6] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity - Part I. System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[7] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[8] R. Ahlswede, N. Cai, S.-Y. Li, and R. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, no. 4, pp. 1204–1216, Jul. 2000.

[9] R. Koetter and M. Medard, "An algebraic approach to network coding," *IEEE/ACM Trans. Netw.*, vol. 11, no. 5, pp. 782–795, Oct. 2003.

[10] D. Nguyen, T. Tran, T. Nguyen, and B. Bose, "Wireless broadcast using network coding," *IEEE Trans. Veh. Technol.*, vol. 58, no. 2, pp. 914–925, Feb. 2009.

[11] C. Fragouli, J. Widmer, and J.-Y. Le Boudec, "Efficient broadcasting using network coding," *IEEE/ACM Trans. Netw.*, vol. 16, no. 2, pp. 450–463, Apr. 2008.

[12] Y. Chen and S. Kishore, "On the tradeoffs of implementing randomized network coding in multicast networks," *IEEE Trans. Commun.*, vol. 58, no. 7, pp. 2107–2115, Jul. 2010.

[13] P. Fan, C. Zhi, C. Wei, and K. Ben Letaief, "Reliable relay assisted wireless multicast using network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 749–762, Jun. 2009.

[14] Y. Liu, W. Chen, J. Ji, and J. Zhang, "Network-coded cooperation for multi-unicast with non-ideal source-relay channels," in *Proc. IEEE ICC 2010*, Cape Town, South Africa, May 2010, pp. 1–5.

[15] J. Liu, D. Goeckel, and D. Towsley, "Bounds on the throughput gain of network coding in unicast and multicast wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 582–592, Jun. 2009.

[16] M. Ju and I.-M. Kim, "Error performance analysis of BPSK modulation in physical-layer network-coded bidirectional relay networks," *IEEE Trans. Commun.*, vol. 58, no. 10, pp. 2770–2775, Oct. 2010.

[17] J. M. Park, S.-L. Kim, and J. Choi, "Hierarchically modulated network coding for asymmetric two-way relay systems," *IEEE Trans. Veh. Technol.*, vol. 59, no. 5, pp. 2179–2184, Jun. 2010.

[18] S. Katti, D. Katabi, W. Hu, H. Rahul, and M. Medard, "The importance of being opportunistic: Practical network coding for wireless environments," in *Proc. Allerton'05*, Montecillo, Illinois, USA, Sep. 2005.

[19] S. Zhang, S. C. Liew, and P. P. Lam, "Hot topic: Physical-layer network coding," in *Proc. ACM MobiCom'06*, Los Angeles, CA, USA, Sep. 2006, pp. 358–365.

[20] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *Proc. ACM SIGCOMM'07*, Kyoto, Japan, Aug. 2007, pp. 397–408.

[21] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the air: Practical wireless network coding," *IEEE/ACM Trans. Netw.*, vol. 16, no. 3, pp. 497–510, Jun. 2008.

[22] R. Louie, Y. Li, and B. Vucetic, "Practical physical layer network coding for two-way relay channels: Performance analysis and comparison," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 764–777, Feb. 2010.

[23] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699–712, Jun. 2009.

[24] B. Jiang, F. Gao, X. Gao, and A. Nallanathan, "Channel estimation and training design for two-way relay networks with power allocation," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2022–2032, Jun. 2010.

[25] *IEEE Standard for Local and Metropolitan Area Networks - Part 16: Air Interface for Broadband Wireless Access Systems*, IEEE Std. 802.16-2009, May 2009.

[26] Z. Yong, L. Jun, X. Youyun, and C. Yueming, "An adaptive non-orthogonal cooperation scheme based on channel quality information," in *Proc. WiCom'07*, Shanghai, China, Sep. 2007, pp. 988–991.

[27] J.-B. Kim and D. Kim, "Comparison of two SNR-based feedback schemes in multiuser dual-hop amplify-and-forward relaying networks," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 557–559, Aug. 2008.

[28] R. Schoenen, D. Bultmann, and Z. Xu, "Channel quality indication for adaptive resource scheduling in multihop OFDMA systems," in *Proc. EW'09*, Aalborg, Denmark, May 2009, pp. 58–62.

[29] H. Yomo and E. de Carvalho, "A CSI estimation method for wireless relay network," *IEEE Commun. Lett.*, vol. 11, no. 6, pp. 480–482, Jun. 2007.

