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Network Coding-Based Block Acknowledgement Scheme for Wireless Regenerative Relay Networks

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Abstract

This paper is concerned with block acknowledgement (ACK) mechanisms in wireless regenerative relay networks. In an N-relay network, a total of $(2N+1)$ block ACK packets is required to acknowledge the data transmission between source and destination nodes via the N relay nodes. In this paper, we propose a block ACK scheme based on network coding (NC) to significantly reduce the acknowledgement overheads by N block ACK packets. In addition, this achieves a reduction of $N(N-1)$ computational operators. Particularly, we derive the error probability of the determination of the packets to be retransmitted at the source and relays, which shows that the NC-based scheme also improves the reliability of block ACK transmissions. Furthermore, asymptotic signal-to-noise (SNR) scenarios for forward links are considered and a general expression of error probability in multi-relay networks is derived for each SNR scenario. Finally, simulation results are presented to verify the analytical findings and demonstrate a lower number of data retransmissions for a higher system throughput.

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Index Terms

Block acknowledgement, network coding, wireless relay network.

I. INTRODUCTION

Positive acknowledgement (ACK) with retransmission is a communication protocol designed to assure the reliability of data packet transmission over wireless channels that suffer from fading and background noise. This protocol requires the receiver to send an ACK packet to the transmitter to confirm the successful reception of each data packet. Although the transmission reliability is improved by using the ACK protocol, overall throughput is significantly reduced due to frequent transmissions of small-sized ACK packets [1]. To address this issue, a block ACK mechanism is employed in the IEEE 802.11e standard to reduce the overhead required at each node [2]. A block ACK aggregates multiple ACK packets into a single ACK packet to acknowledge a group of received data packets. This aggregation of block ACK packets improves the overall throughput by reducing the arbitrary inter-frame spacing periods, the backoff counter time, and the acknowledgement time. Recently, new medium access control (MAC) amendments based on package aggregation techniques [3] and block ACK mechanisms [4] have been proposed for the IEEE 802.11n standard [5] to improve further the throughput. However, existing block ACK schemes are generally restricted to one-to-one communications.

In parallel, there has been a growing interest in relaying techniques which are aimed to extend the coverage of wireless networks with spatial diversity gains [6]–[8]. In wireless relay networks, assume the transmission from source node $\mathcal{S}$ to destination node $\mathcal{D}$ is carried out with the aid of N relay nodes $\mathcal{R}^{(N)} = \{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_N\}$ in an orthogonal decode-and-forward manner, where $\mathcal{R}_n$ denotes the nth relay node. While block ACK mechanisms were originally proposed for one-to-one communications, using block ACK in wireless relay networks is more complicated because each relay node in $\mathcal{R}^{(N)}$ has to send block ACK packets for links $\mathcal{S} \to \mathcal{R}^{(N)}$ to $\mathcal{S}$, and $\mathcal{D}$ has to send block ACK packets for links $\mathcal{R}^{(N)} \to \mathcal{D}$ to $\mathcal{R}^{(N)}$ and send a block ACK packet for link $\mathcal{S} \to \mathcal{D}$ to $\mathcal{S}$ [9], [10]. These will result in a total of $(2N + 1)$ block ACK packets. Furthermore, the resulting simultaneous retransmissions of the same packets at $\mathcal{S}$ and $\mathcal{R}^{(N)}$ can considerably degrade the network throughput. To solve this problem, a cooperative retransmission scheme was proposed in [11], i.e., $\mathcal{S}$ only retransmits the corrupted packets at both $\mathcal{R}^{(N)}$ and $\mathcal{D}$, and, $\mathcal{R}^{(N)}$ helps $\mathcal{S}$ retransmit the rest of the corrupted packets at $\mathcal{D}$. However, the overall throughput of this cooperative network still suffers from having to send and process $(2N+1)$ block ACK packets at $\mathcal{S}$, $\mathcal{R}^{(N)}$, and $\mathcal{D}$.

In this paper, we propose a new block ACK scheme based on network coding (NC) for wireless regenerative relay networks. Our proposed NC-based block ACK scheme will not only reduce the number of block ACK packets but also improve the reliability of determination of packets to be retransmitted[1]. This NC-based scheme will thus minimize the number of data retransmissions for an improved system throughput with a lower complexity in comparison with the non-NC-based block ACK scheme[2]. NC was initially used to increase the system throughput for a lossless network [12], and was later applied to two-way relay channels [13] and peer-to-peer communications [14]. The basic idea of our proposed NC-based scheme is that $\mathcal{D}$ combines all the block ACK packets for links $\mathcal{R}^{(N)} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{D}$ to create a combined block ACK packet. Thus, the total number of block ACK packets decreases to $(N+1)$ through this combination. After this combined block ACK packet is received along with the block ACK packets for links $\mathcal{S} \rightarrow \mathcal{R}^{(N)}$, the question becomes - *How can $\mathcal{S}$ and $\mathcal{R}^{(N)}$ determine the packets to be retransmitted to $\mathcal{D}$?* As we will show later, thanks to NC, the packets to be retransmitted can be determined by performing simple bitwise XOR and/or AND operators on the received block ACK packets at $\mathcal{S}$ and $\mathcal{R}^{(N)}$. Our analysis will also show that the reduction of the number of block ACK packets not only improves the reliability of the determination of packets to be retransmitted at the source and relay nodes, but also incurs a lower complexity compared to the non-NC-based block ACK scheme by a reduction of $N(N-1)$ computational operations.

Another contribution of this paper is that we will derive closed-form expressions for the probability of error in the determination of the packets to be retransmitted at $\mathcal{S}$ and $\mathcal{R}_1$ over Rayleigh flat fading channels in a one-relay network. To the best of our knowledge, this has not yet been derived. The error probabilities are derived with respect to the signal-to-noise ratio (SNR) of the forward and backward links. The derived closed-form expressions manifest not only the effect of channel links on the determination of packets to be retransmitted but also the

[1]We limit our work to the phases of generation and detection of acknowledgement information only. For full MAC protocols, readers are referred to standard references, e.g., [5].

[2]The non-NC-based block ACK scheme is referred to as a scheme where $\mathcal{R}^{(N)}$ sends N block ACK packets to $\mathcal{S}$, and $\mathcal{D}$ sends $(N+1)$ block ACK packets to $\mathcal{R}^{(N)}$ and $\mathcal{S}$.

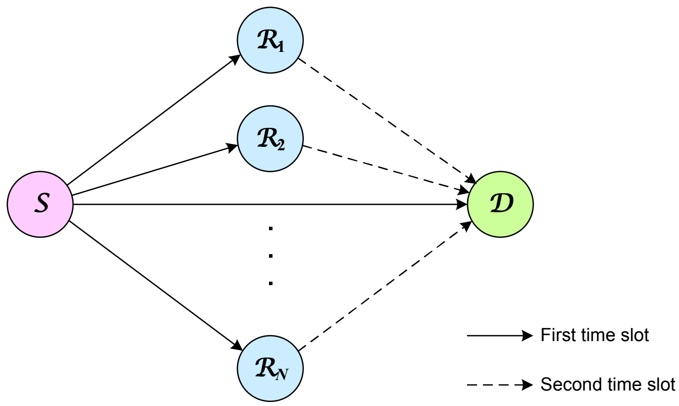


Fig. 1. Two-hop relay network.

higher reliability of our proposed NC-based block ACK scheme over the non-NC-based scheme. In order to gain insights into our proposed NC-based block ACK scheme, we will consider some extreme scenarios for the forward links. For each scenario, we will derive an approximate general expression for the error probability in multi-relay networks. Simulations are then presented to verify the advantages of the proposed NC-based block ACK scheme. The simulation results are shown to be consistent with the numerical results in the three extreme scenarios and reflect the improved reliability in the determination of packets to be retransmitted using our proposed NC-based block ACK scheme compared with the non-NC-based scheme. Furthermore, the higher reliability of our proposed NC-based block ACK scheme results in a significant reduction in the average number of data retransmissions at all nodes, which is verified through the simulation results.

The rest of this paper is organized as follows: In Section II, we describe the system model of a typical two-hop regenerative relay network. The fundamental of our proposed NC-based block ACK scheme is presented in Section III in contrast with the non-NC-based block ACK scheme. Section IV presents an analysis of the probability of error in the determination of packets to be retransmitted at $\mathcal{S}$ and $\mathcal{R}$. Numerical results are given in Section V and Section VI draws the main conclusions of the paper.

