Correspondence

SIC-Based Detection With List and Lattice Reduction for MIMO Channels

Jinho Choi and Huan Nguyen

***Abstract*—To derive low-complexity multiple-input–multiple-output (MIMO) detectors, we combine two complementary approaches, i.e., lattice reduction (LR) and list within the framework of the successive interference cancellation (SIC)-based detection. It is shown that the performance of the proposed detector, which is called the SIC-based detector with list and LR, can approach that of the maximum-likelihood (ML) detector with a short list length. For example, the signal-to-noise ratio (SNR) loss of the proposed detector, compared with that of the ML detector, is less than 1 dB for a 4 $\times$ 4 MIMO system with 16-state quadrature amplitude modulation (QAM) at a bit error rate (BER) of 10^{-3} with a list length of 8.**

***Index Terms*—Lattice reduction (LR)-based detection, list detection, successive interference cancellation (SIC), multiple-input–multiple-output (MIMO) detection.**

I. Introduction

In wireless communications, it is well known that the channel capacity can linearly increase with the number of antennas (provided that the numbers of transmit and receive antennas are the same) [1], [2]. Thus, to increase the channel capacity, the transmitter and receiver can be equipped with multiple antennas, and the resulting channel becomes a multiple-input–multiple-output (MIMO) channel. Various space-time architectures for signal transmission over MIMO channels are proposed to effectively exploit spatial and temporal diversity gain in [3] and [4].

In general, since more symbols are transmitted in MIMO systems, the detection complexity can be high. For example, the complexity of maximum-likelihood (ML) detection exponentially increases with the number of transmit antennas. Thus, various approaches are devised to reduce the complexity. The successive interference cancellation (SIC) approach is employed in [4]. The relation between SIC-based MIMO detection and the decision feedback equalizer (DFE) is exploited in [5]. In [6], the partial maximum *a posteriori* probability (MAP) principle is derived to discuss the optimality of SIC-based detection. List detectors are also considered for MIMO detection to obtain a soft decision in [7] and [8] based on [9].

In [10], a lattice reduction (LR)-based MIMO detector used as a low-complexity MIMO detector is first discussed. In [11], more LR-based MIMO detectors are proposed. It is shown that the performance of LR-based MIMO detectors using minimum mean square error (MMSE)-SIC approaches ML performance. An overview of LR-based detection can be found in [12]. In [13] and [14], it is shown that LR-based detection can achieve full diversity. This is an important observation as most low-complexity suboptimal MIMO detectors could not exploit full diversity. It is noteworthy that a soft decision can also be obtained from the LR-based detection [15].

Although the Lenstra–Lenstra–Lovasz (LLL) algorithm, which is one of the LR algorithms, has a polynomial (average) complexity (for a certain class of random channel matrices) [16], [17], the complexity increases relatively rapidly with the number of basis vectors (or the number of transmit antennas). Thus, for a large MIMO system, the computational complexity of the LR-based detection would still be high. To further reduce the complexity, we can decompose a large MIMO detection problem into multiple small MIMO subdetection problems with SIC, as in [6]. Due to SIC, this approach would suffer from error propagation. To mitigate error propagation, the list detection approach can be adopted. The resulting detector has low complexity as the number of basis vectors in the subdetection problem is small. Due to list detection, the proposed detector can enjoy the tradeoff between complexity and performance, i.e., it has better mitigation against error propagation as the list length increases at the expense of increasing complexity.

II. System Model

Suppose that there are K transmit antennas and N receive antennas. The $N \times 1$ received signal vector $\mathbf{r}$ is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{H}$, $\mathbf{s}$, and $\mathbf{n}$ are the $N \times K$ channel matrix, $K \times 1$ transmitted signal vector, and $N \times 1$ noise vector, respectively. We assume that $\mathbf{n}$ is a zero-mean circular complex Gaussian random vector with $E[\mathbf{n}\mathbf{n}^H] = N_0\mathbf{I}$. Let $\mathcal{S}$ denote the signal alphabet for symbols, i.e., $s_k \in \mathcal{S}$, where s_k is the kth element of $\mathbf{s}$, and its size is denoted by M, i.e., $M = |\mathcal{S}|$.

We assume that $N \geq K$ and consider the QR factorization of the channel matrix as $\mathbf{H} = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is unitary, and $\mathbf{R}$ is upper triangular. We have

$$\mathbf{x} = \mathbf{Q}^H\mathbf{r} = \mathbf{R}\mathbf{s} + \mathbf{Q}^H\mathbf{n}. \tag{2}$$

Since the statistical properties of $\mathbf{Q}^H\mathbf{n}$ are identical to those of $\mathbf{n}$, $\mathbf{Q}^H\mathbf{n}$ will be denoted by $\mathbf{n}$. If $N = K$, there are no zero rows in $\mathbf{R}$; otherwise, the last $N - K$ rows become zero. Thus, the last $N - K$ elements of $\mathbf{x}$ would be ignored for the detection if $N > K$. If there is no risk of confusion, hereinafter, we assume that the sizes of $\mathbf{x}$, $\mathbf{R}$, and $\mathbf{n}$ are $K \times 1$, $K \times K$, and $K \times 1$, respectively.