[30] W. Xu, X. Dong, and Y. Huang, "Asymptotic achievable rate analysis for selection strategies in amplify-and-forward MIMO two-hop networks with feedback," *IEEE Trans. Veh. Technol.*, vol. 59, no. 7, pp. 3662–3668, Sep. 2010.

[31] H. Wu, Y. Wang, G. Wei, and D. Yang, "A relay selection scheme for the downlink of cellular networks," in *Proc. IEEE WiCom'09*, Beijing, China, Sep. 2009, pp. 1–4.

[32] F. Brouwer, I. de Bruin, J. Silva, N. Souto, F. Cercas, and A. Correia, "Usage of link-level performance indicators for HSDPA network-level simulations in E-UMTS," in *Proc. IEEE ISSSTA'04*, Sydney, Australia, Aug. 2004, pp. 844–848.

[33] K. Freudenthaler, A. Springer, and J. Wehinger, "Novel SINR-to-CQI mapping maximizing the throughput in HSDPA," in *Proc. IEEE WCNC'07*, Hong Kong, China, Mar. 2007, pp. 2231–2235.

[34] F. Tsuzuki and T. Ohtsuki, "Channel estimation with selective superimposed pilot sequences under fast fading environments," in *Proc. IEEE VTC 2004-Fall*, vol. 1, Los Angeles, CA, USA, Sep. 2004, pp. 62–66.

[35] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Academic Press, 2007.

[36] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. John Wiley & Sons, 2005.

[37] H. A. David and H. N. Nagaraja, *Order Statistics*, 3rd ed. Wiley, 2003.

TABLE I

BOUND-BASED RELAY SELECTION SCHEME

For $n = 1 : N$
Calculate max-MSE(n).
If max-MSE$(n) \leqslant$ MSE$_u(n^*_{sub})$
$n^*_{bound-based\ sub} = n$
max-MSE$(n^*_{bound-based\ sub})$ = max-MSE(n)
Exit For
End If
End For

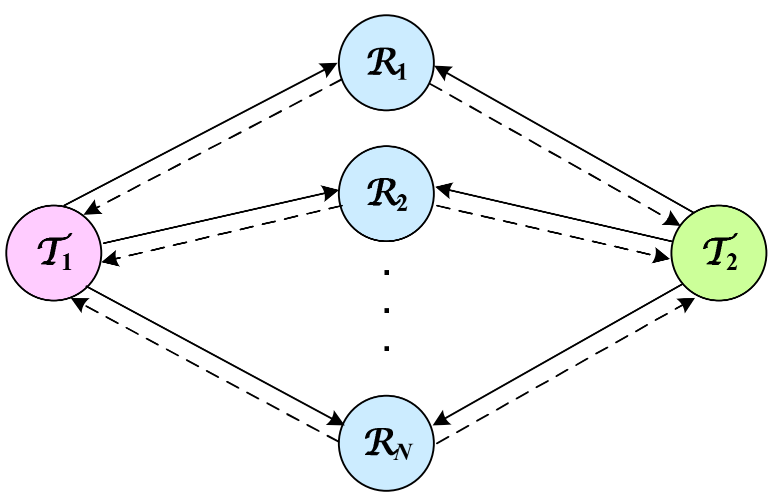

Fig. 1. System Model.