II. System Model

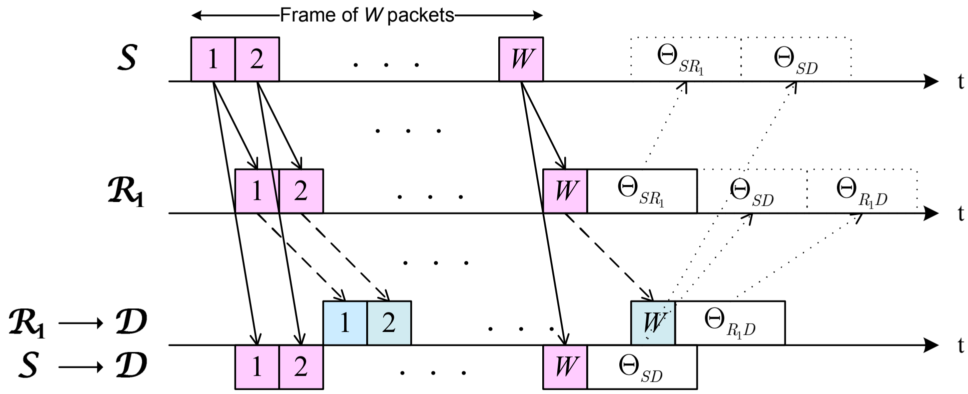
Fig. 1 illustrates a typical two-hop regenerative relay network where the data transmission from source node $\mathcal{S}$ to destination node $\mathcal{D}$ is accomplished by a two-hop protocol with the assistance of a group of N relays $\mathcal{R}^{(N)} = \{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_N\}$. In this two-hop regenerative cooperation scheme, $\mathcal{S}$ transmits data sequences continuously to $\mathcal{R}^{(N)}$ and $\mathcal{D}$ in the first hop. In the second hop, $\mathcal{R}^{(N)}$ decode and forward the received data sequences to $\mathcal{D}$. We assume that $\mathcal{S}$ sends data sequences in the form of aggregated frames, each consisting of W data packets. An aggregated ACK packet, i.e., block ACK packet, of length K (in bits) is used to report the status of each frame where bits '0' and '1' represent the data packet being correctly received and the packet being lost or erroneously received, respectively. For the sake of simplicity, we omit the bits used for overhead and other signalling information in block ACK packets, and assume that the length of each block ACK packet in bits is equal to the number of packets in a data frame, i.e., $K = W$. For convenience, let Θ_{AB} denote the W-bit block ACK packet that is generated at node B and sent to node A to acknowledge a frame of W packets that are sent from A to B, where $A, B \in \{\mathcal{S}, \mathcal{D}, \mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_N\}$.

Fig. 2 illustrates the process of data transmission and block ACK reporting for a one-relay network. The transmission protocol can be readily extended for multi-relay networks. In the first hop, $\mathcal{S}$ transmits W packets sequentially to $\mathcal{R}^{(N)}$ and $\mathcal{D}$. Then, $\mathcal{R}^{(N)}$ forwards the correctly received packet to $\mathcal{D}$ in the second hop. After decoding and error-checking all the W packets received from $\mathcal{S}$, relay nodes $\mathcal{R}_j$, $j \in \{1, 2, \ldots, N\}$, and destination node $\mathcal{D}$ generate block ACK packets Θ_{SR_j} and Θ_{SD}, respectively. Meanwhile, $\mathcal{D}$ also attempts to decode signals forwarded from $\{\mathcal{R}_j\}$ and then generates $\{\Theta_{R_jD}\}$ after checking all the W data packets.

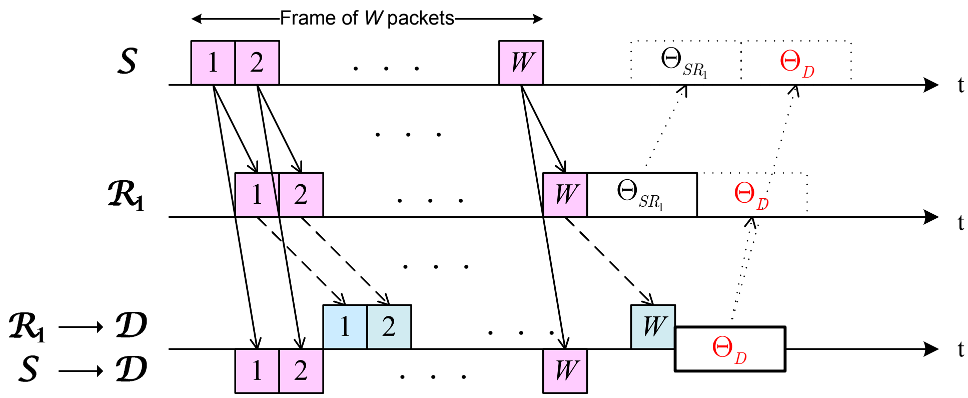
In the non-NC-based block ACK scheme as shown in Fig. 2(a), $\mathcal{R}_j$ and $\mathcal{D}$ send the block ACK packets Θ_{SR_j} and Θ_{SD}, respectively, to $\mathcal{S}$ to acknowledge their receipt of the data packets. Similarly, $\mathcal{D}$ sends Θ_{R_jD} to $\mathcal{R}_j$ to acknowledge the receipt of the packets forwarded by $\mathcal{R}_j$. For the purpose of cooperative retransmission, $\mathcal{R}_j$ needs to know which packets $\mathcal{D}$ has received correctly from $\mathcal{S}$. Thus, Θ_{SD} is additionally sent to all $\{\mathcal{R}_j\}$. In our proposed NC-based block ACK scheme as shown in Fig. 2(b), instead of sending Θ_{SD} and Θ_{R_jD} separately, $\mathcal{D}$ generates *only one* combined block ACK packet, denoted as Θ_D, and broadcasts it to $\mathcal{R}_j$ and $\mathcal{S}$. Thus, the number of block ACK packets to be sent from $\mathcal{D}$ decreases.

III. BLOCK ACK: NON-NC-BASED AND PROPOSED NC-BASED SCHEMES

In this section, we present the fundamentals of our proposed NC-based block ACK scheme in contrast with the non-NC-based block ACK scheme.



(a)



(b)

Fig. 2. Protocol sequence with: (a) non-NC-based block ACK scheme, (b) proposed NC-based block ACK scheme.

Non-NC-based Block ACK Scheme

After decoding a frame of W packets, each relay node $\mathcal{R}_j, j = 1, 2, \ldots, N$, generates block ACK packet Θ_{SR_j} while $\mathcal{D}$ generates $(N+1)$ block ACK packets $\Theta_{R_1D}, \Theta_{R_2D}, \ldots, \Theta_{R_ND}$, and Θ_{SD}. Note that the length of each block ACK packet is W bits. Let Ω_S and Ω_{R_j} denote the W-bit retransmission indication packets (RIPs) generated at $\mathcal{S}$ and $\mathcal{R}_j$, respectively, in which bit '1' indicates that the corresponding data packet needs to be retransmitted while bit '0' indicates otherwise. The RIPs can be obtained as follows:

$$\Omega_S = \Theta_{SR_1} \otimes \Theta_{SR_2} \otimes \cdots \otimes \Theta_{SR_N} \otimes \Theta_{SD}, \tag{1a}$$

$$\Omega_{R_j} = \Theta_{R_1D} \otimes \Theta_{R_2D} \otimes \cdots \otimes \Theta_{R_ND} \otimes \Theta_{SD} \otimes \overline{\Theta}_{SR_j}, \tag{1b}$$

respectively, where $\otimes$ denotes the bitwise AND operator and $\overline{\Theta}_{AB}$ is the bitwise complement of Θ_{AB}. Note that (1a) and (1b) are based on the principle of cooperative retransmission, i.e., the source node retransmits the packets that are lost at all the relay and destination nodes, whereas each relay node retransmits only those packets that it correctly decodes but the destination node fails to do so.

Our Proposed NC-based Block ACK Scheme

Instead of sending $(2N+1)$ block ACK packets, $\{\Theta_{SR_j}\}$, $\{\Theta_{R_jD}\}$, and Θ_{SD}, as in the non-NC-based block ACK scheme, our proposed NC-based block ACK scheme only needs to send $(N+1)$ block ACK packets, $\{\Theta_{SR_j}\}$ and Θ_D, at $\mathcal{R}^{(N)}$ and $\mathcal{D}$, respectively. While $\{\Theta_{SR_j}\}$ is generated at $\mathcal{R}^{(N)}$ as in the non-NC-based scheme, Θ_D is created at $\mathcal{D}$ as follows:

$$\Theta_D = \Theta_{R_1D} \otimes \Theta_{R_2D} \otimes \cdots \otimes \Theta_{R_ND} \otimes \Theta_{SD}. \tag{2}$$

The RIPs, Ω_S and $\Omega_{R_j}, j = 1, 2, \ldots, N$, can be obtained by

$$\Omega_S = \Theta_{SR_1} \otimes \Theta_{SR_2} \otimes \cdots \otimes \Theta_{SR_N} \otimes \Theta_D, \tag{3a}$$

$$\Omega_{R_j} = \Theta_D \oplus \left(\Theta_{SR_j} \otimes \Theta_D\right), \tag{3b}$$

respectively, where $\oplus$ denotes the bitwise XOR operator. In (3a), the determination of packets to be retransmitted at $\mathcal{S}$ follows the principle that the source node retransmits the packets that are lost at all $\mathcal{R}^{(N)}$ and $\mathcal{D}$. Particularly, the idea behind (3b) is originated from NC in the sense that $\mathcal{R}_j$ resends those packets that are correctly decoded at $\mathcal{R}_j$ but fails to be decoded at $\mathcal{D}$ and that are not to be resent by $\mathcal{S}$. Thus, the packets that $\mathcal{R}_j$ needs to retransmit are determined by an XOR operation of Θ_D and $\left(\Theta_{SR_j} \otimes \Theta_D\right)$. It is noted that (3b) is different from (1b).

