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High-Rate Groupwise STBC Using Low-Complexity SIC Based Receiver

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***Abstract*—In this paper, using diagonal signal repetition with Alamouti code employed as building blocks, we propose a high-rate groupwise space-time block code (GSTBC) which can be effectively decoded by a low-complexity successive interference cancellation (SIC) based receiver. The proposed GSTBC and SIC based receiver are jointly designed such that the diversity repetition in a GSTBC can induce the dimension expansion to suppress interfering signals as well as to obtain diversity gain. Our proposed scheme can be easily applied to the case of large number of antennas while keeping a reasonably low complexity at the receiver. It is found that the required minimum number of receive antennas is only two for the SIC based receiver to avoid the error floor in performance. The simulation results show that the proposed GSTBC with SIC based receiver obtains a near maximum likelihood (ML) performance while having a significant performance gain over other codes equipped with linear decoders.**

***Index Terms*—MIMO system, layered array processing, successive interference cancellation (SIC), groupwise STBC, dimension expansion.**

I. INTRODUCTION

Various space-time codes (STC) have been proposed to improve data rate and bit error rate (BER) performance of multiple input multiple output (MIMO) systems [1]-[5]. While larger number of transmit antennas can potentially provide higher diversity/multiplexing gains, fully exploiting these potential gains remains to be investigated. One of the requirements for full exploitation is that the number of receive antennas is proportional to the number of transmit antennas [6], which results in a large MIMO system. Thus, design of STCs for a large MIMO system with reasonable decoding complexity becomes important for high data rate and reliable MIMO communications.

To reduce decoding complexity, the interference cancellation based decoding[1] can be employed, e.g., [7]-[9], where iterative decoding and array processing techniques are used to reduce the spatial interference. Alternatively, groupwise STC is considered in [10]-[12] for a trade-off of performance and complexity, where transmit antennas are divided into multiple groups to transmit smaller-size STC symbol blocks. To avoid the exhaustive search as in the maximum likelihood (ML) approach, a joint design of STC grouping and groupwise interference cancelling is generally required, as shown in [13], [14]. In [13], iterative groupwise interference cancellation is employed for decoding of the groupwise space-time block code (GSTBC). However, this GSTBC provides a limited diversity gain, especially in the case of large MIMO systems, as each group of symbols is always passed through a fixed set of four transmit antennas only.

Usually, an STC that is decodable by a linear receiver is preferable due to complexity. This direction of STC design has been considered in [15]-[19]. In [15], [16], a class of space-time block code (STBC) called Toeplitz codes of rate $L/(L+K-1)$ is proposed using the Toeplitz structure. Here, L is the number of symbols encoded in the codeword matrix and K is the number of transmit antennas. Another class of STBC called overlapped Alamouti code (OAC) is proposed in [17], [18] by linearly embedding information symbols and their complex conjugates. Both classes allow a full diversity performance with linear decoding, however, at the expense of low code rate. Usually, the code rate of these classes is less than 1 symbol per channel use (sym/channel). Without achieving full diversity, some other STCs allow higher code rates and can also be decoded by linear receivers. Examples of these classes are double space-time transmit diversity (DSTTD) [20], [21] and linear dispersion code (LDC) [22]. While the DSTTD scheme utilizes two independent Alamouti blocks for high rate purpose, the LDC takes into account both diversity gain and spectral efficiency by optimizing the mutual information between the transmitted and received signals. Although both DSTTD and LDC can employ low-complexity linear decoders, the LDC requires additional pre-processing computation to find the codeword matrix.

In this paper, we aim to design a high-rate GSTBC (rate $>$ 1 sym/channel) for large MIMO systems which can be decodable by using a low-complexity successive interference cancellation (SIC) based receiver. This work generalizes the

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[1]We use *decoding* to refer to processing of the received signals for the estimation and the decision of the transmitted symbols in each codeword. This process can be similarly referred to as *detection* in uncoded MIMO systems.

case of 2×4 MIMO system considered in [14] and allows us to build a GSTBC for a large number of transmit antennas. The Alamouti code in [2] is used as a building block for the proposed GSTBC. The diagonal repetition of Alamouti blocks is employed to facilitate the SIC and reduce the complexity at the receiver. Compared to the LDC which also has a high rate, the GSTBC has some conceptual differences as follows: i) *codeword structure*: The GSTBC uses layered diagonal structure to transmit data signals while the LDC transmits them in linear combinations over space and time; ii) *channel information* is required to find the LDC codeword matrix; and iii) *no additional pre-processing* is required to find the GSTBC codeword matrix. Our performance analysis shows that the SIC-based receiver can exploit the dimension expansion to suppress interference across layers using a minimum mean square error (MMSE) filter and, at the same time, obtain the coding/diversity gains of the GSTBC. Simulation results show that the SIC based decoding for the layered GSTBC has a marginal performance degradation compared to the ML approach with an affordable complexity. It is also shown that a significant performance gain over full-diversity codes is obtained thanks to the high rate of the proposed GSTBC. With respect to higher rate codes such as LDC and DSTTD, the proposed GSTBC provides a better coding gain while using the same SIC based receiver structure.

The rest of the paper is organized as follows: Section II describes the proposed GSTBC and discusses key properties. The SIC receiver is then designed in Section III. Section IV provides the performance and diversity analyses while complexity is discussed in Section V. Simulation results are discussed in Section VI. Section VII provides some concluding remarks.

Notations: $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote complex conjugation, transpose and Hermitian transpose, respectively; $\mathrm{Diag}(\mathbf{x})$ represents a diagonal matrix whose diagonal is vector $\mathbf{x}$; $\mathbf{I}$ represents the identity matrix; $\mathbf{0}_{U \times V}$ denotes a $U \times V$ matrix with entire zero entries while $\mathbf{0}$ (without subscription) stands for a 2×2 zero matrix.

II. Layered GSTBC Design

We consider a MIMO system with N receive antennas and K transmit antennas where K can assumably be divided equally into M groups. Each group conveys a $P \times P$ STC building block over P transmit antennas and P time slots. It can be seen that $K = MP$. Our code design is illustrated in Fig. 1, where $2M-1$ building blocks $\{\mathbf{S}_1, \mathbf{S}_2, \cdots, \mathbf{S}_{2M-1}\}$ are transmitted over K transmit antennas and K time slots. The codematrix includes M layers. Layer 1 is the main diagonal which carries signal block $\mathbf{S}_1$ and its repetitions. Layer m ($2 \leq m \leq M$) is formed by two symmetric diagonals which consist of the multiple copies of $\mathbf{S}_{2m-2}$ and $\mathbf{S}_{2m-1}$. In our layered GSTBC design, signals $\mathbf{S}_{2m-2}$ and $\mathbf{S}_{2m-1}$ in Layer m are repeated $M-m+1$ times along the mth diagonal(s) of the codeword matrix and therefore experiences $M-m+1$ different channel groups with respect to each receive antenna. Here, each channel group is formed by a set of channel gains corresponding to each antenna group (i.e., there are M channel groups for each receive antenna).

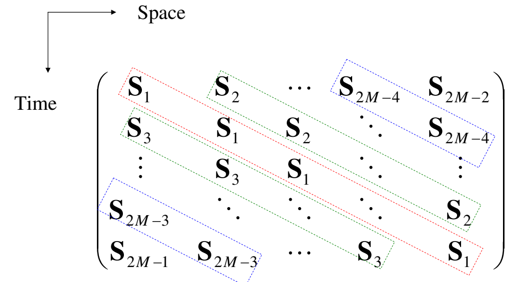

Fig. 1. Codeword matrix with M layers: Layer 1 - main diagonal consisting of $\mathbf{S}_1$. Layer m ($2 \leq m \leq M$) - two symmetric diagonals consisting of $\mathbf{S}_{2m-2}$ and $\mathbf{S}_{2m-1}$.