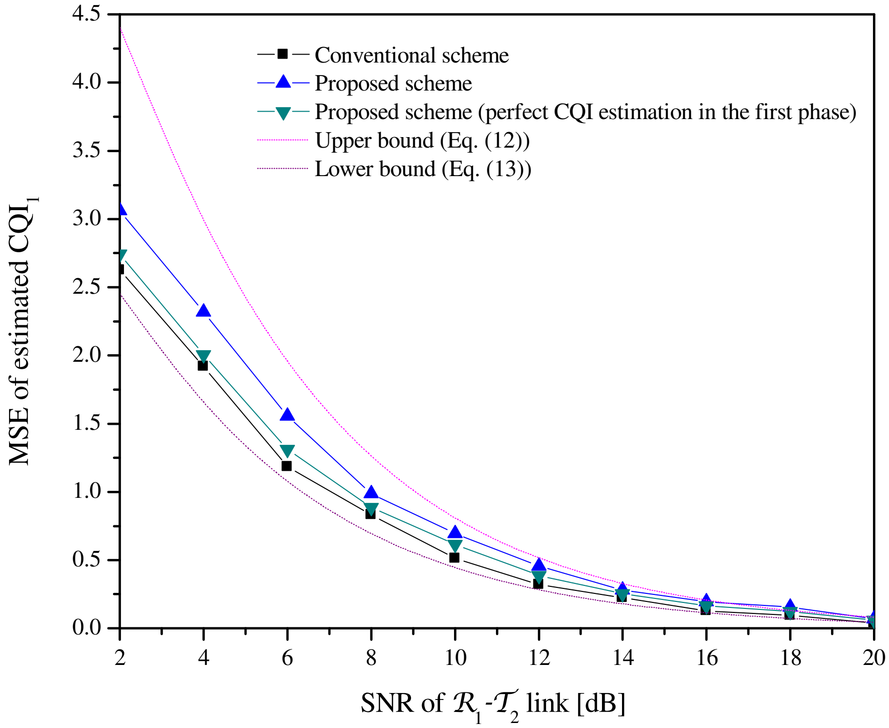

Fig. 2. MSE of estimated ρ_1 at $\mathcal{T}_2$ versus SNR of $\mathcal{R}_1 - \mathcal{T}_2$ link with different schemes.

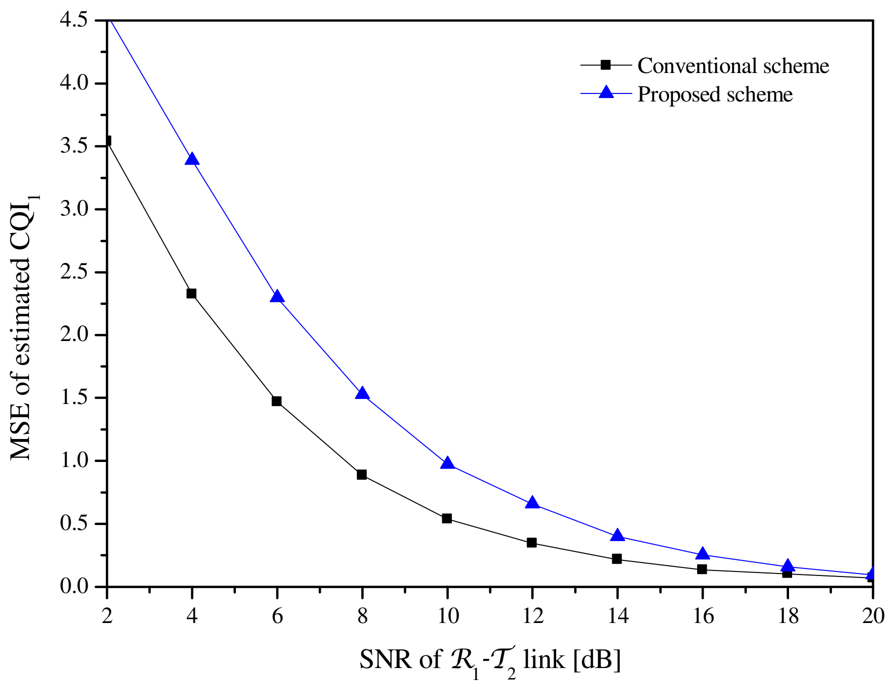

Fig. 3. MSE of estimated ρ_1 at $\mathcal{T}_2$ versus SNR of $\mathcal{R}_1 - \mathcal{T}_2$ link when the SNRs of $\mathcal{T}_1 - \mathcal{R}_1$ and $\mathcal{R}_1 - \mathcal{T}_2$ links are equal.