III. SIC-List-LR Based Detection

The LR-based detectors in [10] and [11] have near-ML performance with relatively low complexity. It is shown that those LR-based detectors can achieve full diversity gain, just like the ML detector in [13] and [14]. Unfortunately, however, the complexity of LR can rapidly increase with the number of basis vectors, which implies that the complexity of the LR-based detectors may not be reasonably low for a large MIMO system. To avoid this problem, in this section, we propose an SIC-list-LR-based detection method within the framework of the partial MAP detection in [6]. The main idea of this method is to break a high-dimensional MIMO detection problem into multiple lower dimensional MIMO subdetection problems so that the complexity

Manuscript received June 29, 2008; revised November 14, 2008, December 19, 2008, and January 12, 2009. The review of this paper was coordinated by Dr. A. Ghrayeb.

The authors are with the Institute of Advanced Telecommunications, University of Wales Swansea, SA2 8PP Swansea, U.K. (e-mail: j.choi@swansea.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2009.2013472

0018-9545/$25.00 © 2009 IEEE

associated with LR can be reduced. The notion of the partial MAP detection [6] is applied to include multiple lower dimensional MIMO subdetection problems, together with the list detection approach.

To perform the proposed LR and list-based detection, we consider the partition of $\mathbf{x}$ as follows:

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_3 \\ \mathbf{0} & \mathbf{R}_2 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix} \tag{3}$$

where $\mathbf{x}_i$, $\mathbf{s}_i$, and $\mathbf{n}_i$ are the $K_i \times 1$ ith subvectors of $\mathbf{x}$, $\mathbf{s}$, and $\mathbf{n}$, $i = 1, 2$, respectively. Note that $K_1 + K_2 = K$. From (3), we can have two lower dimensional MIMO subdetection problems to detect $\mathbf{s}_1$ and $\mathbf{s}_2$. It is straightforward to extend the partition into more than two groups. However, for the sake of simplicity, we only consider the partition into two groups, as in (3).

A. Algorithm Description

In the proposed SIC-list-LR-based detection, the subdetection of $\mathbf{s}_2$ is carried out first using the LR-based detector. Then, a list of candidate vectors of $\mathbf{s}_2$ is generated. With the list of $\mathbf{s}_2$, the subdetection of $\mathbf{s}_1$ is performed with the LR-based detector. The candidate vector in the list is used for the SIC to mitigate the interference from $\mathbf{s}_2$. The proposed SIC-list-LR-based detection is summarized here.

S1) The LR-based detection of $\mathbf{s}_2$ is performed with the received signal $\mathbf{x}_2$, i.e.,

$$\tilde{\mathbf{c}}_2 = \mathrm{LRDet}(\mathbf{x}_2) \tag{4}$$

where LRDet is the function of the LR detection operation, which will be discussed in Section III-B, and $\tilde{\mathbf{c}}_2$ is the estimated vector of $\mathbf{s}_2$ in the corresponding LR domain. Note that there is no interference from $\mathbf{s}_1$ in detecting $\mathbf{s}_2$.

S2) A list of candidate vectors in the LR domain is generated by

$$\mathcal{C}_2 = \mathrm{List}(\tilde{\mathbf{c}}_2) \tag{5}$$

where List is a function that chooses the Q closest vectors to $\tilde{\mathbf{c}}_2 (1 \le Q \le M^{K_2})$ in the LR domain. We will discuss the list generation in Section III-C.

S3) The list of candidates of $\mathbf{s}_2$, which is denoted by $\mathcal{S}_2$, can be converted from $\mathcal{C}_2$. For convenience, denote $\mathcal{S}_2 = \{\tilde{\mathbf{s}}_2^{(1)}, \tilde{\mathbf{s}}_2^{(2)}, \ldots, \tilde{\mathbf{s}}_2^{(Q)}\}$.

S4) Once $\mathcal{S}_2$ is available, the LR-based detection of $\mathbf{s}_1$ can be carried out with SIC, i.e.,

$$\mathbf{c}_1^{(q)} = \mathrm{LRDet}\left(\mathbf{x}_1 - \mathbf{R}_3 \tilde{\mathbf{s}}_2^{(q)}\right) \tag{6}$$

where $\tilde{\mathbf{s}}_2^{(q)}$ is the qth decision vector of $\mathbf{s}_2$ from list $\mathcal{S}_2$.