A. Building Block

The building blocks $\{\mathbf{S}_1, \mathbf{S}_2, \cdots, \mathbf{S}_{2M-1}\}$ are the $P \times P$ coded matrices, which should be properly chosen to maximize the code rate and diversity gain. The Alamouti STBC [2] perfectly matches the requirement as it is the only rate-1 code with full diversity. In this case, the block size is $P = 2$. Note that if the single symbol (i.e., $P = 1$) is chosen as the building block, the full rate remains but the diversity is not fully exploited. In addition, the smaller the block size, the larger the number of layers we need for the same data rate. This leads to a severer error propagation for the SIC based decoding. On the other hand, if a larger building block is selected (i.e., $P > 2$), it is not possible to obtain a full diversity STC with full rate and consequently the data rate would decrease in this case. Simulation results in Fig. 2 verify the superior performance of Alamouti building blocks compared to the other cases (further discussion on this is included in Section VI). We therefore choose the Alamouti STBC as the building block under the assumption that the channel is invariant over two consecutive time slots. The Alamouti building block is expressed as

$$\mathbf{S}_u = \begin{bmatrix} s_{u,1} & s_{u,2} \\ -s_{u,2}^* & s_{u,1}^* \end{bmatrix}, \; u = 1, 2, \cdots, 2M-1,$$

where $s_{u,1}, s_{u,2}, \in \mathcal{S}$ are equally likely data symbols. Here, $\mathcal{S}$ denotes the symbol alphabet. For convenience, $E[|s_{u,i}|^2] = 1$ is considered throughout the paper. Hereafter, we assume $P = 2$.

B. Code Rate

The code rate, denoted by R, is defined as the number of symbols over the number of channel uses. From the codeword matrix in Fig. 1, the code rate is

$$R = \frac{2M-1}{M}. \tag{1}$$

The code rate R increases with M and approaches 2 symbols pcu as M increases. The conventional Alamouti code is retained if $M = 1$. It can be seen that the code design can be extended to a large value of $K = 2M$ by increasing M. In our work, we assume $M \geq 2$. The maximal rate and the minimal delay are basic design criteria of STBCs [23]. Note that the rate of complex orthogonal STBCs (OSTBCs) [5], [24]-[26] is upper bounded by 4/5 sym/channel when the number of transmit antennas is more than 2 [27] while the quasi OSTBCs (QOSTBCs) [4] have a rate of no more than 1. The existing codes with rate of more than 1 would usually require the ML decoding for a satisfactory performance. It is therefore a motivation to design a high-rate code which is decodable by a linear low-complexity receiver and provides a near ML performance. However, as a nature of high-rate code, it is expected that the diversity order of the proposed code is lower than that of the OSTBC and QOSTBC. It should also be noted that the delay resulted from the proposed GSTBC is equal to the number of transmit antennas, which is a significant advantage compared to the orthogonal codes.

C. Layered Structure

Stacking $2M$ consecutive received signals, the received signal vector $\mathbf{v}_i = [v_{i,1}, v_{i,2}, \cdots, v_{i,2M}]^{\mathrm{T}}$ at the ith receive antenna ($i = 1, 2, \cdots, N$) becomes

$$\mathbf{v}_i = \mathbf{S}\mathbf{h}_i + \mathbf{n}_i, \tag{2}$$

where $\mathbf{S}$ is the codeword matrix shown in Fig. 1, $\mathbf{h}_i = [\mathbf{h}_{i,1}^{\mathrm{T}}, \mathbf{h}_{i,2}^{\mathrm{T}}, \cdots, \mathbf{h}_{i,M}^{\mathrm{T}}]^{\mathrm{T}}$, and $\mathbf{n}_i$ is a white zero-mean Gaussian noise vector with $E[\mathbf{n}_i\mathbf{n}_i^{\mathrm{H}}] = N_0\mathbf{I}$. Here, $\mathbf{h}_{i,m} = [h_{i,m,1}, h_{i,m,2}]^{\mathrm{T}}$ is the channel vector over two consecutive time slots from the transmit antennas of the mth group ($m = 1, 2, \cdots, M$) to the ith receive antenna. Each path gain here is assumed to be a circularly symmetric complex zero-mean Gaussian variable with variance σ_h^2. The channel is assumed to be flat fading. By letting

$$\begin{aligned} \mathbf{r}_{i,m} &= [v_{i,2m-1}, -v_{i,2m}^*]^{\mathrm{T}}, \\ \mathbf{r}_i &= [\mathbf{r}_{i,1}^{\mathrm{T}}, \mathbf{r}_{i,2}^{\mathrm{T}}, \cdots, \mathbf{r}_{i,M}^{\mathrm{T}}]^{\mathrm{T}}, \\ \mathbf{H}_{i,m} &= \begin{bmatrix} h_{i,m,1} & h_{i,m,2} \\ -h_{i,m,2}^* & h_{i,m,1}^* \end{bmatrix}, \end{aligned}$$

after some re-arrangements, the received signal vector $\mathbf{r}_i$ at the ith receive antenna can be written as

$$\mathbf{r}_i = \mathbf{G}_{i,1}\mathbf{x}_1 + \mathbf{G}_{i,2}\mathbf{x}_2 + \cdots + \mathbf{G}_{i,M}\mathbf{x}_M + \mathbf{w}_i, \tag{3}$$

where $\mathbf{x}_m$ is the transmitted signal vector in Layer m and given as

$$\mathbf{x}_m = \begin{cases} \mathbf{s}_1 & \text{for } m = 1, \\ [\mathbf{s}_{2m-2}^{\mathrm{T}}, \mathbf{s}_{2m-1}^{\mathrm{T}}]^{\mathrm{T}} & \text{for } m = 2, 3, \cdots, M. \end{cases}$$

Here, $\mathbf{s}_u = [s_{u,1}, s_{u,2}]^{\mathrm{T}}$ for $u = 1, 2, \cdots, 2M-1$. The noise vector $\mathbf{w}_i$ has the same statistical characteristics as those of $\mathbf{n}_i$. The rearranged channel matrices $\{\mathbf{G}_{i,1}, \mathbf{G}_{i,2}, \cdots, \mathbf{G}_{i,M}\}$ are given by

$$\mathbf{G}_{i,1} = \left[\mathbf{H}_{i,1}^{\mathrm{T}}, \mathbf{H}_{i,2}^{\mathrm{T}}, \cdots, \mathbf{H}_{i,M}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{4}$$

$$\mathbf{G}_{i,m} = \begin{bmatrix} \mathbf{H}_{i,m} & \mathbf{0}_{2\times 2(m-1)} \\ \vdots & \mathbf{H}_{i,1} \\ \mathbf{H}_{i,M} & \vdots \\ \mathbf{0}_{2\times 2(m-1)} & \mathbf{H}_{i,M-m+1} \end{bmatrix} \text{ for } m \geq 2. \tag{5}$$

One example of (3) for the case of $M = 3$ is as follows

$$\underbrace{\begin{bmatrix} \mathbf{r}_{i,1} \\ \mathbf{r}_{i,2} \\ \mathbf{r}_{i,3} \end{bmatrix}}_{\mathbf{r}_i} = \underbrace{\begin{bmatrix} \mathbf{H}_{i,1} \\ \mathbf{H}_{i,2} \\ \mathbf{H}_{i,3} \end{bmatrix}}_{\mathbf{G}_{i,1}} \mathbf{s}_1 + \underbrace{\begin{bmatrix} \mathbf{H}_{i,2} & \mathbf{0} \\ \mathbf{H}_{i,3} & \mathbf{H}_{i,1} \\ \mathbf{0} & \mathbf{H}_{i,2} \end{bmatrix}}_{\mathbf{G}_{i,2}} \begin{bmatrix} \mathbf{s}_2 \\ \mathbf{s}_3 \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{H}_{i,3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{i,1} \end{bmatrix}}_{\mathbf{G}_{i,3}} \begin{bmatrix} \mathbf{s}_4 \\ \mathbf{s}_5 \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{w}_{i,1} \\ \mathbf{w}_{i,2} \\ \mathbf{w}_{i,3} \end{bmatrix}}_{\mathbf{w}_i}.$$