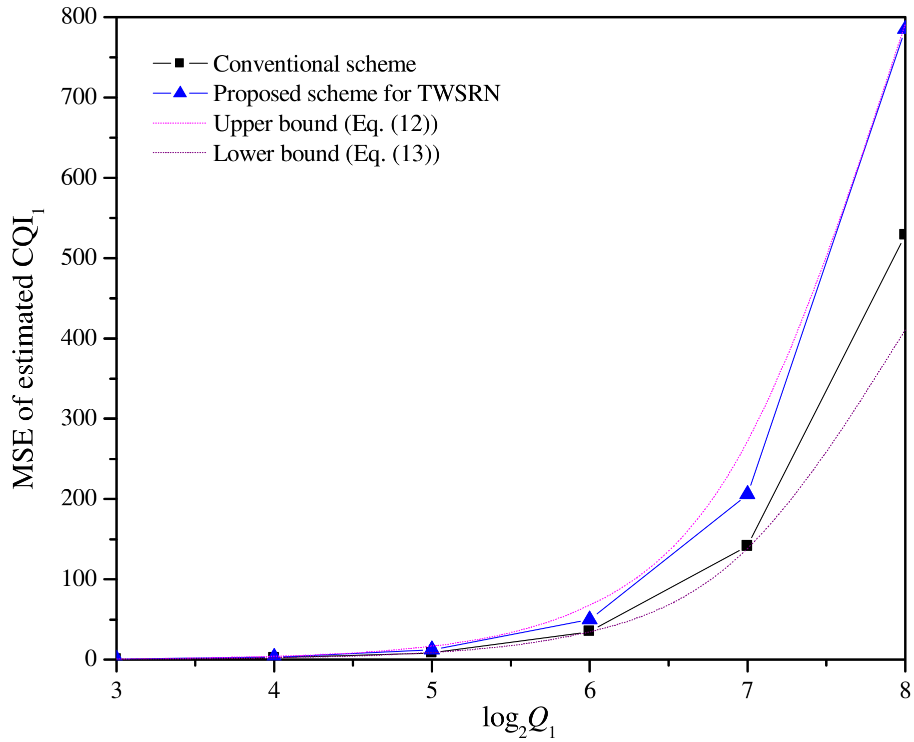

Fig. 4. MSE of estimated ρ_1 at $\mathcal{T}_2$ versus different CQI levels (Q_1).

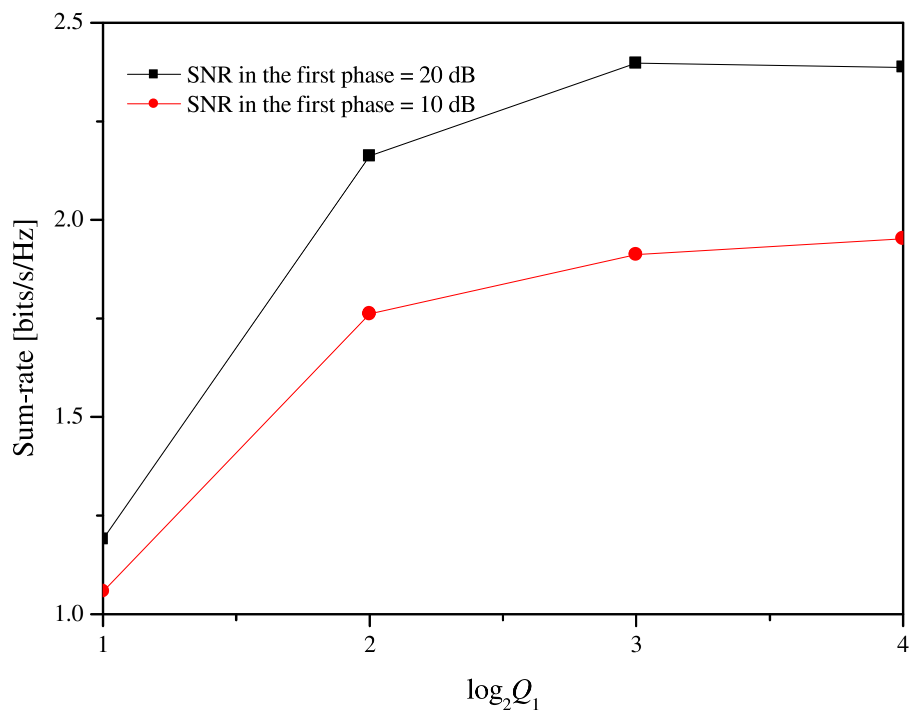

Fig. 5. Sum-rate versus different CQI levels with adaptive modulation and coding scheme.