S5) Let $\tilde{\mathbf{s}}_1^{(q)}$ denote the signal vector corresponding to $\tilde{\mathbf{c}}_1^{(q)}$ in the LR domain and $\tilde{\mathbf{s}}^{(q)} = [(\tilde{\mathbf{s}}_1^{(q)})^T \quad (\tilde{\mathbf{s}}_2^{(q)})^T]^T$; the final decision of $\mathbf{s}$ is found as

$$\tilde{\mathbf{s}} = \arg \min_{q=1,2,\ldots,Q} \left\| \mathbf{x} - \mathbf{R}\tilde{\mathbf{s}}^{(q)} \right\|^2 . \tag{7}$$

Note that a soft decision is also available from the list generated in S5. There are Q candidate vectors for $\mathbf{s}$, and they can be used to approximate the log-likelihood ratio as a soft decision, as in [18]. In the succeeding sections, we will explain the proposed detection in detail.

B. LR-Based Detection

In this section, we describe the LR-based detection used in steps S1) and S4).

TABLE I
SIGNALS AND PARAMETERS FOR THE LR-BASED DETECTION IN (4) AND (6)

Steps	$\mathbf{y}$	$\mathbf{A}$	$\mathbf{z}$	$\tilde{\mathbf{c}}$	K_i
S1)	$\mathbf{x}_2$	$\mathbf{R}_2$	$\mathbf{s}_2$	$\tilde{\mathbf{c}}_2$	K_2
S4)	$\mathbf{x}_1 - \mathbf{R}_2\tilde{\mathbf{s}}_2^{(q)}$	$\mathbf{R}_1$	$\mathbf{s}_1$	$\tilde{\mathbf{c}}_1^{(q)}$	K_1

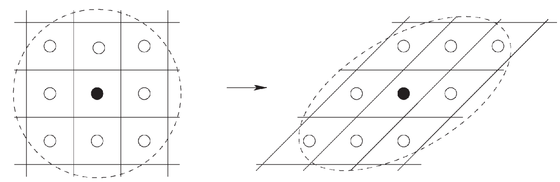

Fig. 1. List in different domains. (Left) $\mathcal{C}_2$ in the LR domain, which is orthogonal and where the black dot represents $\tilde{\mathbf{c}}_2$. (Right) $\mathcal{S}_2$ in the original domain, where the black dot represents $\tilde{\mathbf{s}}_2$.

Let $\mathbb{C}$ denote the set of complex integers or Gaussian integers $\mathbb{C} = \mathbb{Z} + j\mathbb{Z}$, where $\mathbb{Z}$ is the set of integers, and $j = \sqrt{-1}$. We assume that $\{\alpha s + \beta | s \in \mathcal{S}\} \subseteq \mathbb{C}$, where α and β are the scaling and shifting coefficients, respectively. For example, for M-QAM, if $M = 2^{2m}$, we have

$$\mathcal{S} = \{s = a + jb | a, b \in \{\pm A, \pm 3A, \ldots, \pm(2m-1)A\}\}$$

where $A = \sqrt{(3E_s/2(M-1))}$, and $E_s = E[|s|^2]$ is the symbol energy. Thus, $\alpha = 1/(2A)$, and $\beta = ((2m-1)/2)(1+j)$. Note that the pair of α and β is not uniquely decided.

Consider the MIMO detection with the following signal:

$$\mathbf{y} = \mathbf{A}\mathbf{z} + \mathbf{v} \tag{8}$$

where $\mathbf{A}$ is a MIMO channel matrix, $\mathbf{z} \in \mathcal{S}^{K_i}$ is the signal vector, and $\mathbf{v}$ is a zero-mean Gaussian noise with $E[\mathbf{v}\mathbf{v}^H] = N_0\mathbf{I}$. We scale and shift $\mathbf{y}$ as

$$\begin{aligned} \mathbf{d} &= \alpha\mathbf{y} + \beta\mathbf{A}\mathbf{1} \\ &= \mathbf{A}(\alpha\mathbf{z} + \beta\mathbf{1}) + \alpha\mathbf{v} \\ &= \mathbf{A}\mathbf{b} + \alpha\mathbf{v} \end{aligned} \tag{9}$$

where $\mathbf{1} = [1 \quad 1 \quad \cdots \quad 1]^T$, and $\mathbf{b} = \alpha\mathbf{z} + \beta\mathbf{1} \in \mathbb{C}^{K_i}$. Let

$$\bar{\mathbf{A}} = \mathbf{A}\mathbf{U} \tag{10}$$

where $\mathbf{U}$ is a unimodular matrix.[1] Using any LR algorithm, including the LLL algorithm [16], we can find the value of $\mathbf{U}$ that makes the column vectors of $\bar{\mathbf{A}}$ shorter. It follows that

$$\begin{aligned} \mathbf{d} &= \mathbf{A}\mathbf{U}\mathbf{U}^{-1}\mathbf{b} + \alpha\mathbf{v} \\ &= \bar{\mathbf{A}}\mathbf{c} + \alpha\mathbf{v} \end{aligned} \tag{11}$$

where $\mathbf{c} = \mathbf{U}^{-1}\mathbf{b}$. Note that, as the basis vectors are complex, we can use complex LR algorithms [19] or convert a complex matrix into a real matrix, as in [11]. The MMSE filter for estimating $\mathbf{c}$ is given by