Remark 1 (*Higher Reliability*)**.** The proposed NC-based scheme can determine the packets to be retransmitted more reliably than the non-NC-based scheme. In our proposed NC-based scheme as shown in (3a), to determine Ω_S, besides N block ACK packets from $\mathcal{R}^{(N)}$, i.e., Θ_{SR_1}, Θ_{SR_2}, $\ldots, \Theta_{SR_N}$, a block ACK packet Θ_D is required from $\mathcal{D}$ instead of Θ_{SD} as in the non-NC-based scheme shown in (1a). From (2), Θ_D is determined by combining the block ACK packets of links $\mathcal{R}^{(N)} \to \mathcal{D}$ and $\mathcal{S} \to \mathcal{D}$. This means that the creation of Θ_D depends on decisions of various links, and thus, we can improve the decision reliability of the packets to be retransmitted at $\mathcal{S}$. Additionally, in the non-NC-based scheme as shown in (1b), to determine Ω_{R_j} at each $\mathcal{R}_j$, a total of $(N+1)$ block ACK packets, Θ_{R_1D}, Θ_{R_2D}, $\ldots, \Theta_{R_ND}$, and Θ_{SD}, are required. Contrastingly, in our proposed NC-based scheme as shown in (3b), only one packet, Θ_D, needs to be known to determine Ω_{R_j} at $\mathcal{R}_j$. Therefore, our proposed NC-based scheme has a lower probability of error in the determination of packets to be retransmitted at $\mathcal{R}_j$ since only one packet, Θ_D, has to be detected correctly. Furthermore, it can be seen that the number of packets to be retransmitted depends on the quality of backward links and block ACK schemes. Compared with the non-NC-based block ACK scheme over the same backward environment, our proposed NC-based scheme achieves a higher reliability in the determination of packets to be retransmitted, and thus less data retransmissions are needed.

Remark 2 (*Lower Complexity*)**.** If the computational complexity is measured by the number of binary operations (e.g., XOR, AND, and complement) to determine the packets to be retransmitted at the relays and the source, i.e., the number of required operations to compute Ω_S and Ω_{R_j}, $j \in \{1, 2, \ldots, N\}$, our proposed NC-based scheme has lower complexity than the non-NC-based scheme. It can be seen from (1a) and (1b) that the numbers of operations performed at $\mathcal{S}$ and $\mathcal{R}_j$ are N and $(N + 2)$, respectively. Thus, a total of $(N^2 + 3N)$ operations is required in the non-NC-based block ACK scheme. In our proposed NC-based scheme, N operations are required at $\mathcal{D}$, while no operation is performed at $\mathcal{D}$ in the non-NC-based scheme. However, in our proposed NC-based scheme, the complexity at $\mathcal{R}_j$ is significantly low since only 2 operations are required at $\mathcal{R}_j$ (see (3b)). In addition, N operations are required at $\mathcal{S}$ according to (3a). Thus, a total of $4N$ operations is required in our proposed NC-based scheme, which results in a quadratic reduction of $(N^2 - N)$ operations compared to the non-NC-based block ACK scheme. This reduction is substantial when N increases. For example, only 20 operations are required when $N = 5$ (i.e., 50% reduced), while only 8 operations are required when $N = 2$ (i.e., 20% reduced).

IV. Analysis of Error Probability of Block Acknowledgement Transmission

In this section, we first present signal models for the transmission of block ACK packets through backward links. Then, we will derive the probability of error in the determination of packets to be retransmitted, i.e., the retransmission decision error probability (RDEP), at the relay and source nodes in our proposed NC-based scheme.

We assume that the channels for all links are Rayleigh flat fading channels. The channel gains for forward links $\mathcal{S} \to \mathcal{R}_j$, $\mathcal{R}_j \to \mathcal{D}$, $j \in \{1, 2, \ldots, N\}$, and $\mathcal{S} \to \mathcal{D}$ are denoted by

h_{SR_j}, h_{R_jD}, and h_{SD}, respectively. Similarly, the channel gains for backward links $\mathcal{R}_j \to \mathcal{S}$, $\mathcal{D} \to \mathcal{R}_j$, and $\mathcal{D} \to \mathcal{S}$ are denoted by h_{R_jS}, h_{DR_j}, and h_{DS}, respectively. After receiving a frame of W packets from $\mathcal{S}$ in the first hop of the transmission, each $\mathcal{R}_j$ creates a block ACK packet $\Theta_{SR_j}, j = 1, \ldots, N$, and sends it back to $\mathcal{S}$. The signal received at $\mathcal{S}$ from $\mathcal{R}_j$ can be written as

$$\mathbf{y}_{R_jS} = \sqrt{\Gamma_{R_jS}} h_{R_jS} \mathbf{x}_{SR_j} + \mathbf{n}_{R_jS}, \tag{4}$$

where Γ_{R_jS} is the power level for the block ACK signal of link $\mathcal{R}_j \to \mathcal{S}$, $\mathbf{x}_{SR_j}$ is the binary phase shift keying (BPSK) modulated signal of Θ_{SR_j}, and $\mathbf{n}_{R_jS}$ is an independent circularly symmetric complex Gaussian (CSCG) noise vector with each entry having zero mean and variance of N_0. From $\mathbf{y}_{R_jS}$, $\mathcal{S}$ can detect Θ_{SR_j}. Let $\hat{\Theta}_{SR_j}$ denote the detected Θ_{SR_j}. Assume that the channels for the backward links are invariant over the whole transmission of block ACK sequences and known to all the nodes in the network.

At the same time, $\mathcal{D}$ generates Θ_{SD} corresponding to the error of the packets transmitted from $\mathcal{S}$. The data packets forwarded from each $\mathcal{R}_j$ in the second hop of the transmission are acknowledged by packet Θ_{R_jD}. Thus, we have $(N+1)$ block ACK packets generated at $\mathcal{D}$, i.e., Θ_{SD} and $\{\Theta_{R_jD}\}$. Then, $\mathcal{D}$ generates a new combined block ACK packet, denoted as Θ_D, as described in (2). Θ_D is sent to $\mathcal{S}$ and all $\{\mathcal{R}_j\}$. The received signals at $\mathcal{S}$ and $\mathcal{R}_j, j = 1, \ldots, N$, can be written as

$$\mathbf{y}_{DS} = \sqrt{\Gamma_{DS}} h_{DS} \mathbf{x}_D + \mathbf{n}_{DS}, \tag{5}$$

$$\mathbf{y}_{DR_j} = \sqrt{\Gamma_{DR_j}} h_{DR_j} \mathbf{x}_D + \mathbf{n}_{DR_j}, \tag{6}$$

respectively. Here, Γ_{DS} and Γ_{DR_j} are the power levels for the block ACK signals of the two links $\mathcal{D} \to \mathcal{S}$ and $\mathcal{D} \to \mathcal{R}_j$, respectively, $\mathbf{x}_D$ is the BPSK modulated signal of Θ_D, and $\mathbf{n}_{DS}$ and $\mathbf{n}_{DR_j}$ are independent CSCG noise vectors with each entry having zero mean and variance of N_0. From (5) and (6), $\mathcal{S}$ and $\mathcal{R}_j$ can detect Θ_D as $\hat{\Theta}_{D,0}$ and $\hat{\Theta}_{D,j}$, respectively.

The RIPs at $\mathcal{S}$ and $\mathcal{R}_j$ are given, respectively, by

$$\hat{\Omega}_S = \hat{\Theta}_{SR_1} \otimes \hat{\Theta}_{SR_2} \otimes \cdots \otimes \hat{\Theta}_{SR_N} \otimes \hat{\Theta}_{D,0}, \tag{7}$$

$$\hat{\Omega}_{R_j} = \hat{\Theta}_{D,j} \oplus \left(\Theta_{SR_j} \otimes \hat{\Theta}_{D,j} \right). \tag{8}$$

Next, we derive a closed-form expression for the RDEP at both $\mathcal{S}$ and $\mathcal{R}_1$ in our proposed NC-based scheme for the one-relay network ($N = 1$). Eqs. (7) and (8) now become:

$$\hat{\Omega}_S = \hat{\Theta}_{SR_1} \otimes \hat{\Theta}_{D,0}, \tag{9}$$

$$\hat{\Omega}_{R_1} = \hat{\Theta}_{D,1} \oplus \left(\Theta_{SR_1} \otimes \hat{\Theta}_{D,1}\right), \tag{10}$$

respectively. The RDEP at $\mathcal{S}$ and $\mathcal{R}_1$ can be defined as the bit error probability (BEP) of Ω_S given by (9) and BEP of Ω_{R_1} given by (10), respectively.