Taking into account all the N receive antennas, a more compact expression for the received signal in (3) can be given by

$$\mathbf{r} = \sum_{m=1}^{M} \mathbf{G}_m\mathbf{x}_m + \mathbf{w}, \tag{6}$$

where

$$\begin{aligned} \mathbf{r} &= [\mathbf{r}_1^{\mathrm{T}}, \mathbf{r}_2^{\mathrm{T}}, \cdots, \mathbf{r}_N^{\mathrm{T}}]^{\mathrm{T}}, \\ \mathbf{G}_m &= [\mathbf{G}_{1,m}^{\mathrm{T}}, \mathbf{G}_{2,m}^{\mathrm{T}}, \cdots, \mathbf{G}_{N,m}^{\mathrm{T}}]^{\mathrm{T}}. \end{aligned} \tag{7}$$

D. Dimension Expansion

From (6), it is observed that the signals can be detected layer by layer instead of one signal vector of large dimension. This plays a key role in reducing complexity of the receiver regardless the number of transmit antennas. Thus, our approach is easily applicable to a large MIMO system. In (6), the signal from one layer is seen as the interference to the other layers. To detect the signal in each layer, the interference signals from the other layers should be mitigated or cancelled.

It is noteworthy that the diagonal repetition structure of the codeword matrix in Fig. 1 is interpreted by the dimension expansion of signal $\mathbf{x}_m$ through $\mathbf{G}_m$ in (6). For example, the signal $\mathbf{x}_1$ of 2-dimension in Layer 1 is fully expanded to that of $2NM$-dimension whereas the other signals $\{\mathbf{x}_m, m \neq 1\}$ of 4-dimension are expanded to those of $2N(M-m+1)$-dimension. The diagonal repetition structure of the codeword matrix also leads to the fact that each signal $\mathbf{x}_m$ is transmitted by different channel groups. As such, the diagonal repetition helps induce the dimension expansion and exploit spatial diversity. Layer 1, to be detected first, has the largest dimension expansion. The dimension expansion gradually decreases towards Layer M. This design strategy well suits the low-complexity layered SIC based decoding at the receiver, where the reliability of the decoding in the first layers is extremely important to avoid error propagation. The proposed GSTBC is inspired by [28] where a layered transmission can achieve the channel capacity with an SIC based receiver.

III. Decoding with Layered Successive Interference Cancellation

For low-complexity decoding, we derive an SIC based receiver. The transmitted signals are successively estimated layer by layer (from Layer 1 to Layer m). The estimated signals from the previous layers will be used to cancel their contribution from the received signal in detecting the subsequent signals. As the MMSE filtering is used for the signal estimation, the overall complexity is reasonably low compared to the ML approach. Note that the signal decoding in Layer 1 is the most difficult as no interference cancellation has been carried out. The signals in Layer M after cancellation suffers the least interference as all the signals from other layers can be cancelled.

A. Soft Estimation of Signal in Layer 1

As mentioned earlier, the repetition of $\mathbf{x}_1$ induces the full dimension expansion. From this, the MMSE filter can be applied for the decoding of $\mathbf{x}_1$ with the suppression of the interfering signals $\{\mathbf{x}_2, \mathbf{x}_3, \cdots, \mathbf{x}_M\}$. The output of the MMSE filter, denoted by $\tilde{\mathbf{x}}_1 = [\tilde{s}_{1,1}, \tilde{s}_{1,2}]^{\mathrm{T}}$, is

$$\tilde{\mathbf{x}}_1 = \mathbf{G}_1^{\mathrm{H}} \mathbf{R}^{-1} \mathbf{r}, \tag{8}$$

where

$$\mathbf{R} = \sum_{t=1}^{M} \mathbf{G}_t \mathbf{G}_t^{\mathrm{H}} + N_0 \mathbf{I}. \tag{9}$$

Due to the diagonal repetition, the MMSE decoding[2] exploits the diversity gain and suppresses interfering signals effectively. As the estimation of $\mathbf{x}_1$ can be reliable, there would be much less error propagation in the decoding of the subsequent layers.

***Lemma** 1:* The MMSE filtering exploiting the dimension expansion does not observe an error floor in the asymptotic case where the noise variance approaches zero if the number of receive antennas, N, is not less than 2.

Proof: Equation (6) can be rewritten as

$$\mathbf{r} = \mathbf{G}_1 \mathbf{x}_1 + \mathbf{v}, \tag{10}$$

where

$$\mathbf{v} = \sum_{m=2}^{M} \mathbf{G}_m \mathbf{x}_m + \mathbf{w}. \tag{11}$$

Letting $\mathbf{G}_I = [\mathbf{G}_2, \mathbf{G}_3, \cdots, \mathbf{G}_M]$ where the size of $\mathbf{G}_I$ is $2MN \times 4(M-1)$, the covariance matrix of the noise-plus-interference vector in the case of $N_0 \to 0$ becomes

$$\mathbf{C} = \sum_{m=2}^{M} \mathbf{G}_m \mathbf{G}_m^{\mathrm{H}} = \mathbf{G}_I \mathbf{G}_I^{\mathrm{H}}. \tag{12}$$

Note that the Toeplitz structure of the codeword matrix results in a unique channel matrix in each layer as can be seen from (4) and (5). Thus, each column (or row) of channel matrix $\mathbf{G}_m$ in Layer m is linearly independent from that in the other layers. This means that $\mathbf{G}_I$ has either full column or full row rank. It is therefore observed that

$$\begin{aligned} \mathrm{Rank}(\mathbf{C}) = \mathrm{Rank}(\mathbf{G}_I) &= \min(2MN,\ 4(M-1)) \\ &= \begin{cases} 2MN & \text{if } N = 1, \\ 4(M-1) & \text{if } N \geq 2. \end{cases} \end{aligned}$$

From this, we can see that the rank of the covariance matrix $\mathbf{C}$ is independent of the number of receive antennas N if $N \geq 2$. Since $\mathbf{C}$ has a size of $2MN \times 2MN$, it is always rank deficient if $N \geq 2$. $\mathbf{C}$ is full rank only if $N = 1$. Since the eigenvalues of matrix $\mathbf{C}$ are proportional to the power of interference, the linear MMSE filter can only perfectly suppress the interference if some of the eigenvalues are zero (i.e., $\mathbf{C}$ is rank deficient). It can therefore be seen that there would be no error floor in the MMSE decoder's performance if $N \geq 2$. ■

It is well known that the distribution of the MMSE filter output can be well approximated by a Gaussian distribution [29]. We make the following approximation:

$$\tilde{s}_{1,j} = \mu_j s_{1,j} + \eta_j, \tag{13}$$

where $j = 1, 2$, μ_j is the equivalent amplitude of the jth symbol and η_j is a zero-mean complex Gaussian noise variable with $E[\eta_j \eta_j^*] = \nu_j^2$. It can be obtained that

$$\begin{aligned} \mu_j &= \mathbf{g}_{1,j}^{\mathrm{H}} \mathbf{R}^{-1} \mathbf{g}_{1,j}, \\ \nu_j^2 &= \mu_j - |\mu_j|^2, \end{aligned}$$

where $\mathbf{g}_{1,j}$ is the jth column of $\mathbf{G}_1$. Under the Gaussian assumption, the log-likelihood ratio (LLR) can be found as a soft-decision.