$$\begin{aligned} \mathbf{W}_{\mathrm{MMSE}} &= \min_{\mathbf{W}} E\left[\left\|\mathbf{W}^H(\mathbf{d} - \bar{\mathbf{d}}) - (\mathbf{c} - \bar{\mathbf{c}})\right\|^2\right] \\ &= \left(\bar{\mathbf{A}}\mathrm{cov}(\mathbf{c})\bar{\mathbf{A}}^H + |\alpha|^2 N_0 \mathbf{I}\right)^{-1} \bar{\mathbf{A}}\mathrm{cov}(\mathbf{c}) \\ &= \left(\mathbf{A}\mathbf{A}^H \alpha^2 E_s + |\alpha|^2 N_0 \mathbf{I}\right)^{-1} \mathbf{A}\mathbf{U}^{-H} \alpha^2 E_s \end{aligned} \tag{12}$$

[1] A unimodular matrix is a square integer matrix with determinant ± 1.

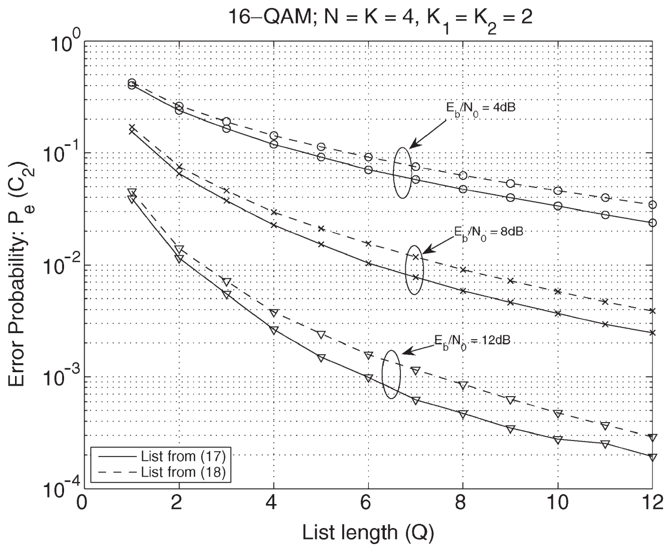

Fig. 2. Error probability with the list of $\mathbf{c}_2$ for various list lengths.

where $\bar{\mathbf{d}} = E[\mathbf{d}] = \beta\mathbf{A}\mathbf{1}$, $\bar{\mathbf{c}} = E[\mathbf{c}] = \mathbf{U}^{-1}\beta\mathbf{1}$, and $\mathrm{cov}(\mathbf{c}) = |\alpha|^2\mathbf{U}^{-1}\mathbf{U}^{-H}E_s$ since

$$\mathrm{cov}(\mathbf{c}) = \mathrm{cov}(\mathbf{U}^{-1}\mathbf{b}) = \alpha^2 E_s \mathbf{I}.$$

The estimate of $\mathbf{c}$ is given by

$$\tilde{\mathbf{c}} = \bar{\mathbf{c}} + \mathbf{W}_{\mathrm{MMSE}}^H(\mathbf{d} - \bar{\mathbf{d}}). \quad (13)$$

In Table I, the signals and parameters for the LR-based MMSE detection for each step are shown. Note that other approaches, including the LR-based MMSE-SIC detector in [11] or non-LR-based detectors, can also be used for subdetection.

C. List Generation in the LR Domain

To avoid or mitigate the error propagation, the use of a list of candidate vectors of $\mathbf{s}_2$ in detecting $\mathbf{s}_1$ is crucial. Using the ML metric, we can find the candidate vectors for the list $\mathcal{S}_2$. Let

$$f\left(\mathbf{r}\middle|\hat{\mathbf{s}}_2^{(1)}\right) \geq f\left(\mathbf{r}\middle|\hat{\mathbf{s}}_2^{(2)}\right) \geq \cdots \geq f\left(\mathbf{r}\middle|\hat{\mathbf{s}}_2^{(M^{K_2})}\right)$$

where $f(\mathbf{r}|\mathbf{s})$ is the likelihood function of $\mathbf{s}$ for a given $\mathbf{r}$, and $\hat{\mathbf{s}}_2^{(q)}$ is the symbol vector that corresponds to the qth largest likelihood. With log-likelihood values, we can also find the candidate vectors as follows:

$$\left\|\mathbf{r}-\mathbf{R}_2\hat{\mathbf{s}}_2^{(1)}\right\|^2 \leq \left\|\mathbf{r}-\mathbf{R}_2\hat{\mathbf{s}}_2^{(2)}\right\|^2 \leq \cdots \leq \left\|\mathbf{r}-\mathbf{R}_2\hat{\mathbf{s}}_2^{(M^{K_2})}\right\|^2.$$

Therefore, the ML-based list becomes

$$\mathcal{S}_2 = \left\{\hat{\mathbf{s}}_2^{(1)}, \hat{\mathbf{s}}_2^{(2)}, \ldots, \hat{\mathbf{s}}_2^{(Q)}\right\}. \quad (14)$$

However, for each log-likelihood value, we need to perform a matrix–vector multiplication. Thus, the resulting computational complexity could be high.