Without loss of generality, let us consider only the first bit in each block ACK and RIP packet. In particular, let a_S and a_{R_1} denote the first bits of Ω_S and Ω_{R_1}, respectively. Similarly, b_D and b_{SR_1} represent the first bits of Θ_D and Θ_{SR_1}, respectively. From (9) and (10), the BEP of Ω_S and Ω_{R_1} can be obtained as follows:

$$\begin{aligned} P_b\left(E_{\Omega_S}\right) &= \text{Pr}\left(\hat{a}_S = 0|a_S = 1\right)\text{Pr}\left(a_S = 1\right) + \text{Pr}\left(\hat{a}_S = 1|a_S = 0\right)\text{Pr}\left(a_S = 0\right) \\ &= \text{Pr}\left(\hat{b}_{SR_1} \otimes \hat{b}_{D,0} = 0|b_{SR_1}b_D = 1\right)\text{Pr}\left(b_{SR_1}b_D = 1\right) \\ &\quad + \text{Pr}\left(\hat{b}_{SR_1} \otimes \hat{b}_{D,0} = 1|b_{SR_1}b_D = 0\right)\text{Pr}\left(b_{SR_1}b_D = 0\right), \end{aligned} \tag{11}$$

$$\begin{aligned} P_b\left(E_{\Omega_{R_1}}\right) &= \text{Pr}\left(\hat{a}_{R_1} = 0|a_{R_1} = 1\right)\text{Pr}\left(a_{R_1} = 1\right) + \text{Pr}\left(\hat{a}_{R_1} = 1|a_{R_1} = 0\right)\text{Pr}\left(a_{R_1} = 0\right) \\ &= \text{Pr}\left(\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = 0|\bar{b}_{SR_1}b_D = 1\right)\text{Pr}\left(\bar{b}_{SR_1}b_D = 1\right) \\ &\quad + \text{Pr}\left(\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = 1|\bar{b}_{SR_1}b_D = 0\right)\text{Pr}\left(\bar{b}_{SR_1}b_D = 0\right), \end{aligned} \tag{12}$$

where $\hat{a}_S$, $\hat{a}_{R_1}$, $\hat{b}_{SR_1}$, $\hat{b}_{D,0}$, and $\hat{b}_{D,1}$ denote the first bit in $\hat{\Omega}_S$, $\hat{\Omega}_{R_1}$, $\hat{\Theta}_{SR_1}$, $\hat{\Theta}_{D,0}$, and $\hat{\Theta}_{D,1}$, respectively. We observe that $\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = 0$ if $b_{SR_1} = 1$. Consequently, $\text{Pr}(\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = 1|b_{SR_1} = 1, b_D = 0) = 0$ and $\text{Pr}(\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = 1|b_{SR_1} = 1, b_D = 1) = 0$. Thus, (12) can be rewritten as

$$\begin{aligned} P_b\left(E_{\Omega_{R_1}}\right) &= \text{Pr}\left(\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = 0|b_{SR_1} = 0, b_D = 1\right)\text{Pr}\left(b_{SR_1} = 0\right)\text{Pr}\left(b_D = 1\right) \\ &\quad + \text{Pr}\left(\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = 1|b_{SR_1} = 0, b_D = 0\right)\text{Pr}\left(b_{SR_1} = 0\right)\text{Pr}\left(b_D = 0\right). \end{aligned} \tag{13}$$

For simplicity, we assume that the channels for both forward and backward links are independent Rayleigh flat fading. That is, $h_{AB} \sim \mathcal{CN}(0,1)$, $A, B \in \{S, R_1, D\}$, $A \neq B$ and $h_{AB} \neq h_{BA}$. In this case, the BEP for signal transmission through link $A \to B$, $A, B \in \{S, R_1, D\}$, $A \neq B$, over a Rayleigh flat fading channel is given by [15]

$$P_b(E_{AB}) = \phi(\gamma_{AB}), \tag{14}$$

where γ_{AB} is the average SNR given by $\gamma_{AB} = \Gamma_{AB}/N_0$, Γ_{AB} is the power level of the signal transmitted through the link $A \to B$, and $\phi(x) \triangleq \frac{1}{2}\left(1 - \sqrt{\frac{x}{1+x}}\right)$.

Lemma 1. The RDEPs at $\mathcal{S}$ and $\mathcal{R}_1$ are given by

$$P_b(E_{\Omega_S}) = \zeta_{11}\alpha\beta + \zeta_{01}(1-\alpha)\beta + \zeta_{10}\alpha(1-\beta) + \zeta_{00}(1-\alpha)(1-\beta), \tag{15a}$$

$$P_b(E_{\Omega_{R_1}}) = \xi(1-\alpha), \tag{15b}$$

respectively, where $\alpha = \phi(\gamma_{SR_1})$, $\beta = \phi(\gamma_{R_1D})\phi(\gamma_{SD})$, $\zeta_{00} = \phi(\gamma_{R_1S})\phi(\gamma_{DS})$, $\zeta_{01} = \phi(\gamma_{R_1S})[1-\phi(\gamma_{DS})]$, $\zeta_{10} = [1-\phi(\gamma_{R_1S})]\phi(\gamma_{DS})$, $\zeta_{11} = \phi(\gamma_{R_1S})+\phi(\gamma_{DS})-\phi(\gamma_{R_1S})\phi(\gamma_{DS})$, and $\xi = \phi(\gamma_{DR_1})$.

Proof: For convenience, let $\alpha' = \Pr(b_{SR_1} = 1)$, $\beta' = \Pr(b_D = 1)$, $\zeta'_{ij} = \Pr(\hat{b}_{SR_1} \otimes \hat{b}_{D,0} = \overline{i \otimes j} | b_{SR_1} = i, b_D = j)$, and $\xi'_i = \Pr(\hat{b}_{D,1} \oplus (b_{SR_1} \otimes \hat{b}_{D,1}) = \bar{i} | b_{SR_1} = 0, b_D = i)$, $\{i,j\} \in \{0,1\}$. Then, (11) and (13) can be rewritten as

$$P_b\left(E_{\Omega_S}\right) = \zeta'_{11}\alpha'\beta' + \zeta'_{01}(1-\alpha')\beta' + \zeta'_{10}\alpha'(1-\beta') + \zeta'_{00}(1-\alpha')(1-\beta'),$$

$$P_b\left(E_{\Omega_{R_1}}\right) = \xi'_1(1-\alpha')\beta' + \xi'_0(1-\alpha')(1-\beta'),$$

respectively. Now, we need to find α', β', ζ'_{ij}, and ξ'_i.

Let us first find α' and β'. We observe that $b_{SR_1} = 1$ if there are errors in the data transmission over forward link $\mathcal{S} \to \mathcal{R}_1$, and $b_D = 1$ if $b_{SD} = 1$ and $b_{R_1D} = 1$, i.e., if the data transmission over both links $\mathcal{S} \to \mathcal{D}$ and $\mathcal{R}_1 \to \mathcal{D}$ has errors. Thus, α' and β' can be given by

$$\alpha' = P_b(E_{SR_1}) = \phi(\gamma_{SR_1}) = \alpha,$$

$$\beta' = P_b(E_{R_1D})P_b(E_{SD}) = \phi(\gamma_{R_1D})\phi(\gamma_{SD}) = \beta.$$

Here, ζ'_{ij}, $\{i,j\} \in \{0,1\}$ can be found as

$$\zeta'_{00} = P_b(E_{\Theta_{SR_1}})P_b(E_{\Theta_{D,0}}),$$

$$\zeta'_{01} = P_b(E_{\Theta_{SR_1}})(1 - P_b(E_{\Theta_{D,0}})),$$

$$\zeta'_{10} = (1 - P_b(E_{\Theta_{SR_1}}))P_b(E_{\Theta_{D,0}}),$$

$$\zeta'_{11} = \zeta'_{00} + \zeta'_{01} + \zeta'_{10} = P_b(E_{\Theta_{SR_1}}) + P_b(E_{\Theta_{D,0}}) - P_b(E_{\Theta_{SR_1}})P_b(E_{\Theta_{D,0}}),$$

where $P_b(E_{\Theta_{SR_1}})$ and $P_b(E_{\Theta_{D,0}})$ denote the BEPs of Θ_{SR_1} and Θ_D, respectively, at $\mathcal{S}$. Applying (14), $P_b(E_{\Theta_{SR_1}})$ and $P_b(E_{\Theta_{D,0}})$ can be given by

$$P_b(E_{\Theta_{SR_1}}) = \phi(\gamma_{R_1S}),$$

$$P_b(E_{\Theta_{D,0}}) = \phi(\gamma_{DS}).$$

Therefore, we obtain

$$\zeta'_{00} = \phi(\gamma_{R_1S})\phi(\gamma_{DS}) = \zeta_{00},$$

$$\zeta'_{01} = \phi(\gamma_{R_1S})[1-\phi(\gamma_{DS})] = \zeta_{01},$$

$$\zeta'_{10} = [1-\phi(\gamma_{R_1S})]\phi(\gamma_{DS}) = \zeta_{10},$$

$$\zeta'_{11} = \phi(\gamma_{R_1S}) + \phi(\gamma_{DS}) - \phi(\gamma_{R_1S})\phi(\gamma_{DS}) = \zeta_{11}.$$