B. SIC Based Detection of Signals in Layer m ($2 \leq m \leq M$)

Assume that the soft decision vectors of the signals in the previous layers are available. These vectors are denoted by $\{\bar{\mathbf{x}}_1, \cdots, \bar{\mathbf{x}}_{m-1}\}$. Let $\bar{s}_{u,j}$ ($u = 1, 2, \cdots, M$ and $j = 1, 2$) denote the soft-decision of $s_{u,j}$. The SIC based decoding can now be applied for the decoding of signal $\mathbf{x}_m$ in Layer m. The received signal vector after cancelling the interference from the previous layers becomes

$$\begin{aligned} \bar{\mathbf{r}}_m &= \mathbf{r} - \sum_{p=1}^{m-1} \mathbf{G}_p \bar{\mathbf{x}}_p \\ &= \sum_{q=m}^{M} \mathbf{G}_q \mathbf{x}_q + \bar{\mathbf{w}}_m, \end{aligned} \tag{14}$$

where

$$\bar{\mathbf{w}}_m = \sum_{p=1}^{m-1} \mathbf{G}_p (\mathbf{x}_p - \bar{\mathbf{x}}_p) + \mathbf{w}.$$

Here, $\bar{\mathbf{w}}_m$ is assumed to be a Gaussian vector with

$$\begin{aligned} E[\bar{\mathbf{w}}_m] &= \mathbf{0}_{2M \times 1}, \\ E[\bar{\mathbf{w}}_m \bar{\mathbf{w}}_m^{\mathrm{H}}] &= \sum_{p=1}^{m-1} \mathbf{G}_p \mathbf{Q}_p \mathbf{G}_p^{\mathrm{H}} + N_0 \mathbf{I}, \end{aligned}$$

where

$$\begin{aligned} \mathbf{Q}_p &= E[(\mathbf{x}_p - \bar{\mathbf{x}}_p)(\mathbf{x}_p - \bar{\mathbf{x}}_p)^{\mathrm{H}}] \\ &= \mathrm{Diag}(\mathbf{q}_p), \end{aligned} \tag{15}$$

and $\mathbf{q}_p$ is given by

$$\mathbf{q}_p = \begin{cases} \begin{bmatrix} 1 - |\bar{s}_{2p-1,1}|^2 \\ 1 - |\bar{s}_{2p-1,2}|^2 \end{bmatrix} & \text{for } p = 1, \\ \\ \begin{bmatrix} 1 - |\bar{s}_{2p-2,1}|^2 \\ 1 - |\bar{s}_{2p-2,2}|^2 \\ 1 - |\bar{s}_{2p-1,1}|^2 \\ 1 - |\bar{s}_{2p-1,2}|^2 \end{bmatrix} & \text{for } p \geq 2. \end{cases} \tag{16}$$

[2] *MMSE decoding* is referred to a decoding process using linear MMSE filtering.

Applying the MMSE filter to the received signal in (14), the output denoted by $\tilde{\mathbf{x}}_m$ is obtained as

$$\tilde{\mathbf{x}}_m = \mathbf{G}_m^{\mathrm{H}} \mathbf{R}_m^{-1} \mathbf{r}_m, \tag{17}$$

where

$$\mathbf{R}_m = \sum_{p=1}^{m-1} \mathbf{G}_p \mathbf{Q}_p \mathbf{G}_p^{\mathrm{H}} + \sum_{t=m}^{M} \mathbf{G}_t \mathbf{G}_t^{\mathrm{H}} + N_0 \mathbf{I}. \tag{18}$$

***Lemma** 2:* The statement in Lemma 1 still holds for the soft SIC based signal decoding in any Layer m ($m \geq 2$).

Proof: We consider the case $N \geq 2$. The covariance matrix of the noise-plus-interference vector in the case of $N_0 \to 0$ is

$$\mathbf{C}_m = \sum_{p=1}^{m-1} \mathbf{G}_p \mathbf{Q}_p \mathbf{G}_p^{\mathrm{H}} + \sum_{t=m+1}^{M} \mathbf{G}_t \mathbf{G}_t^{\mathrm{H}}. \tag{19}$$

The following inequalities can be applied:

$$\begin{aligned} \mathrm{Rank}(\mathbf{C}_m) &\leq \sum_{p=1}^{m-1} \mathrm{Rank}(\mathbf{G}_p \mathbf{Q}_p \mathbf{G}_p^{\mathrm{H}}) \\ &\qquad + \sum_{t=m+1}^{M} \mathrm{Rank}(\mathbf{G}_t \mathbf{G}_t^{\mathrm{H}}) \\ &\leq \sum_{t=1, t\neq m}^{M} \mathrm{Rank}(\mathbf{G}_t \mathbf{G}_t^{\mathrm{H}}) \\ &= \sum_{t=1, t\neq m}^{M} \mathrm{Rank}(\mathbf{G}_t) \\ &= 2(2M-3). \end{aligned} \tag{20}$$

Thus, matrix $\mathbf{C}_m$ of size $2MN \times 2MN$ is always rank deficient with $N \geq 2$. With the special case where the cancellation is perfectly correct (i.e., $\mathbf{Q}_p = \mathbf{0}$ for all $p = 1, 2, \cdots, m-1$), the rank of matrix $\mathbf{C}_m$ is $4(M-m)$ which decreases with m. ■

The Gaussian approximation of $\tilde{\mathbf{x}}_m$ can then be similarly assumed as in (13) to find its soft-decision $\bar{\mathbf{x}}_m$.

IV. Performance Analysis

In this section, we consider the probability of error in each layer and find out the diversity gain resulted from the dimension expansion.

A. Probability of Error in Each Layer

For convenience, we make the following assumption:

***Assumption** 1*: Perfect cancellation is carried out at each layer.[3]

Note that the property of error of signal vectors $\mathbf{s}_{2m-2}$ and $\mathbf{s}_{2m-1}$ ($m = 2, 3, \cdots, M$) in Layer m are the same due to symmetry. Thus, for analysis purpose, we consider $\mathbf{s}_{2m-2}$ only as the signal vector of Layer m. Under Assumption 1, the received signal vector at the ith receive antenna in Layer m is given by

$$\mathbf{r}_{i,m} = \mathbf{\Theta}_{i,m} \mathbf{s}_{\kappa_m - 1} + \sum_{u=\kappa_m}^{2M-1} \mathbf{\Upsilon}_{i,u,m} \mathbf{s}_u + \mathbf{v}_{i,m}, \tag{21}$$

where

$$\begin{aligned} \kappa_m &= \begin{cases} 2m & \text{for} \quad m = 1 \\ 2m-1 & \text{for} \quad m = 2, 3, \cdots, M, \end{cases} \\ \mathbf{\Theta}_{i,m} &= [\mathbf{H}_{i,m}^{\mathrm{T}}, \cdots, \mathbf{H}_{i,M}^{\mathrm{T}}]^{\mathrm{T}}, \\ \mathbf{\Upsilon}_{i,u,m} &= \begin{cases} [\mathbf{H}_{i,\frac{u}{2}+1}^{\mathrm{T}}, \cdots, \mathbf{H}_{i,M}^{\mathrm{T}}, \mathbf{0}_{2\times u-2m+2}]^{\mathrm{T}}, & u\text{: even} \\ [\mathbf{0}_{2\times u-1}, \mathbf{H}_{i,1}^{\mathrm{T}}, \cdots, \mathbf{H}_{i,M-\frac{u-3}{2}-m}^{\mathrm{T}}]^{\mathrm{T}}, & u\text{: odd.} \end{cases} \end{aligned}$$