To avoid high computational complexity in generating the list, we can find a suboptimal list in the LR domain that can be obtained with a low complexity. Consider (9). According to Table I, let $\mathbf{A} = \mathbf{R}_2$, $\mathbf{d} = \alpha\mathbf{x}_2 + \beta\mathbf{A}\mathbf{1}$, and $\mathbf{b} = \alpha\mathbf{s}_2 + \beta\mathbf{1}$. Then, from (10), we have

$$\|\mathbf{r} - \mathbf{R}_2\mathbf{s}_2\| \propto \|\mathbf{d} - \mathbf{A}\mathbf{b}\| = \|\mathbf{d} - \bar{\mathbf{A}}\mathbf{c}\|. \quad (15)$$

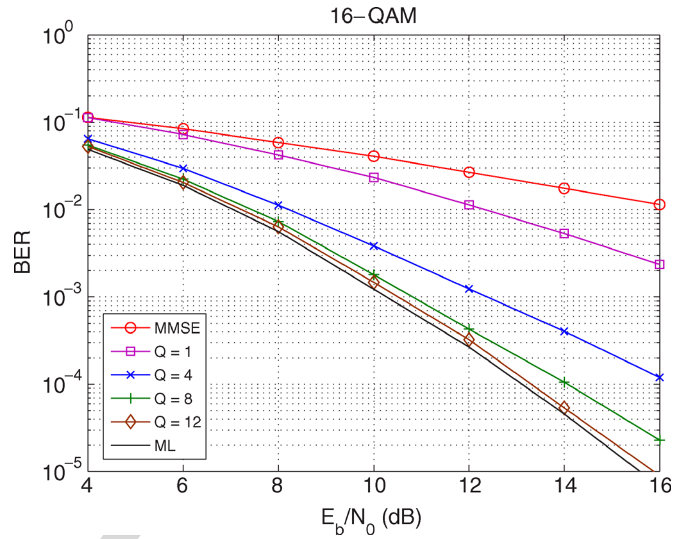

Fig. 3. BER performance of a 4 × 4 MIMO system with 16-QAM signaling.

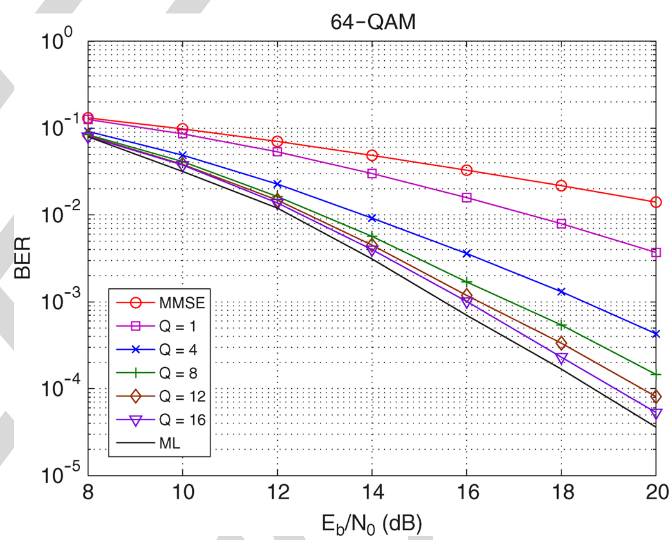

Fig. 4. BER performance of a 4 × 4 MIMO system with 64-QAM signaling.

It is noteworthy that the metric on the right-hand side of (15) is defined in the LR domain. Let $\tilde{\mathbf{s}}_2$ be the signal vector in $\mathcal{S}^{K_2}$ corresponding to $\tilde{\mathbf{c}}_2$, and assume that $\tilde{\mathbf{s}}_2$ is sufficiently close to $\hat{\mathbf{s}}_2^{(1)}$. Then, we can have $\mathbf{d} \simeq \bar{\mathbf{A}}\tilde{\mathbf{c}}_2$. From this, the ML metric (ignoring a scaling factor) for constructing the list in the LR domain becomes

$$\|\mathbf{d} - \bar{\mathbf{A}}\mathbf{c}\| \simeq \|\bar{\mathbf{A}}\tilde{\mathbf{c}}_2 - \bar{\mathbf{A}}\mathbf{c}\| = \|\tilde{\mathbf{c}}_2 - \mathbf{c}\|_{\bar{\mathbf{A}}^H\bar{\mathbf{A}}} \quad (16)$$

where $\|\mathbf{x}\|_{\mathbf{A}} = \sqrt{\mathbf{x}^H\mathbf{A}\mathbf{x}}$ is a weighted norm. The list in the LR domain becomes