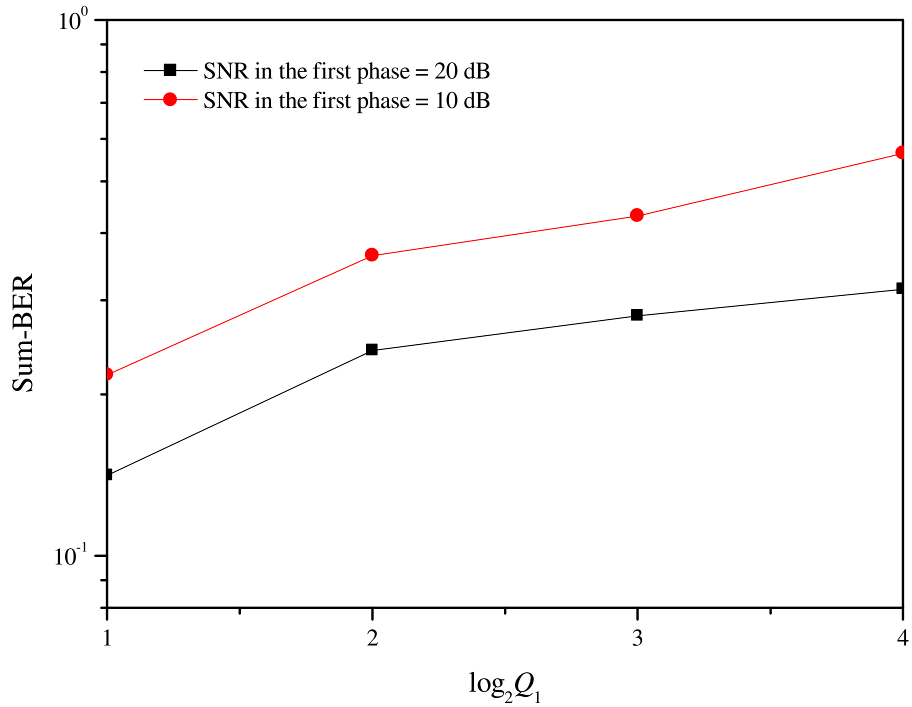

Fig. 6. Sum-BER versus different CQI levels with adaptive modulation and coding scheme.

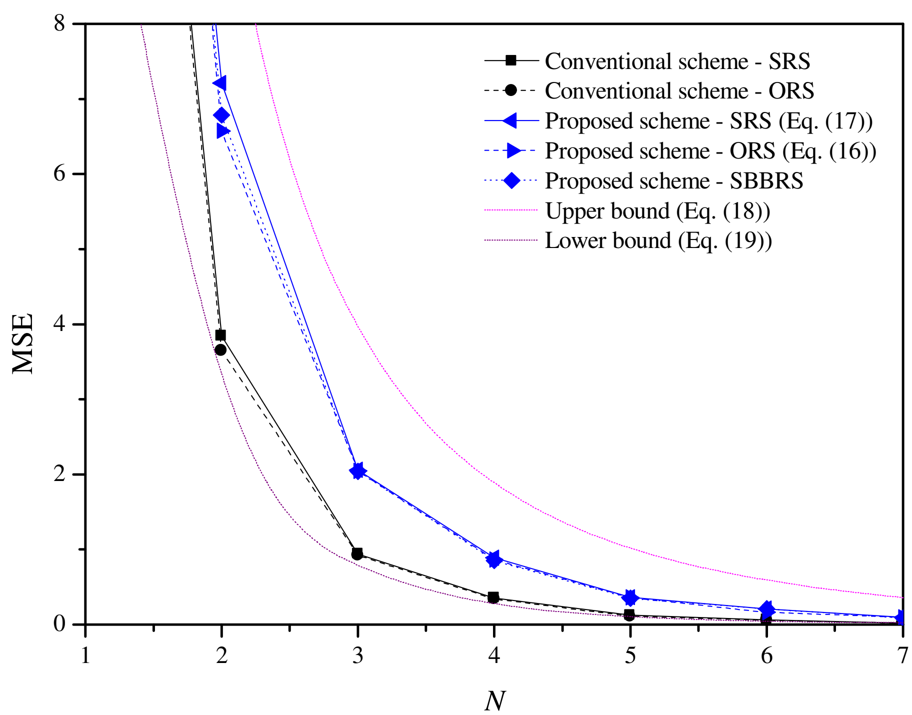

Fig. 7. MSE versus number of relays (N) with different relay selection schemes.