Here, $\mathbf{v}_{i,m}$ is the noise vector with the same statistical properties as of $\mathbf{w}_i$. By letting

$$\begin{aligned} \mathbf{r}_m &= [\mathbf{r}_{1,m}^{\mathrm{T}}, \mathbf{r}_{2,m}^{\mathrm{T}}, \cdots, \mathbf{r}_{N,m}^{\mathrm{T}}]^{\mathrm{T}}, \\ \mathbf{v}_m &= [\mathbf{v}_{1,m}^{\mathrm{T}}, \mathbf{v}_{2,m}^{\mathrm{T}}, \cdots, \mathbf{v}_{N,m}^{\mathrm{T}}]^{\mathrm{T}}, \\ \mathbf{\Upsilon}_{u,m} &= [\mathbf{\Upsilon}_{1,u,m}^{\mathrm{T}}, \mathbf{\Upsilon}_{2,u,m}^{\mathrm{T}}, \cdots, \mathbf{\Upsilon}_{N,u,m}^{\mathrm{T}}]^{\mathrm{T}}, \\ \mathbf{\Theta}_m &= [\mathbf{\Theta}_{1,m}^{\mathrm{T}}, \mathbf{\Theta}_{2,m}^{\mathrm{T}}, \cdots, \mathbf{\Theta}_{N,m}^{\mathrm{T}}]^{\mathrm{T}}, \end{aligned}$$

the received signal vector in Layer m for all N receive antennas is written as

$$\mathbf{r}_m = \mathbf{\Theta}_m \mathbf{s}_{\kappa_m - 1} + \mathbf{u}_m, \tag{22}$$

where $\mathbf{u}_m$ is the interference-plus-noise vector given as:

$$\mathbf{u}_m = \sum_{u=\kappa_m}^{2M-1} \mathbf{\Upsilon}_{u,m} \mathbf{s}_u + \mathbf{v}_m. \tag{23}$$

Let

$$\mathbf{\Upsilon}_m = [\mathbf{\Upsilon}_{\kappa_m,m}, \mathbf{\Upsilon}_{\kappa_m+1,m}, \cdots, \mathbf{\Upsilon}_{2M-1,m}].$$

***Assumption** 2*: i) $\mathbf{u}_m$ is Gaussian with mean zero and covariance $(\mathbf{\Upsilon}_m \mathbf{\Upsilon}_m^{\mathrm{H}} + N_0 \mathbf{I})$; and ii) $\mathbf{u}_m$ is independent of $\mathbf{\Theta}_m$ and $\mathbf{s}_{\kappa_m - 1}$.

Since two symbols $s_{\kappa_m-1,1}$ and $s_{\kappa_m-1,2}$ are orthogonally designed in Alamouti building block $\mathbf{S}_{\kappa_m-1}$, they do not interfere with each other (i.e., two columns of $\mathbf{\Theta}_m$ are orthogonal). Thus, the signal to interference-plus-noise ratio (SINR) at the output of the MMSE filtering with respect to $s_{\kappa_m-1,j}$ ($j = 1, 2$) becomes

$$\begin{aligned} \gamma_{m,j} &= \mathbf{a}_{m,j}^{H} (\mathbf{\Upsilon}_m \mathbf{\Upsilon}_m^{\mathrm{H}} + N_0 \mathbf{I})^{-1} \mathbf{a}_{m,j} \\ &= \mathbf{a}_{m,j}^{H} \mathbf{\Phi}_m^{-1} \mathbf{a}_{m,j}, \end{aligned} \tag{24}$$

where $\mathbf{a}_{m,j}$ is the jth column of $\mathbf{\Theta}_m$ and $\mathbf{\Phi}_m = \mathbf{\Upsilon}_m \mathbf{\Upsilon}_m^{\mathrm{H}} + N_0 \mathbf{I}$. Since $s_{\kappa_m-1,1}$ and $s_{\kappa_m-1,2}$ are equally likely, the probability of error at Layer m can be given by

$$P_{e,m} = \mathrm{E}_{\gamma_{m,1}} \left[\Pr\left(e|\gamma_{m,1}\right) \right] = \mathrm{E}_{\gamma_{m,2}} \left[\Pr\left(e|\gamma_{m,2}\right) \right]. \tag{25}$$

For convenience, we drop index j. From the Gaussian assumption in Assumption 2, the probability of error is upper-bounded by [31]

$$\begin{aligned} P_{e,m} &= \mathrm{E}_{\gamma_m} [Q(\sqrt{\gamma_m})] \\ &\leq \mathrm{E} \left[\frac{1}{12} \exp\left(-\frac{\mathbf{a}_m^{H} (\mathbf{\Phi}_m)^{-1} \mathbf{a}_m}{2} \right) \right. \\ &\qquad \left. + \frac{1}{4} \exp\left(-\frac{2\mathbf{a}_m^{H} (\mathbf{\Phi}_m)^{-1} \mathbf{a}_m}{3} \right) \right], \end{aligned} \tag{26}$$

[3]Readers can be referred to [30] for error propagation analysis of the basic MIMO-MMSE detection which also uses an SIC based structure.

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}\mathrm{d}t$. Using unitary transformation $\mathbf{U}_m$, the covariance matrix can be diagonalized as

$$\mathbf{\Phi}_m = \mathbf{\Upsilon}_m\mathbf{\Upsilon}_m^{\mathrm{H}} + N_0\mathbf{I} = \mathbf{U}_m^{\mathrm{H}}(\mathbf{\Lambda}_m + N_0\mathbf{I})\mathbf{U}_m,$$

where $\mathbf{\Lambda}_m = \mathrm{Diag}([\lambda_{m,1}, \lambda_{m,2}, \cdots, \lambda_{m,2(M-m+1)N}])$ and $\lambda_{m,t}$ is the tth eigenvalue of $\mathbf{\Upsilon}_m\mathbf{\Upsilon}_m^{\mathrm{H}}$. Here, we assume that $\lambda_{m,1}$, $\lambda_{m,2}$, $\cdots$, and $\lambda_{m,2(M-m+1)N}$ are sorted in ascending order. Let

$$\begin{aligned}\mathbf{\Psi}_m &\triangleq [\psi_{m,1}, \psi_{m,2}, \cdots, \psi_{m,2(M-m+1)N}]^{\mathrm{T}} \\ &= \mathbf{a}_m\mathbf{U}_m. \end{aligned} \tag{27}$$

Since $\mathbf{a_m}$ (and subsequently $\mathbf{\Psi}_m$) is zero-mean uncorrelated Gaussian, the expectation with respect to γ_m can be carried out separately. Thus, the probability of error can be rewritten as

$$\begin{aligned} P_{e,m} &\leq \mathrm{E}_{\gamma_m}\left[\frac{1}{12}\exp\left(-\sum_{t=1}^{2(M-m+1)N}\frac{|\psi_{m,t}|^2}{2(\lambda_{m,t}+N_0)}\right)\right. \\ &\qquad \left. +\frac{1}{4}\exp\left(-\sum_{t=1}^{2(M-m+1)N}\frac{2|\psi_{m,t}|^2}{3(\lambda_{m,t}+N_0)}\right)\right] \\ &= \mathrm{E}_{\lambda_{m,t}}\left[\frac{1}{12}\prod_{t=1}^{2(M-m+1)N}\left(1+\frac{\sigma_h^2}{2(\lambda_{m,t}+N_0)}\right)^{-1}\right. \\ &\qquad \left. +\frac{1}{4}\prod_{t=1}^{2(M-m+1)N}\left(1+\frac{2\sigma_h^2}{3(\lambda_{m,t}+N_0)}\right)^{-1}\right]. \end{aligned}$$

***Property** 1*: τ_m eigenvalues of $\mathbf{\Upsilon}_m\mathbf{\Upsilon}_m^{\mathrm{H}}$ are zeros if number of receive antennas $N \geq 2$, where $\tau_m = 2((M-m+1)N - (2M-\kappa_m))$.