$$\mathcal{C}_2 = \left\{\mathbf{c}_2 \middle| \|\tilde{\mathbf{c}}_2 - \mathbf{c}\|_{\bar{\mathbf{A}}^H\bar{\mathbf{A}}} < r_{\bar{\mathbf{A}}}(Q)\right\} \quad (17)$$

where $r_{\bar{\mathbf{A}}}(Q) > 0$ is the radius of an ellipsoid centered at $\tilde{\mathbf{c}}_2$, which contains Q elements in the LR domain. If the column vectors of $\bar{\mathbf{A}}$ or the basis vectors in the LR domain are orthogonal, $\bar{\mathbf{A}}^H\bar{\mathbf{A}}$ becomes diagonal. Furthermore, if they have the same norm, $\bar{\mathbf{A}}^H\bar{\mathbf{A}} \propto \mathbf{I}$. Thus, for nearly orthogonal basis vectors of almost equal norm, the list of $\mathbf{c}_2$ can be approximated as

$$\mathcal{C}_2 \simeq \tilde{\mathcal{C}}_2 = \left\{\mathbf{c}_2 \middle| \|\tilde{\mathbf{c}}_2 - \mathbf{c}\| < r(Q)\right\} \quad (18)$$

TABLE II
IMPACT OF THE MAXIMUM NUMBER OF COLUMN SWAPS IN THE LR ON BER (BER RAPIDLY DECREASES WITH THE NUMBER OF COLUMN SWAPS. ONLY A FEW NUMBER OF COLUMN SWAPS IS REQUIRED)

E_b/N_0	$N_{\text{cs}} = 1$	$N_{\text{cs}} = 2$	$N_{\text{cs}} = 3$	$N_{\text{cs}} = 4$
6 dB	2.2472×10^{-2}	2.2459×10^{-2}	2.2459×10^{-2}	2.2459×10^{-2}
10 dB	1.9068×10^{-3}	1.9062×10^{-3}	1.9062×10^{-3}	1.9062×10^{-3}

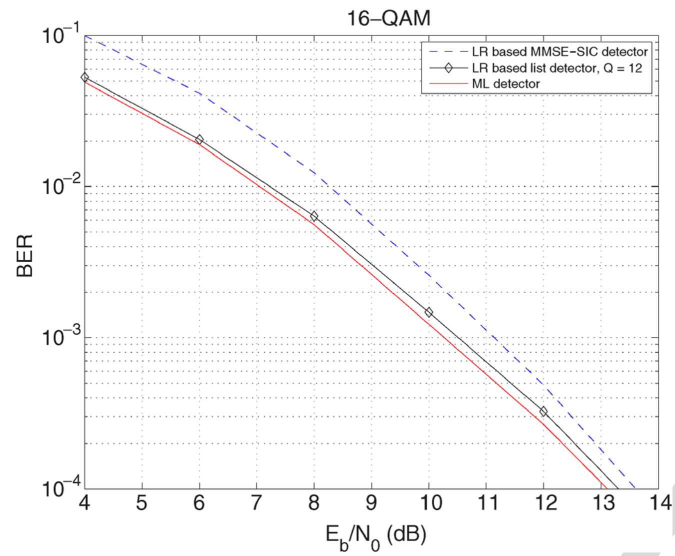

Fig. 5. BER performance comparison of a 4 × 4 MIMO system with 16-QAM signaling.

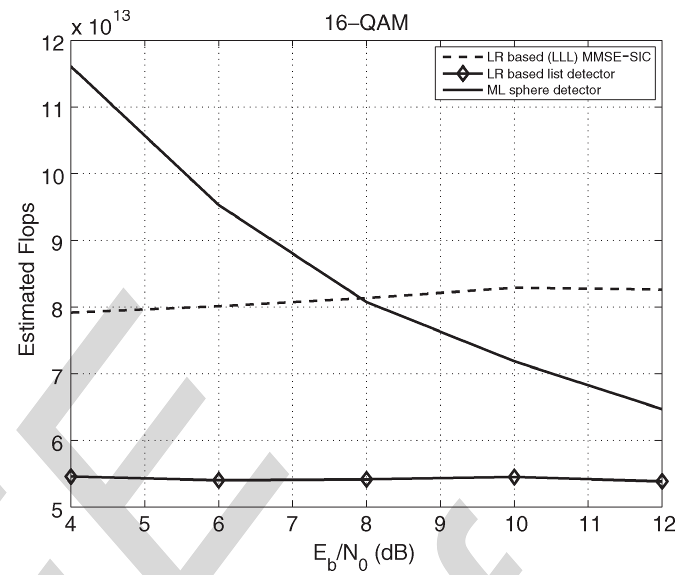

Fig. 6. Complexity comparison of a 4 × 4 MIMO system with 16-QAM signaling.

where $r(Q) > 0$ is the radius of a sphere centered at $\tilde{\mathbf{c}}_2$, which contains Q elements. Since the LR provides a set of nearly orthogonal basis vectors for the LR-based detection, we can see that the column vectors in $\bar{\mathbf{A}}$ in (10) are nearly orthogonal, as shown in Fig. 1, with a two-basis system. Let $\tilde{\mathcal{S}}_2$ denote the list in the original domain obtained from $\tilde{\mathcal{C}}_2$ as in step S3). Since no matrix–vector multiplications are required to generate $\tilde{\mathcal{C}}_2$ or $\tilde{\mathcal{S}}_2$, we can use $\tilde{\mathcal{S}}_2$ as the list in the proposed detector to reduce computational complexity.