We observe that $\xi'_i, i = 0, 1$, depends only on the estimation of Θ_D at $\mathcal{R}_1$. Thus, ξ'_i can be given by

$$\xi'_0 = \xi'_1 = P_b(E_{\Theta_{D,1}}),$$

where $P_b(E_{\Theta_{D,1}})$ denotes the BEP of Θ_D at $\mathcal{R}_1$. From (14), we obtain

$$\xi'_0 = \xi'_1 = \phi(\gamma_{DR_1}) = \xi.$$

Finally, we obtain a closed-form expression for the RDEP at $\mathcal{S}$ and $\mathcal{R}_1$ as (15a) and (15b), respectively. ■

Remark 3 (*Impact of Transmission Links on RDEP at* $\mathcal{S}$)**.** As seen from (15a), RDEP at $\mathcal{S}$ is influenced by the qualities of all outgoing forward links (i.e., $\mathcal{S} \to \mathcal{R}_1$, $\mathcal{R}_1 \to \mathcal{D}$, and $\mathcal{S} \to \mathcal{D}$) and two incoming backward links (i.e., $\mathcal{R}_1 \to \mathcal{S}$ and $\mathcal{D} \to \mathcal{S}$). Specifically, $P_b(E_{\Omega_S})$ monotonically increases over α, β, $\phi(\gamma_{R_1S})$, or $\phi(\gamma_{DS})$. This can be seen by taking the derivative of $P_b(E_{\Omega_S})$ with respect to α, β, $\phi(\gamma_{R_1S})$, and $\phi(\gamma_{DS})$ as follows:

$$\frac{\partial P_b(E_{\Omega_S})}{\partial \alpha} = \phi(\gamma_{DS})\beta + \phi(\gamma_{DS})[1-2\phi(\gamma_{R_1S})](1-\beta) \geqslant 0,$$
$$\frac{\partial P_b(E_{\Omega_S})}{\partial \beta} = \phi(\gamma_{R_1S})\alpha + \phi(\gamma_{R_1S})[1-2\phi(\gamma_{DS})](1-\alpha) \geqslant 0,$$
$$\frac{\partial P_b(E_{\Omega_S})}{\partial \phi(\gamma_{R_1S})} = [1-\phi(\gamma_{DS})]\beta + \phi(\gamma_{DS})(1-\beta)(1-2\alpha) \geqslant 0,$$
$$\frac{\partial P_b(E_{\Omega_S})}{\partial \phi(\gamma_{DS})} = [1-\phi(\gamma_{R_1S})]\alpha + \phi(\gamma_{R_1S})(1-\alpha)(1-2\beta) \geqslant 0.$$

This implies that if the quality of any forward and backward links $\mathcal{S} \to \mathcal{R}_1$, $\mathcal{R}_1 \to \mathcal{D}$, $\mathcal{S} \to \mathcal{D}$, $\mathcal{R}_1 \to \mathcal{S}$, and $\mathcal{D} \to \mathcal{S}$ is improved, lower determination error of retransmissions at $\mathcal{S}$ is expected.

In fact, it can be drawn from an intuitive observation that the quality of any outgoing and incoming links at $\mathcal{S}$ influences, in a monotonically increasing manner, the RDEP at $\mathcal{S}$.

Remark 4 (*Impact of Transmission Links on RDEP at $\mathcal{R}_1$*)**.** As seen from (15b), RDEP at $\mathcal{R}_1$ is influenced by the qualities of two incoming links including a forward link $\mathcal{S} \to \mathcal{R}_1$ and a backward link $\mathcal{D} \to \mathcal{R}_1$. However, RDEP at $\mathcal{R}_1$ is independent of the outgoing links (i.e., $\mathcal{R}_1 \to \mathcal{S}$ and $\mathcal{R}_1 \to \mathcal{D}$). Specifically, $P_b(E_{\Omega_{R_1}})$ monotonically increases over ξ but monotonically decreases over α. This means that the reliability of the determination of packets to be retransmitted at $\mathcal{R}_1$ would be improved if either the quality of the backward link $\mathcal{D} \to \mathcal{R}_1$ increases or that of the forward link $\mathcal{S} \to \mathcal{R}_1$ deteriorates. In fact, we can intuitively observe that the increase of the quality of backward link $\mathcal{D} \to \mathcal{R}_1$ obviously improves the RDEP at $\mathcal{R}_1$, and $\mathcal{R}_1$ would be released from the responsibility of helping $\mathcal{S}$ retransmit a packet to $\mathcal{D}$ if this packet received from $\mathcal{S}$ is corrupted. Thus, if the number of corrupted packets received at $\mathcal{R}_1$ from $\mathcal{S}$ increases, i.e., α increases, the RDEP at $\mathcal{R}_1$ would decrease. However, it should be noted that if α increases, $P_b(E_{\Omega_S})$ would increase as well, as discussed in Remark 3.

Remark 5 (*Lower RDEP at $\mathcal{S}$ and $\mathcal{R}_1$*)**.** Our proposed NC-based block ACK scheme has a lower RDEP at $\mathcal{S}$ and $\mathcal{R}_1$ than the non-NC-based scheme. This observation confirms the statement in Remark 1. Following the non-NC-based block ACK scheme, the BEPs of Ω_S and Ω_{R_1} can be derived as

$$P_b(E_{\Omega'_S}) = \zeta_{11}\alpha\mu + \zeta_{01}(1-\alpha)\mu + \zeta_{10}\alpha(1-\mu) + \zeta_{00}(1-\alpha)(1-\mu), \tag{16a}$$

$$P_b(E_{\Omega'_{R_1}}) = \nu_1(1-\alpha)\beta + \nu_0(1-\alpha)(1-\beta), \tag{16b}$$

respectively, where

$$\mu \triangleq \Pr(b_{SD} = 1),$$

$$\nu_1 \triangleq \Pr(\hat{b}_{R_1D} \otimes \hat{b}_{SD} \otimes \bar{b}_{SR_1} = 0 | b_{SR_1} = 0, b_{R_1D} = 1, b_{SD} = 1),$$

$$\nu_0 \triangleq \Pr(\hat{b}_{R_1D} \otimes \hat{b}_{SD} \otimes \bar{b}_{SR_1} = 1 | b_{SR_1} = 0, b_{R_1D} \otimes b_{SD} = 0).$$

Similar to the proof of Lemma 1, ν_1 and ν_0 can be found as

$$\nu_0 = \nu_1 = \nu = \phi(\gamma_{DR_1})[1-\phi(\gamma_{DR_1})] + [1-\phi(\gamma_{DR_1})]\phi(\gamma_{DR_1}) + [\phi(\gamma_{DR_1})]^2 = 2\phi(\gamma_{DR_1}) - [\phi(\gamma_{DR_1})]^2.$$

Thus, (16b) can be rewritten as

$$P_b(E_{\Omega'_{R_1}}) = \nu(1-\alpha).$$

It can be seen that Pr$(b_{SD}=1) >$ Pr$(b_D=1) =$ Pr$(b_{SD}=1)$Pr$(b_{R_1D}=1)$, i.e., $\mu > \beta$, and $2\phi(\gamma_{DR_1}) - [\phi(\gamma_{DR_1})]^2 > \phi(\gamma_{DR_1})$, i.e., $\nu > \xi$. Thus, $P_b(E_{\Omega'_S})$ and $P_b(E_{\Omega'_{R_1}})$ in (16a) and (16b) are greater than $P_b(E_{\Omega_S})$ and $P_b(E_{\Omega_{R_1}})$ in (15a) and (15b), respectively.

To understand further the behaviour of the error probabilities in (15a) and (15b), we consider some asymptotic scenarios of forward links where links $\mathcal{S} \to \mathcal{R}_1$ and $\mathcal{R}_1 \to \mathcal{D}$ are considered at either very low or high SNR (see Table I)[3]. We assume that the direct link, $\mathcal{S} \to \mathcal{D}$, has a very low SNR (i.e., $\gamma_{SD} \to 0$) (as this is the main motivation for using relay-assisted cooperative transmissions). These asymptotic scenarios allow us to extend our error probability analysis to an N-relay network.