This can be seen from the fact that the matrix $\mathbf{\Upsilon}_m$ of size $2(M-m+1)N \times 2(2M-\kappa_m)$ is a full column rank matrix if $N \geq 2$. Thus, there are only $2(2M-\kappa_m)$ non-zero eigenvalues of $\mathbf{\Upsilon}_m\mathbf{\Upsilon}_m^{\mathrm{H}}$. In other words, there are $\tau_m = 2((M-m+1)N-(2M-\kappa_m))$ zero eigenvalues. From Property 1, the probability of error can be written as

$$\begin{aligned} P_{e,m} &\leq \frac{1}{12}\left(1+\frac{\sigma_h^2}{2N_0}\right)^{-\tau_m} \\ &\quad \times\mathrm{E}_{\lambda_{m,t}}\left[\prod_{t=\tau_m+1}^{2(M-m+1)N}\left(1+\frac{\sigma_h^2}{2(\lambda_{m,t}+N_0)}\right)^{-1}\right] \\ &\quad +\frac{1}{4}\left(1+\frac{2\sigma_h^2}{3N_0}\right)^{-\tau_m} \\ &\quad \times\mathrm{E}_{\lambda_{m,t}}\left[\prod_{t=\tau_m+1}^{2(M-m+1)N}\left(1+\frac{2\sigma_h^2}{3(\lambda_{m,t}+N_0)}\right)^{-1}\right]. \end{aligned}$$

The upper bound can then be approximated as

$$\begin{aligned} P_{e,m} &\approx \frac{1}{12}\left(1+\frac{\sigma_h^2}{2N_0}\right)^{-\tau_m} \\ &\quad \times\prod_{t=\tau_m+1}^{2(M-m+1)N}\left(1+\frac{\sigma_h^2}{2(\bar{\lambda}_{m,t}+N_0)}\right)^{-1} \\ &\quad +\frac{1}{4}\left(1+\frac{2\sigma_h^2}{3N_0}\right)^{-\tau_m} \\ &\quad \times\prod_{t=\tau_m+1}^{2(M-m+1)N}\left(1+\frac{2\sigma_h^2}{3(\bar{\lambda}_{m,t}+N_0)}\right)^{-1}, \end{aligned} \tag{28}$$

where $\bar{\lambda}_{m,t} = \mathrm{E}[\lambda_{m,t}]$ and $\bar{\lambda}_{m,t}$ can be found via closed-form solutions (in some special cases) or Monte Carlo simulations as discussed in [32].

B. Diversity in Each Layer

For a high signal to noise (SNR) scenario, it is reasonable to assume that $N_0 \ll \bar{\lambda}_{m,t}$. Thus, the probability of error in (28) can be approximated as

$$\begin{aligned} P_{e,m} &\approx \frac{1}{12}\left(1+\frac{\sigma_h^2}{2N_0}\right)^{-\tau_m} \\ &\qquad \times\prod_{t=\tau_m+1}^{2(M-m+1)N}\left(1+\frac{\sigma_h^2}{2\bar{\lambda}_{m,t}}\right)^{-1} \\ &\quad +\frac{1}{4}\left(1+\frac{2\sigma_h^2}{3N_0}\right)^{-\tau_m} \\ &\qquad \times\prod_{t=\tau_m+1}^{2(M-m+1)N}\left(1+\frac{2\sigma_h^2}{3\bar{\lambda}_{m,t}}\right)^{-1} \end{aligned} \tag{29}$$

From (29), we have the following observations:

***Observation** 1*: The diversity order in Layer m is $\tau_m = 2((M-m+1)N-(2M-\kappa_m))$. The diversity order is actually the signal dimension $(2(M-m+1)N)$ subtracted by the number of interferers $(2(2M-\kappa_m))$. This was previously discussed in [33] for the case of optimum combining in an independent Rayleigh fading channel.

***Observation** 2*: As m decreases, the signal dimension increases faster than the number of interferers does. This means that the signal repetition in each layer contributes significantly to the diversity gain, especially at low layers. Since SIC-based decoding is subsequently applied, this is important to obtain reliable decoding in the first layers to avoid error propagation in the subsequent layers.

***Observation** 3*: The diversity order τ_m decreases with layer m. The decrease is more significant if the number of receive antenna N is larger. The highest diversity order is in Layer 1: $\tau_1 = 2MN - 4M + 4$. The lowest diversity order is in Layer M: $\tau_M = 2N$. The diversity gain in Layer M is obtained solely from the receive antennas (N) and the building block (2), which means that there is no diversity gain from signal repetition. Thus, it is expected the performance of Layer M is the worst compared to the other lower layers.

TABLE I
COMPLEXITY COMPARISON OF DIFFERENT DECODERS WITH QPSK SIGNALLING

Decoder	**Complex multiplications**	$K=4$, $N=4$	$K=6$, $N=6$	$K=8$, $N=8$
MMSE	$2K^3N^3+2K^2N^2-KN$	8688	95868	532416
ML	$4(K-1)^2\|\mathcal{S}\|^{2(K-1)}$	147456	$\sim 10.4\times10^6$	$\sim 5.2\times10^{10}$
SIC-based	$2K^3N^3+K^2N^2+(4-K)N+2\sum_{k=2}^{K/2}((K-2k+2)^3N^3+(K-2k+2)^2N^2+8(K-2k+2)N)$	9728	127740	832032

TABLE II
COMPARISON WITH DIFFERENT LINEAR CODES: RATE, DELAY AND COMPLEX MULTIPLICATIONS

Code	**Rate**	**Delay**	**Complex multiplications**	$K=4$, $N=4$	$K=8$, $N=4$
OAC	L/Z (with $Z=L+K-1$ or $L+K-2$)	Z	$Z^3N^3+Z^2N^2+L(2LN-L+1)N$	115200 ($L=9$)	11357024 ($L=49$)
Toeplitz	L/Y (with $Y=L+K-1$)	Y	$Y^3N^3+Y^2N^2+L(2LN-L+1)N$	115200 ($L=9$)	11357024 ($L=49$)
GSTBC	$2(K-1)/K$	K	$2K^3N^3+K^2N^2+(4-K)N+2\sum_{k=2}^{K/2}((K-2k+2)^3N^3+(K-2k+2)^2N^2+8(K-2k+2)N)$	9728	105968
QOSTBC	1	$K=4$	$6(6N+1)\|\mathcal{S}\|^2$	9600 ($\|\mathcal{S}\|=8$)	—

V. COMPLEXITY ANALYSIS

First, we discuss the complexity of different decoders (ML, MMSE and SIC-based decoders) applied to the same proposed GSTBC code. The complexity is measured by the number of complex multiplications (CMs). Table I shows the details of the complexity comparison. Although the exact complexity of the $Q\times Q$ matrix inversion can vary depending on their implementation, we approximate its number of CMs as Q^3. Note that the complexity of SIC-based decoding in each layer decreases with layer m because of the decrease of signal dimension. It can be seen that the SIC-based decoder has a significantly lower complexity compared to the exhaustive ML search and just slightly more complex compared to the MMSE decoder. For example, the ratio of CMs between the SIC-based and MMSE decoders are $1.1197, 1.3325$, and 1.5627 for $N=K=4,6$, and 8, respectively.