IV. SIMULATION RESULT

In this section, we present simulation results. We mainly focus on the case of $K = 4$, particularly the case of $K_1 = K_2 = 2$. The elements of $\mathbf{H}$ are independent zero-mean circular complex Gaussian random variables with unit variance. This case is particularly interesting as the Gaussian reduction, which can find the two shortest vectors in two-basis systems [10], [20], can be used for LR.

In the proposed SIC-list-LR-based detection, list length Q plays a key role in the tradeoff between complexity and performance. In general, it is desirable that the list has the true transmitted vector of $\mathbf{c}_2$. If not, the proposed detector will have an incorrect decision. If Q increases, the error probability that $\mathcal{S}_2(\mathcal{C}_2)$ does not have the correct vector of $\mathbf{s}_2(\mathbf{c}_2)$, which is denoted by $P_e(\mathcal{S}_2)$ or $P_e(\mathcal{C}_2)$, decreases. Error probability $P_e(\mathcal{C}_2)$ is considered for the MIMO system with 16-state quadrature amplitude modulation (16-QAM), and $N = K = 4$. Simulation results are shown in Fig. 2, where the error probabilities are shown with two different lists in (17) and (18). As the list in (18) is suboptimal, the performance is worse. However, this performance degradation is not significant as the column vectors of $\bar{\mathbf{A}}$ are nearly orthogonal.

The bit error rate (BER) performance of a 4 × 4 MIMO system with 16-QAM signaling is shown in Fig. 3. In this case, a near-ML performance can be achieved when $Q \geq 8$. For example, the signal-to-noise ratio (SNR) loss of the proposed detector, compared with that of the ML detector, is less than 1 dB at a BER of 10^{-3} when $Q = 8$.

Fig. 4 shows the simulation results with 64-state quadrature amplitude modulation (16-QAM). This result again confirms that the proposed SIC-list-LR-based detector can provide a near-ML performance with low complexity. At a BER of 10^{-3}, the SNR loss is less than 1 dB, compared with that of the ML detector when $Q = 12$. As the SNR or E_b/N_0 increases, the SNR loss increases. However, by increasing list length Q, this loss can be reduced as the list length can exploit the tradeoff between performance and complexity. Note that a full diversity may not be achieved by the proposed detector with a fixed list length, as shown in Figs. 3 and 4. The relationship between diversity order and list length needs to be investigated in the near future.

In the LR-based detection, since the number of column swaps in the LR operation is not fixed, the complexity can vary from a channel matrix to another. Thus, in practice, the maximum number of column swaps can be fixed to limit the maximum complexity for two-basis systems. It is shown in [10] that the two shortest vectors can be found within two iterations for more than 99% of 2 × 2 random matrices (of Rayleigh fading). However, when the number of column swaps is limited, the basis vectors may not be properly reduced for some channels, and the BER performance could be degraded because of it. To see the impact of the maximum number of column swaps, a simulation is considered with 16-QAM. Table II presents the BER performance when the maximum number of column swaps N_{cs} is limited. It is shown that the performance degradation is negligible, even though $N_{\text{cs}} = 1$.

For comparison purposes, we consider the BER performance of the LR-based MMSE-SIC detector, which is the best LR-based detector among the LR-based detectors proposed in [11]. The BER performance results are shown in Fig. 5. It is shown that the proposed detector can provide a performance that is better by about 1 dB than the LR-based MMSE-SIC detector at a BER of 10^{-2}. Again, we can confirm that the combination of the LR-based detection and list detection can improve the performance of the LR-based detector and is an effective means to approach the ML performance.

For complexity comparison, we can take the upper bound on the average number of LLL iterations in [17], which is given by $\bar{N}_{\mathrm{cs}} = K^2 \log K/(N-K+1)$. (We ignore some minor terms to simplify the comparison.) For 4×4 MIMO channels, we have $\bar{N}_{\mathrm{cs}} = K^2 \log K/(N-K+1) = 16 \log 4$ for the LR-based MMSE-SIC detector and $\bar{N}_{\mathrm{cs}} = 2(K/2)^2 \log((K/2)/((N/2)-(K/2)+1)) = 8 \log 2$ for the proposed detector. This shows complexity reduction by more than half in terms of LLL iterations. Note that the proposed detector has additional complexity to build a list, which may offset the complexity advantage of the proposed detector over conventional LR-based detectors [11]. To further see the complexity of each detector, simulations are considered under the same environment, as shown in Fig. 5. Fig. 6 shows the estimated flops using MATLAB execution time that was obtained over all operations for each detector through simulations. The execution time is averaged over hundreds of thousands of channel realizations. The Sphere Schnorr–Euchner algorithm [21] is used for the ML decoding, whereas the LLL-reduced algorithm with reduction factor $\delta = 3/4$ [16] is chosen for the LR-based MMSE-SIC detector [11]. (This is the same as that in Fig. 5.) No limitation on the number of iterations is imposed for any LR algorithm. The proposed LR-based list detector clearly requires the lowest execution time. We can also see that the execution time of the proposed detector is slightly higher than half of the execution time of the LR-based MMSE-SIC detector where the LLL-reduced algorithm is used.