TABLE I

SPECIFIC ANALYSIS SCENARIOS

Scenario 1	High-SNR $\mathcal{S} \to \mathcal{R}_1$	High-SNR $\mathcal{R}_1 \to \mathcal{D}$
Scenario 2	High-SNR $\mathcal{S} \to \mathcal{R}_1$	Low-SNR $\mathcal{R}_1 \to \mathcal{D}$
Scenario 3	Low-SNR $\mathcal{S} \to \mathcal{R}_1$	High-SNR $\mathcal{R}_1 \to \mathcal{D}$

A. *Scenario 1: High-SNR $\mathcal{S} \to \mathcal{R}_1$ and high-SNR $\mathcal{R}_1 \to \mathcal{D}$*

In this scenario, $\gamma_{SR_1} \to \infty$, $\gamma_{R_1D} \to \infty$, $\gamma_{SD} \to 0$, $\alpha \approx 0$ and $\beta \approx 0$. Thus, $P_b(E_{\Omega_S})$ and $P_b(E_{\Omega_{R_1}})$ can be approximated as

$$P_b(E_{\Omega_S}) \approx \zeta_{00} = \phi(\gamma_{R_1S})\phi(\gamma_{DS}), \tag{17a}$$

$$P_b(E_{\Omega_{R_1}}) \approx \xi = \phi(\gamma_{DR_1}). \tag{17b}$$

Extended to an N-relay network, $P_b^{(N)}(E_{\Omega_S})$ can be computed by

$$\begin{aligned} P_b^{(N)}(E_{\Omega_S}) \approx \zeta_{00}^{(N)} &= \Pr\{\hat{a}_S = 1 | b_{SR^{(N)}} = 0, b_D = 0\} \\ &= \phi(\gamma_{R_1S})\phi(\gamma_{R_2S})\cdots\phi(\gamma_{R_NS})\phi(\gamma_{DS}). \end{aligned} \tag{18}$$

[3]Note that we do not consider the scenario where both links $\mathcal{S} \to \mathcal{R}_1$ and $\mathcal{R}_1 \to \mathcal{D}$ are at low SNR because it is expected that the relay node is in a reasonable condition for relaying.

Similar to a one-relay network, $P_b^{(N)}(E_{\Omega_{R_j}}), j = 1, 2, \ldots, N$, is given by

$$P_b^{(N)}(E_{\Omega_{R_j}}) \approx \phi(\gamma_{DR_j}). \tag{19}$$

B. Scenario 2: High-SNR $\mathcal{S} \to \mathcal{R}_1$ and low-SNR $\mathcal{R}_1 \to \mathcal{D}$

In this scenario, $\gamma_{SR_1} \to \infty$, $\gamma_{R_1D} \to 0$, $\gamma_{SD} \to 0$, $\alpha \approx 0$ and $\beta \approx 1/4$. Thus, $P_b(E_{\Omega_S})$ and $P_b(E_{\Omega_{R_1}})$ can be approximated as

$$P_b(E_{\Omega_S}) \approx \frac{1}{4}\zeta_{01} + \frac{3}{4}\zeta_{00} = \frac{1}{4}\phi(\gamma_{R_1S})\left[1 + 2\phi(\gamma_{DS})\right], \tag{20a}$$

$$P_b(E_{\Omega_{R_1}}) \approx \xi = \phi(\gamma_{DR_1}). \tag{20b}$$

Extended to an N-relay network, $P_b^{(N)}(E_{\Omega_S})$ can be computed by

$$P_b^{(N)}(E_{\Omega_S}) \approx \frac{1}{2^{N+1}}\zeta_{01}^{(N)} + \frac{2^{N+1}-1}{2^{N+1}}\zeta_{00}^{(N)}, \tag{21}$$

where $\zeta_{00}^{(N)}$ is given by (18) and

$$\begin{aligned}\zeta_{01}^{(N)} &= \Pr\left\{\hat{a}_S = 1 | b_{SR^{(N)}} = 0, b_D = 1\right\} \\ &= \phi(\gamma_{R_1S})\phi(\gamma_{R_2S})\cdots\phi(\gamma_{R_NS})[1 - \phi(\gamma_{DS})].\end{aligned} \tag{22}$$

Similar to a one-relay network, $P_b^{(N)}(E_{\Omega_{R_j}}), j = 1, 2, \ldots, N$, is given by

$$P_b^{(N)}(E_{\Omega_{R_j}}) \approx \phi(\gamma_{DR_j}). \tag{23}$$

C. Scenario 3: Low-SNR $\mathcal{S} \to \mathcal{R}_1$ and High-SNR $\mathcal{R}_1 \to \mathcal{D}$

In this scenario, $\gamma_{SR_1} \to 0$, $\gamma_{R_1D} \to \infty$, $\gamma_{SD} \to 0$, $\alpha \approx 1/2$ and $\beta \approx 0$. Thus, $P_b(E_{\Omega_S})$ and $P_b(E_{\Omega_{R_1}})$ can be approximated as

$$P_b(E_{\Omega_S}) \approx \frac{1}{2}\zeta_{10} + \frac{1}{2}\zeta_{00} = \frac{1}{2}\phi(\gamma_{DS}), \tag{24a}$$

$$P_b(E_{\Omega_{R_1}}) \approx \frac{1}{2}\xi = \frac{1}{2}\phi(\gamma_{DR_1}). \tag{24b}$$

Extended to an N-relay network, $P_b^{(N)}(E_{\Omega_S})$ can be computed by

$$P_b^{(N)}(E_{\Omega_S}) \approx \frac{1}{2^N}\zeta_{10}^{(N)} + \frac{2^N-1}{2^N}\zeta_{00}^{(N)}, \tag{25}$$

where $\zeta_{00}^{(N)}$ is given by (18) and

$$\begin{aligned}\zeta_{10}^{(N)} &= \Pr\left\{\hat{a}_S = 1 | b_{SR^{(N)}} = 1, b_D = 0\right\} \\ &= [1 - \phi(\gamma_{R_1S})][1 - \phi(\gamma_{R_2S})]\cdots[1 - \phi(\gamma_{R_NS})]\phi(\gamma_{DS}).\end{aligned} \tag{26}$$

Similar to a one-relay network, $P_b^{(N)}(E_{\Omega_{R_j}}), j = 1, 2, \ldots, N$, is given by

$$P_b^{(N)}(E_{\Omega_{R_j}}) \approx \frac{1}{2}\phi(\gamma_{DR_j}). \tag{27}$$

Remark 6 (*Comparison of Scenarios*)**.** Let us investigate the sum-RDEP of the whole system defined by $P_b(E) \triangleq P_b(E_{\Omega_S}) + P_b(E_{\Omega_{R_1}})$. It can be observed that a high SNR of the forward links in Scenario 1 leads to a lower $P_b(E)$ compared to Scenario 2. However, this is not always the case when compared to Scenario 3. For convenience, let $P_{b,i}(E)$ denote the $P_b(E)$ of Scenario i and let $\delta_{ij} \triangleq P_{b,i}(E) - P_{b,j}(E), i, j \in \{1, 2, 3\}$. We have

$$\delta_{12} = P_{b,1}(E) - P_{b,2}(E) = \frac{1}{4}\phi(\gamma_{R_1S})\left[2\phi(\gamma_{DS}) - 1\right],$$

$$\delta_{13} = P_{b,1}(E) - P_{b,3}(E) = \frac{1}{2}\phi(\gamma_{DS})[2\phi(\gamma_{R_1S}) - 1] + \frac{1}{2}\phi(\gamma_{DR_1}),$$

$$\delta_{23} = P_{b,2}(E) - P_{b,3}(E) = \frac{1}{4}\phi(\gamma_{R_1S})\left[1 + 2\phi(\gamma_{DS})\right] - \frac{1}{2}\phi(\gamma_{DS}) + \frac{1}{2}\phi(\gamma_{DR_1}).$$

It can be seen that $\delta_{12} < 0$ for all $\gamma_{R_1S}, \gamma_{DS}, \gamma_{DR_1}$. On the other hand, the other two functions, δ_{13} and δ_{23}, can be zero at some values of γ_{R_1S}, γ_{DR_1} or γ_{DS}. In particular, in these equations, $\delta_{13} = 0$ and $\delta_{23} = 0$ have only one root with respect to either γ_{R_1S}, γ_{DR_1} or γ_{DS}. This clearly shows that the sum-RDEP, $P_b(E)$, in Scenario 2 can be lower or higher than that in Scenario 3 depending on the values of γ_{R_1S}, γ_{DR_1}, and γ_{DS}, which is understandable. However, the same cannot be said for the result with δ_{13} which shows that the sum-RDEP in Scenario 3 can be lower than that in Scenario 1, which is surprising. Actually, this is implied by Remark 4, where it is shown that $P_b(E_{\Omega_{R_1}})$ can be lower as the SNR of link $\mathcal{S} \rightarrow \mathcal{R}_1$ is lower. This behaviour will be further confirmed by simulations in Section V.

V. Numerical Results

In this section, we present simulation results of the RDEP and the average number of packets to be retransmitted at the source and relay nodes for different block ACK schemes when both forward and backward channels experience Rayleigh flat fading. Computer simulations are carried out for a typical one-relay network consisting of three nodes $\mathcal{S}$, $\mathcal{R}_1$, and $\mathcal{D}$ with BPSK for signaling and no channel coding. At $\mathcal{S}$ and $\mathcal{R}_1$, errors occur if the packets required to be retransmitted are different from the actually retransmitted packets.