Second, we compare our code to the quasi orthogonal STBC (QOSTBC) [4] (which has a quite low decoding complexity) and recently proposed linear codes - Toeplitz [15], [16] and OAC [17], [18]. A comparative summary is shown in Table II. For the Toeplitz and OAC codes, L is the number of symbols encoded in each codeword matrix. In the Toeplitz code, we restrict the beamforming matrix $\mathbf{B}$ to the case of $\mathbf{B}=\mathbf{I}$ as we focus on linear receiver only. We choose two particular cases of $K=4, L=9$ and $K=8, L=49$ to make sure all codes have the same throughput of 3 bits/s/Hz and 3.5 bits/s/Hz, respectively. Our code provides a much higher rate with a smaller delay compared to the Toeplitz and OAC codes. For example, for the case of $K=4, L=9$, the proposed GSTBC provides a pair of rate and delay of (3/2, 4) while those of Toeplitz (or OAC) codes are (3/4, 12). Those pairs for the case of $K=8, L=49$ would be (7/4, 8) and (7/8, 56) for GSTBC and Toeplitz (or OAC), respectively. Thus, the resultant complexity of the proposed GSTBC is much lower compared to the linear codes. However, the QOSTBC has a lower decoding complexity compared to the GSTBC (if a relatively low order modulation scheme is employed for the QOSTBC) at the expense of a lower code rate.

VI. SIMULATION RESULTS

MIMO systems with different numbers of receive and transmit antennas are used to verify the performance of the proposed GSTBC with SIC based decoding. The Alamouti STBC [2] is used as the building block (i.e., $P=2$). The channel is assumed to be flat fading. Each path gain from a transmit antenna to a receive antenna is assumed to be a circularly symmetric complex zero-mean Gaussian variable with variance $\sigma_h^2=1$. The simulations include error propagation.

Figure 2 shows the BER performance versus the bit energy to noise power density ratio (E_b/N_0) when different building blocks are used in forming the GSTBC codeword matrix. As it is difficult to obtain the same throughput given the same number of transmit/receive antennas for different codeword matrices, we carry out two separate comparisons: i) Alamouti blocks versus single symbols with $K=N=4$; and ii) Alamouti blocks versus QOSTBC blocks with $K=N=8$. Note that block size is $P=1,2$, and 4 for the case of single symbols, Alamouti blocks and QOSTBC blocks, respectively. Either 64-QAM or 128-QAM is used to obtain same throughput of 10.5 bits/s/Hz in all cases. The results in both comparisons confirm our previous arguments in section II-A that choosing Alamouti code as building blocks would lead to a superior performance of the proposed GSTBC.

Figure 3 shows the BER performance of the proposed GSTBC when different numbers of receive antennas ($N=1,2,\cdots,6$) are used. It confirms that the number of receive antennas should be at least 2 for the SIC-based decoder to avoid the error floor problem. The SIC-based receiver works well in all the cases of $N\geq 2$.

We next consider the BER performance of the GSTBC in each layer with the number of layers being $M=3$ (i.e. number of transmit antennas $K=6$). Fig. 4 shows that the theoretical probability of error in each layer has the same

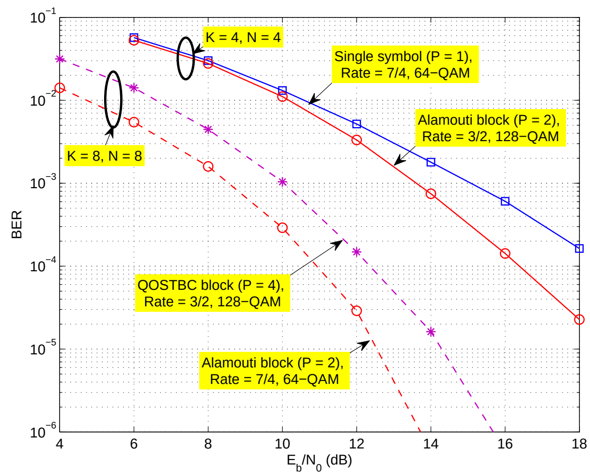

Fig. 2. BER performance of the proposed GSTBC with different building blocks: single symbol, Alamouti block, and QOSTBC block. The throughput of 10.5 bits/s/Hz is achieved in all cases.

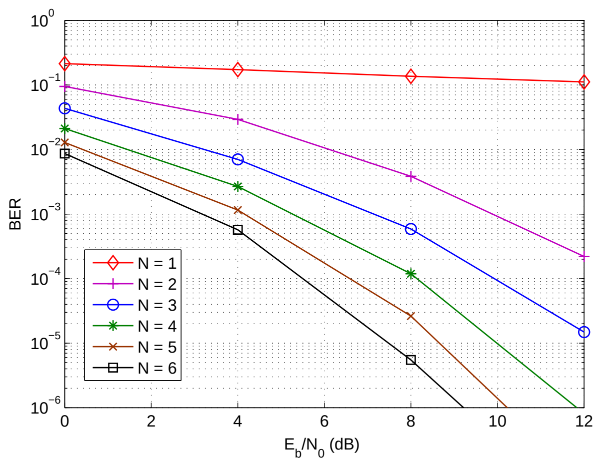

Fig. 3. BER performance of the proposed GSTBC using SIC-based decoder with different numbers of receive antennas for $K = 12$ (i.e., $M = 6$).

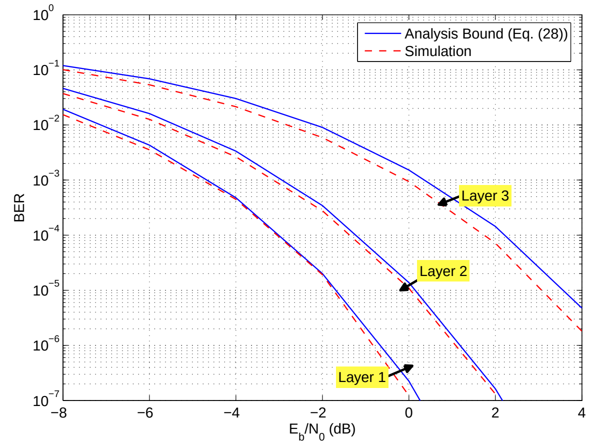

Fig. 4. BER performance of the proposed GSTBC using SIC-based decoder in each layer with $M = 3, N = 10$.

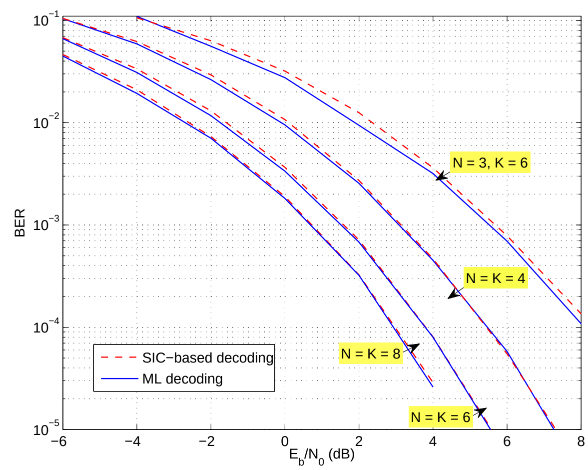

Fig. 5. BER performance of the proposed GSTBC with different numbers of layers $M = 2, 3, 4$.

diversity behaviour compared to its corresponding simulation BER curve. The theoretical bound is proved to be quite tight. It can also be observed that the slopes of curves among layers are noticeably different. The lower layer has a recognisably higher diversity gain. This is consistent with our analysis in section IV-B. It is noteworthy that the BER at a lower layer is always smaller due to the signal repetition design.

In Fig. 5, our simulation results show that the layered SIC based decoder using linear MMSE filtering can obtain near ML performance. This is shown with different combinations of numbers of transmit and receive antennas: $N = K = 4$; $N = 3, K = 6$; $N = K = 6$; and $N = K = 8$. These results are in agreement with the analytical statement in [28] that the successive decoding using a linear MMSE filter allows capacity-approaching performance if the initial decoding of the first signal (in a successive decoding sequence) is sufficiently reliable. It is noteworthy that the performance gap between ML and linear SIC-based decoders is closer when the number of antennas increases.