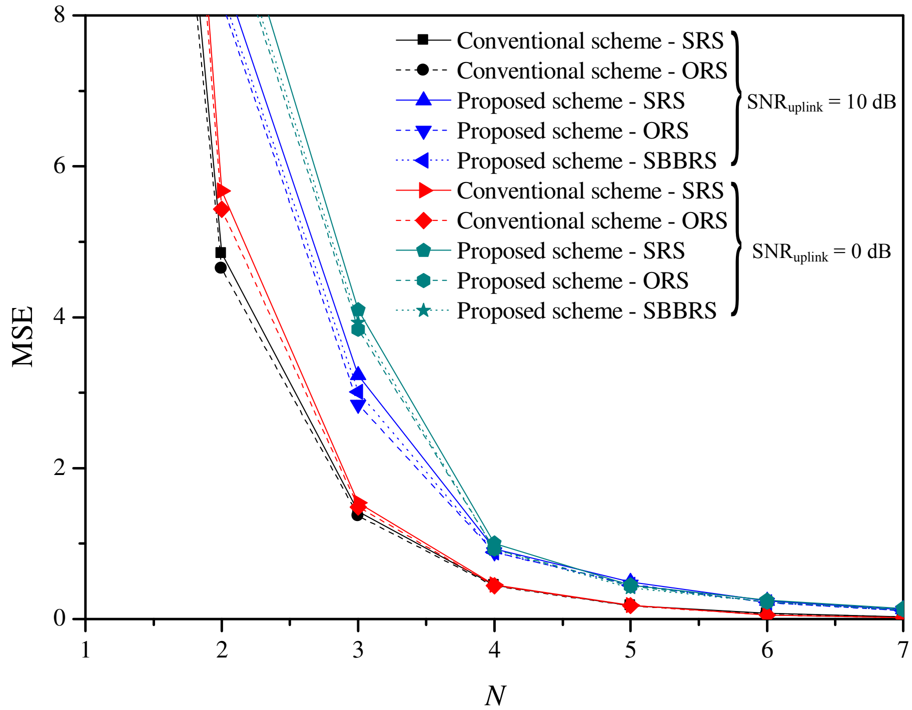

Fig. 8. MSE versus number of relays (N) with different relay selection schemes and different SNR values of uplink transmission.

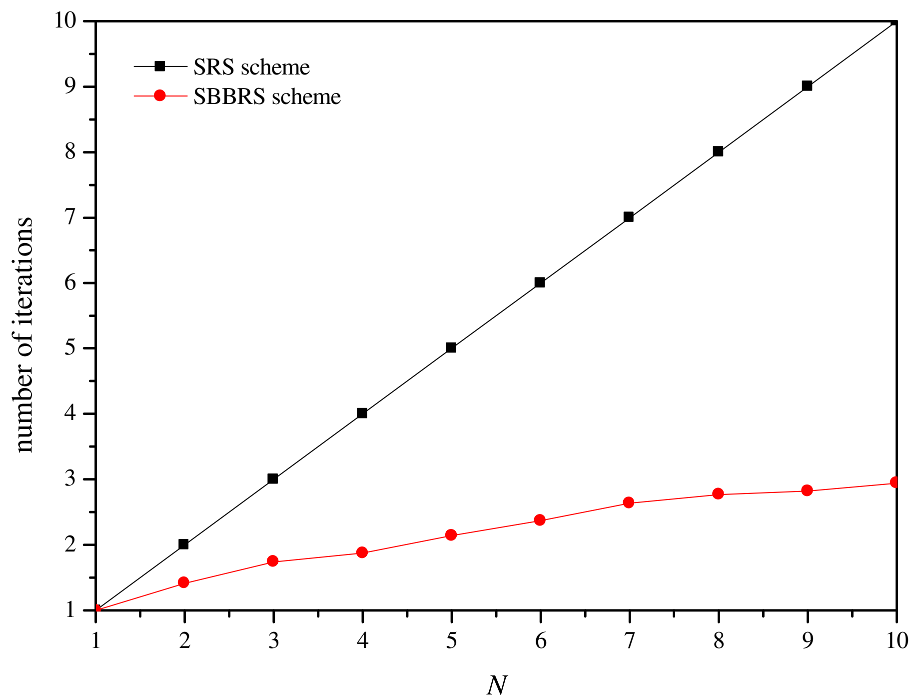

Fig. 9. Number of iterations versus number of relays (N) with SRS and SBBRS schemes.

TABLE II

CQI MAPPING TABLE FOR 16 MCS LEVELS

CQI value	**Modulation**	**Coding rate**	**Bits/Symbol**
0	BPSK	$1/2$	0.50
1	BPSK	$3/4$	0.75
2	QPSK	$1/2$	1.00
3	QPSK	$2/3$	1.33
4	QPSK	$3/4$	1.50
5	QPSK	$5/6$	1.67
6	QPSK	$7/8$	1.75
7	16 QAM	$1/2$	2.00
8	16 QAM	$2/3$	2.67
9	16 QAM	$3/4$	3.00
10	16 QAM	$5/6$	3.33
11	16 QAM	$7/8$	3.50
12	64 QAM	$2/3$	4.00
13	64 QAM	$3/4$	4.50
14	64 QAM	$5/6$	5.00
15	64 QAM	$7/8$	5.25