V. Concluding Remark

In this paper, we have derived an SIC-list-LR-based detector for MIMO detection using two complementary techniques, i.e., LR and list detection, within a framework of SIC-based detection. It was shown that the proposed detector has a near-ML performance with low complexity. The list length plays a key role in the tradeoff between performance and complexity. The performance is improved for a longer list length, whereas the complexity increases with list length Q.

References

[1] I. E. Telatar, "Capacity of multiple-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wirel. Pers. Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[3] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–59, Autumn 1996.

[4] G. J. Foschini, G. Golden, R. Valenzuela, and P. Wolniansky, "Simplified processing for wireless communication at high spectral efficiency," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 11, pp. 1841–1852, Nov. 1999.

[5] W. J. Choi, R. Negi, and J. Cioffi, "Combined ML and DFE decoding for the V-BLAST system," in *Proc. IEEE ICC*, New Orleans, LA, 2000, pp. 1243–1248.

[6] J. Choi, "On the partial MAP detection with applications to MIMO channels," *IEEE Trans. Signal Process.*, vol. 53, no. 1, pp. 158–167, Jan. 2005.

[7] D. J. Love, S. Hosur, A. Batra, and R. W. Heath, "Chase decoding for space-time codes," in *Proc. IEEE VTC*, Nov. 2004, vol. 3, pp. 1663–1667.

[8] D. W. Waters and J. R. Barry, "The Chase family of detection algorithms for multiple-input multiple-output channels," *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 739–747, Feb. 2008.

[9] D. Chase, "A class of algorithms for decoding block codes with channel measurement information," *IEEE Trans. Inf. Theory*, vol. IT-18, no. 1, pp. 170–182, Jan. 1972.

[10] H. Yao and G. W. Wornell, "Lattice-reduction-aided detectors for MIMO communication systems," in *Proc. IEEE Globecom*, Taipei, Taiwan, Nov. 2002, pp. 424–428.

[11] D. Wubben, R. Bohnke, V. Kuhn, and K.-D. Kammeyer, "Near-maximum-likelihood detection of MIMO systems using MMSE-based lattice reduction," in *Proc. IEEE ICC*, Paris, France, Jun. 2004, vol. 2, pp. 798–802.

[12] W. H. Mow, "Universal lattice decoding: Principle and recent advances," *Wireless Commun. Mobile Comput.*, vol. 3, no. 5, pp. 553–569, Aug. 2003.

[13] M. Taherzadeh, A. Mobasher, and A. K. Khandani, "LLL reduction achieves the receive diversity in MIMO decoding," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4801–4805, Dec. 2007.

[14] C. Ling, "On the proximity factors of lattice reduction-aided decoding," *IEEE Trans. Inf. Theory*. to be published. **AQ2**

[15] D. L. Milliner and J. R. Barry"A lattice-reduction-aided soft detector for multiple-input multiple-output channels," in *Proc. IEEE Globecom*, San Francisco, CA, Nov. 2006, pp. 1–5.

[16] A. K. Lenstra, J. H. W. Lenstra, and L. Lovasz, "Factorizing polynomials with rational coefficients," *Math. Ann.*, vol. 261, no. 4, pp. 515–534, Dec. 1982.

[17] J. Jalden, D. Seethaler, and G. Matz, "Worst- and average-case complexity of LLL lattice reduction in MIMO wireless systems," in *Proc. IEEE ICASSP*, Mar./Apr. 2008, pp. 2685–2688.

[18] J. Choi, Y. Hong, and J. Yuan, "An approximate MAP based iterative receiver for MIMO channels using a modified sphere detection," *IEEE Trans. Wireless Commun.*, vol. 5, no. 8, pp. 2119–2126, Aug. 2006.

[19] Y. H. Gan and W. H. Mow, "Complex lattice reduction algorithms for low-complexity MIMO detection," in *Proc. IEEE Globecom*, St. Louis, MO, Nov./Dec. 2005, pp. 2953–2957.

[20] M. C. Cary, *Lattice Basis Reduction Algorithms and Applications*. unpublished manuscript. [Online]. Available: www.cs.washington.edu/homes/cary

[21] C. P. Schnorr and M. Euchner, "Lattice basis reduction: Improved practical algorithms and solving subset sum problems," *Math. Program.*, vol. 66, no. 1–3, pp. 181–191, Aug. 1994.