We also verify the performance of the GSTBC with different numbers of transmit antennas given the same throughput, as shown in Fig. 6. Two cases of $K = 4$ and $K = 8$ are considered while the same number of receive antennas $N = 4$ is used. To obtain the same throughput of 10.5 bits/s/Hz, 64-QAM is used for the case of $K = 8$ while 128-QAM is used for the case of $K = 4$. There is about 1.5 dB performance gain of 8 transmit antennas obtained over 4 transmit antennas at the BER of 10^{-3}. This shows that the proposed GSTBC is efficient in achieving spatial diversity/coding gains when the number of transmit antennas increases.

Figure 7 shows the BER performance comparison with orthogonal codes when $K = N = 4$. Particularly, our code is compared to the orthogonal OSTBC (of rate 3/4) [5] and QOSTBC. Note that for the same throughput of 3 bits/s/Hz, we use 8-phase shift keying (8-PSK) for the QOSTBC, 16-QAM for the OSTBC and QPSK for the proposed GSTBC. Because of the high rate, the proposed GSTBC obtains a significant performance gain over all other codes even though

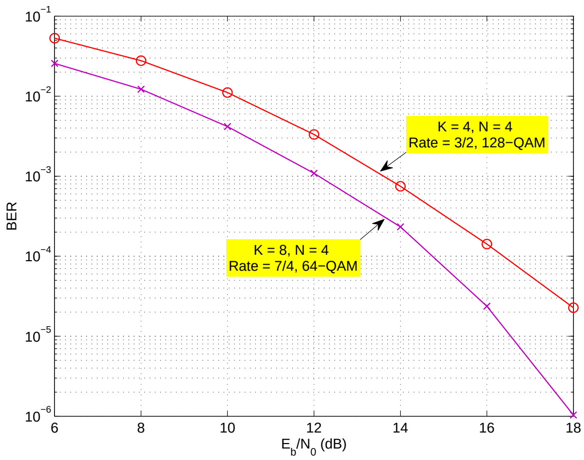

Fig. 6. BER performance of the proposed GSTBC with different numbers of transmit antennas, $K = 4$ and $K = 8$, while using the same $N = 4$ and achieving the same throughput of 10.5 bits/s/Hz.

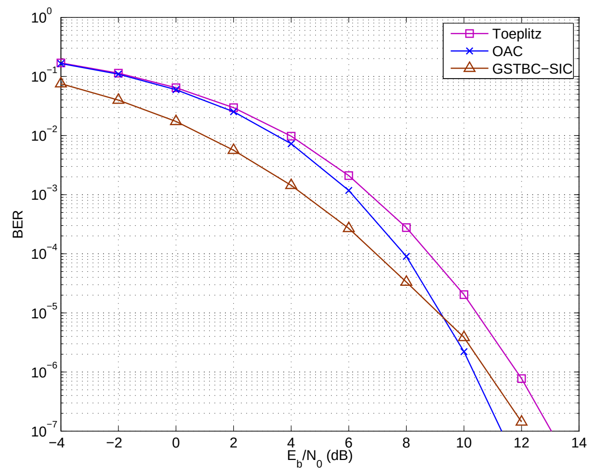

Fig. 8. BER performance comparison with Toeplitz-based codes for $K = 8, N = 4$ and throughput of 3.5 bits/s/Hz.

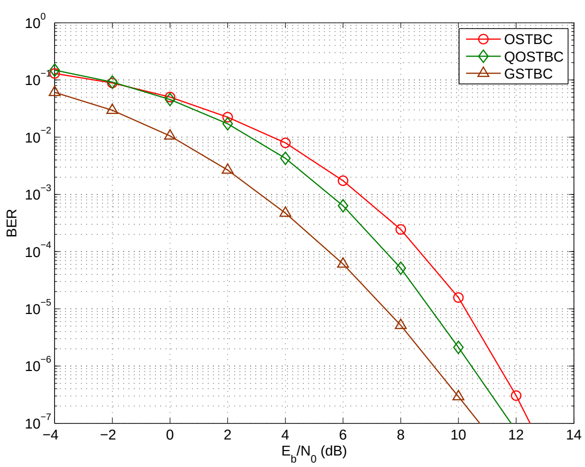

Fig. 7. BER performance comparison with orthogonal codes for $K = N = 4$ and throughput of 3 bits/s/Hz.

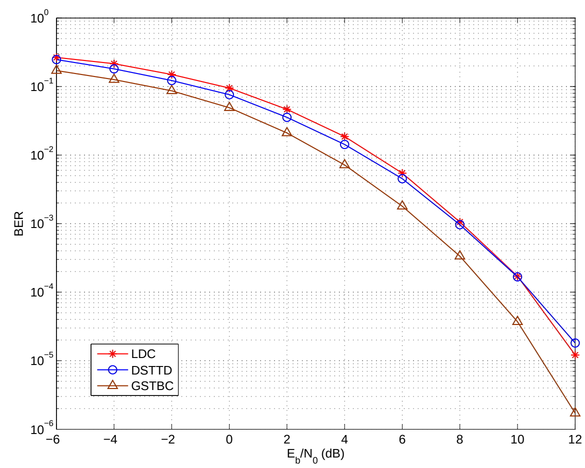

Fig. 9. BER performance comparison with the linear dispersion and DSTTD codes (both with code rate of 2) for $K = 4, N = 4$ and throughput of 6 bits/s/Hz.

the diversity order is lower.

We then compare the proposed GSTBC with the lower rate codes equipped with linear decoders: Toeplitz [15], [16] and OAC [17], [18]. The results are shown in Fig. 8 for the case of $K = 8, N = 4$. Both Toeplitz and OAC have a code rate of 7/8. QPSK is used for GSTBC while 16-QAM is used for Toeplitz and OAC to obtain the same throughput of 3.5 bits/s/Hz. The proposed GSTBC has a performance gain of about 2 dB and 2.5 dB at the BER of 10^{-3} (which is of interest to the users) compared to the OAC and Toeplitz, respectively. However, at a really high E_b/N_0, the OAC and Toeplitz tend to perform better as they have a higher diversity gain.

Finally, we compare the GSTBC with the higher rate codes equipped with similar SIC based receiver: DSTTD and LDC. The results are shown in Fig. 9 for the case of $K = N = 4$. Both the DSTTD and LDC has a code rate of 2 which is higher than rate 3/2 of the GSTBC. For the LDC, we use the same example described in section IV of [22], where the codeword matrix is constructed from 12 data symbols over 6 time slots using two sets of 12 dispersion matrices of size 6×4. To obtain the same throughput of 6 bits/s/Hz, 16-QAM is used for the GSTBC while 8-PSK is used for the DSTTD and the LDC. While there is no obvious diversity gain of the GSTBC obtained over the LDC, the coding gain is noticeable. Both the GSTBC and the LDC have higher diversity gains compared to the DSTTD.

VII. Conclusion

This paper proposed a simple but effective high-rate GSTBC in conjunction with an SIC based receiver. As the GSTBC is designed in a layered manner, the MMSE filter exploiting the dimension expansion of the signal at each layer was used to suppress the interfering signals effectively. For even a large number of transmit antennas, it requires only 2 receive antennas for the SIC-based receiver to avoid error floor problem. Consequently, the proposed scheme can be easily extended to large MIMO systems with near ML performance. The proposed code offers a gain of almost 2 dB over other recently proposed linear codes at a BER of 10^{-3} for the case of 8 transmit antennas.

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