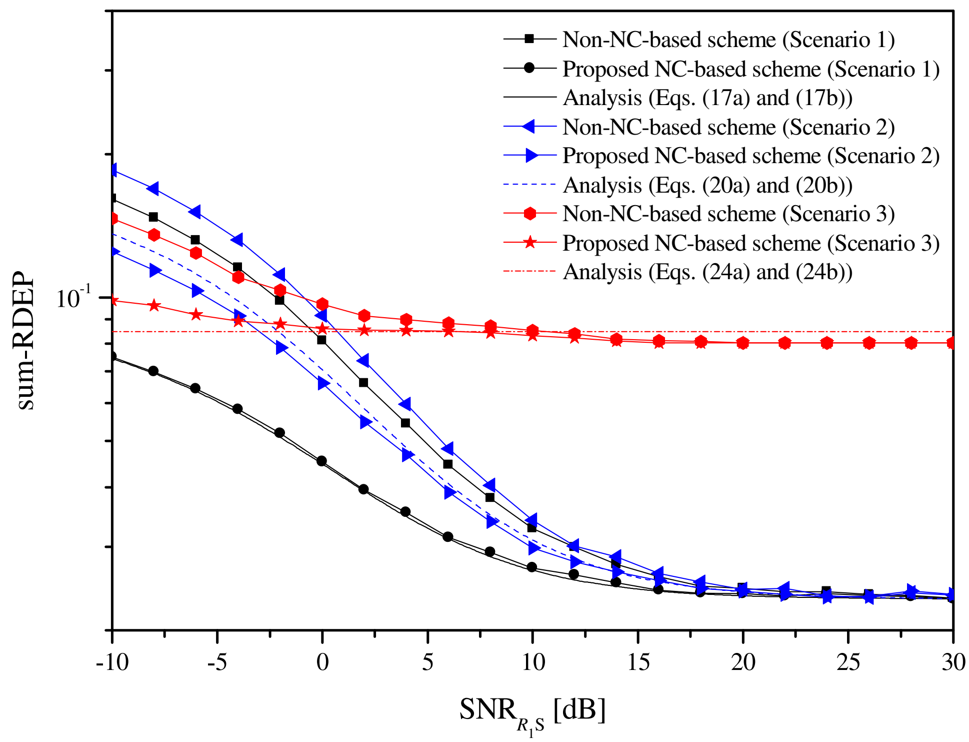


Fig. 3. Sum-RDEP versus SNR_{R_1S}.

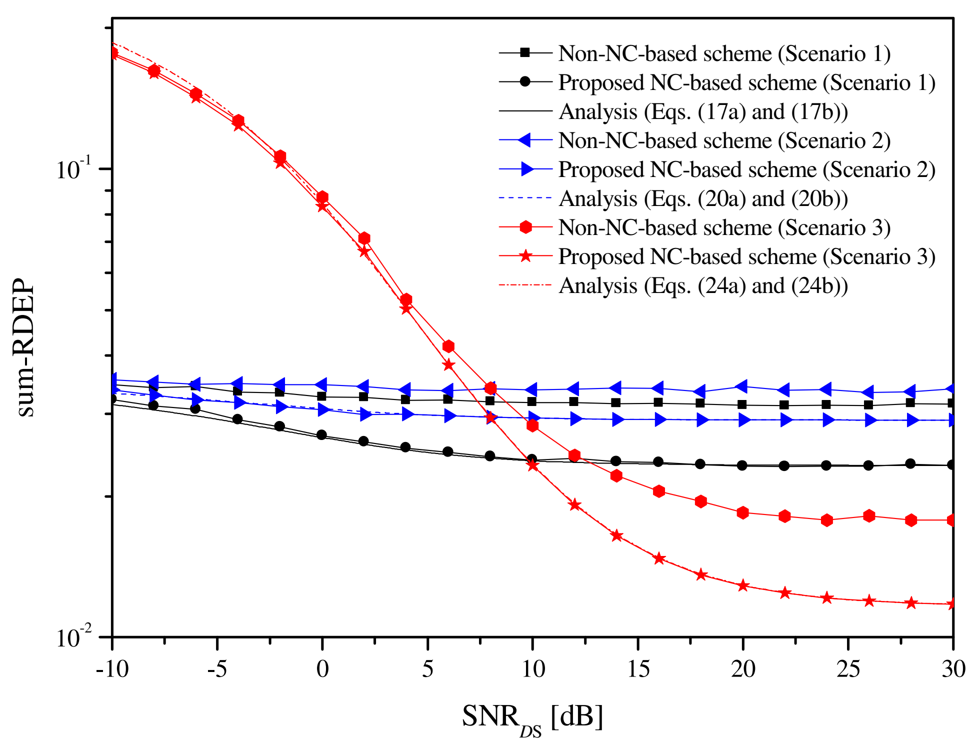


Fig. 4. Sum-RDEP versus SNR_{DS}.

Investigating the whole system, in Fig. 3, the sum-RDEP, i.e., the summation of BEPs of Ω_S and Ω_{R_1}, is shown for various values of the SNR of link $\mathcal{R}_1 \to \mathcal{S}$ with respect to the following scenarios of the forward links.

- Scenario 1: $\gamma_{SR_1} = 20$ dB, $\gamma_{R_1D} = 20$ dB, and $\gamma_{SD} = -20$ dB
- Scenario 2: $\gamma_{SR_1} = 20$ dB, $\gamma_{R_1D} = -20$ dB, and $\gamma_{SD} = -20$ dB
- Scenario 3: $\gamma_{SR_1} = -20$ dB, $\gamma_{R_1D} = 20$ dB, and $\gamma_{SD} = -20$ dB

The SNRs of the other backward links $\mathcal{D} \to \mathcal{R}_1$ and $\mathcal{D} \to \mathcal{S}$ are assumed to be 10 dB and 0 dB, respectively. First of all, it can be observed in Fig. 3 that the sums of the error probabilities at $\mathcal{S}$ and $\mathcal{R}_1$ given by (17a) and (17b), (20a) and (20b), (24a) and (24b) are consistent with the simulation results. As expected, a better performance is achieved in Scenario 1 compared to Scenario 2. However, when comparing Scenario 1 and Scenario 2 with Scenario 3, we can not reach such an explicit conclusion since there are cross-over points among the $P_b(E)$ curves. This observation confirms the statement in Remark 6 where the performance of these three scenarios is theoretically compared, i.e., we always achieve a better performance with Scenario 1 when compared to Scenario 2 but we can not conclude the absolute relationship when comparing Scenario 1 or Scenario 2 with Scenario 3. We can also see that $P_b(E)$ in Scenario 3 does not depend on γ_{R_1S} as shown in (24a) and (24b). Finally, it can be seen that the performance of our proposed NC-based block ACK scheme is better than the non-NC-based scheme for all scenarios. This performance improvement, as explained in Remarks 1 and 5, is achieved by the reduced number of block ACK transmissions in our proposed NC-based block ACK scheme.

The impact of SNR of link $\mathcal{D} \to \mathcal{S}$ on the sum-RDEP is shown in Fig. 4, where the SNRs of the other backward links, $\mathcal{R}_1 \to \mathcal{S}$ and $\mathcal{D} \to \mathcal{R}_1$, are assumed to be equal to 10 dB. The behaviour which was discussed in Remarks 1, 5 and 6 can be observed, i.e., the better performance is achieved with our proposed NC-based scheme for all three scenarios and the relationship between these scenarios are confirmed. In addition, since γ_{R_1S} and γ_{DR_1} are fixed at 10 dB which can be assumed to be a high SNR level, the sum-RDEPs in Scenario 1 and Scenario 2 are close to each other and do not depend particularly on γ_{DS}. On the other hand, the sum-RDEP in Scenario 3 depends only on γ_{DS} and γ_{DR_1}, and as such, there is a significant improvement on the sum-RDEP when γ_{DS} increases.

For the comparison of the average number of data retransmissions required for the whole system to transmit one packet from the source to the destination using different block ACK

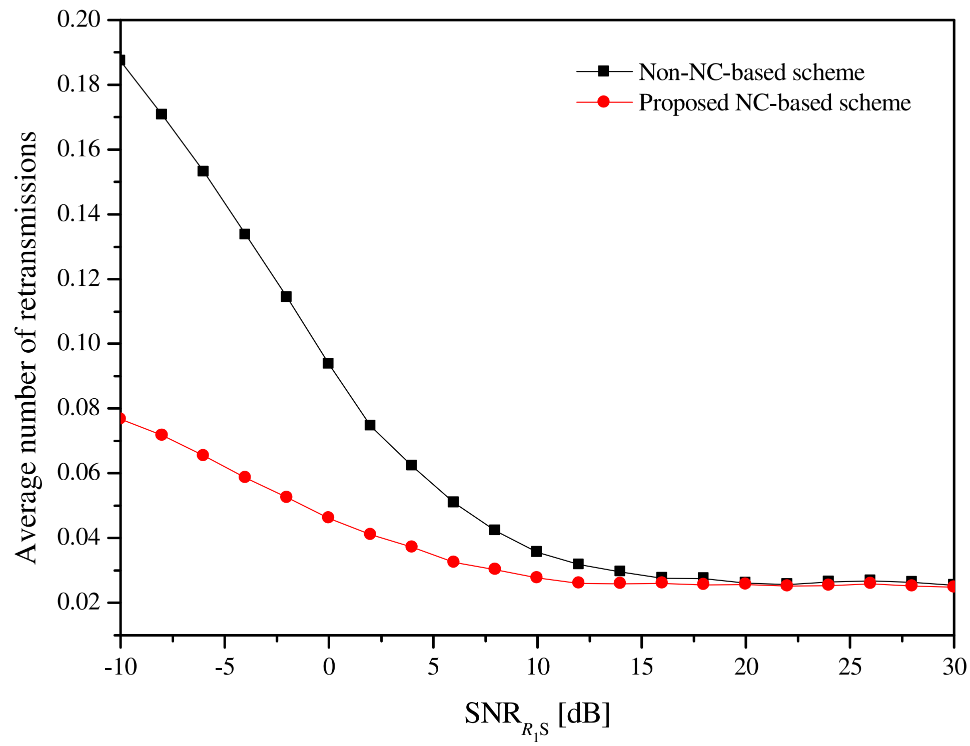


Fig. 5. Average number of data retransmissions per packet versus SNR$_{R_1S}$ for Scenario 1.

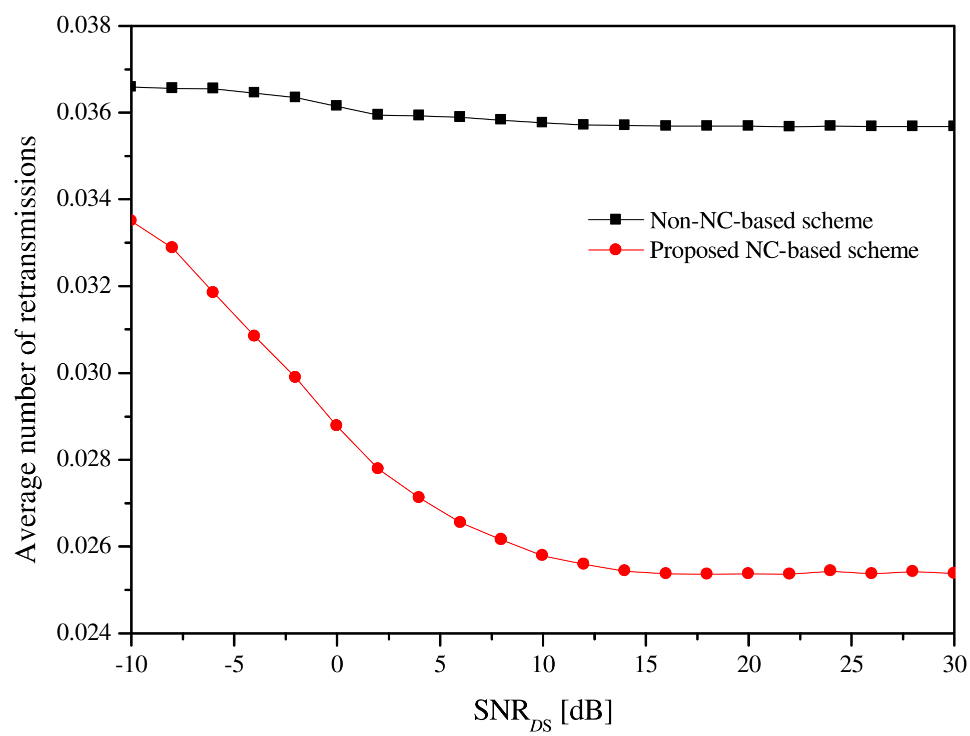


Fig. 6. Average number of data retransmissions per packet versus SNR$_{DS}$ for Scenario 1.

schemes, let us consider Scenario 1 with the similar assumption of the SNRs of the forward and backward links. In Figs. 5 and 6, the average number of data retransmissions is plotted versus the SNR of backward links $\mathcal{R}_1 \rightarrow \mathcal{S}$ and $\mathcal{D} \rightarrow \mathcal{S}$, respectively. It can be seen that our proposed NC-based block ACK scheme reduces the average number of data retransmissions compared with the non-NC-based scheme over the backward links. We observe that the reduction of the number of packets to be retransmitted in Figs. 5 and 6 is corresponding to the lower sum-RDEPs achieved with our proposed NC-based scheme for Scenario 1 in Figs. 3 and 4, respectively. This significant improvement not only reflects the high reliability of our proposed NC-based scheme in the determination of packets to be retransmitted which is stated in Remarks 1 and 5, but also implies the improvement of system throughput with our proposed NC-based block ACK scheme.

VI. Conclusions

In this paper, we have proposed a network coding (NC)-based block acknowledgement scheme for multi-relay based cooperative networks. Using the notion of NC, the source and relay nodes can simultaneously determine the data packets to be retransmitted with a reduced number of block ACK packets. This NC-based block ACK scheme can effectively reduce the number of block ACK packets sent from the destination. This reduction results in a lower computational complexity, a more reliable determination of packets to be retransmitted, and a decreased number of data retransmissions at the source and relays compared to the non-NC-based block ACK scheme. Reduced number of retransmissions actually means freeing up more bandwidth and increasing overall network throughput. An error probability analysis for the determination of packets to be retransmitted has been carried out with respect to the SNR of forward and backward links. The derived expression of error probability reflects well the impact of the quality of both the forward and backward links upon the performance of block ACK schemes. Furthermore, general expressions for multi-relay networks have been derived for three asymptotical scenarios of forward links. In addition, simulations have been carried out which have confirmed the analytical results. For future work, opportunistic block ACK transmission schemes will be investigated for multi-relay networks where relay selection is taken into consideration. Also, we will investigate the delay time performance achieved with our proposed block ACK scheme for specific scenario implementations.

REFERENCES

[1] Kim, Y., Choi, S., Jang, K., Hwang, H.: 'Throughput enhancement of IEEE 802.11 WLAN via frame aggregation,' IEEE VTC'04-Fall, Los Angeles, CA, USA, 2004, 4, pp. 3030–3034.

[2] Cabral, O., Segarra, A., Velez, F., Mihovska, A., Prasad, N.: 'Optimization of multi-service IEEE802.11e block acknowledgement,' IEEE RWS'09, San Diego, CA, USA, 2009, pp. 380–383.

[3] Xiao, Y.: 'Packing mechanisms for the IEEE 802.11n wireless LANs,' IEEE GLOBECOM'04, Dallas, Texas, USA, 2004, 5, pp. 3275–3279.

[4] Nakajima, T., Nabetani, T., Utsunomiya, Y., Adachi, T., Takagi, M.: 'A simple and efficient selective repeat scheme for high throughput WLAN, IEEE802.11n,' IEEE VTC'07-Spring, Dublin, Ireland, 2007, pp. 1302–1306.

[5] *IEEE Standard for Local and Metropolitan Area Networks - Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications - Amendment 5: Enhancements for Higer Throughput*, IEEE Std. 802.11n-2009, 2009.

[6] Sendonaris, A., Erkip, E., Aazhang, B.: 'User cooperation diversity - Part I. System description,' *IEEE Trans. Commun.*, 2003, 51, (11), pp. 1927–1938.

[7] Chang, J.-M., Hsiao, W.-T., Chen, J.-L., Chao, H.-C.: 'Mobile relay stations navigation-based self-optimization handover mechanism in WiMAX networks,' ICUT'09, Fukuoka, Japan, 2009.

[8] Chang, R.-S., Wang, S.-H.: 'Deploying relay nodes in WSNs by considering uneven energy consumption,' *Journal of Internet Technology*, 2012/3, 13, (2), pp. 273–280.

[9] Liu, Y.: 'A low complexity protocol for relay channels employing rateless codes and acknowledgement,' IEEE ISIT'06, Seattle, Washington, USA, 2006, pp. 1244–1248.

[10] Nagao, K., Kadowaki, Y., Yamao, Y.: 'Multi-hop transmission performance of cognitive temporary bypassing for wireless ad hoc networks,' IEEE CCNC'09, Las Vegas, Nevada, USA, 2009, pp. 1–5.

[11] Lu, K., Fu, S., Qian, Y.: 'Increasing the throughput of wireless LANs via cooperative retransmission,' IEEE GLOBECOM'07, Washington, DC, USA, 2007, pp. 5231–5235.

[12] Ahlswede, R., Cai, N., Li, S.-Y., Yeung, R.: 'Network information flow,' *IEEE Trans. Inf. Theory*, 2000, 46, (4), pp. 1204–1216.

[13] Zhang, S., Liew, S. C., Lam, P. P.: 'Hot topic: Physical-layer network coding,'ACM MobiCom'06, Los Angeles, CA, USA, 2006, pp. 358–365.

[14] Chung, T.-Y., Wang, C.-C., Chen, Y.-M., Chang, Y.-H.: 'PNECOS: A peer-to-peer network coding streaming system,' *Journal of Internet Technology*, 2009/7, 10, (3), pp. 261–270.

[15] Simon, M. K., Alouini, M.-S.: *Digital Communication over Fading Channels*, 2nd ed. John Wiley & Sons, 2005.