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# Exploring Markov models for gate-limited service and their application to network-based services

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# Outline of Talk

- Context
- Types of Service
- Exhaustive-Limited Service
- Gated-Limited Service
- An Approximate Solution
- Application to Network-Based Services
- Conclusion and Future Work

# Context: Queuing Theory

- Customers being served in some way
  - Teller at bank, roving salesman, etc
- Servers do work on customers after which customers may leave the system or join another queue for further service.
- Servers may serve other queues
- Different ways of serving
  - Exhaustive
  - Gated

# Types of Service

- Exhaustive – the server serves until the queue is empty. So it serves customers who arrive after service on the queue has started.
- Gated Service – the server only serves the customers in the queue at the start of service. Customers arriving after service has begun are served on subsequent server visits.

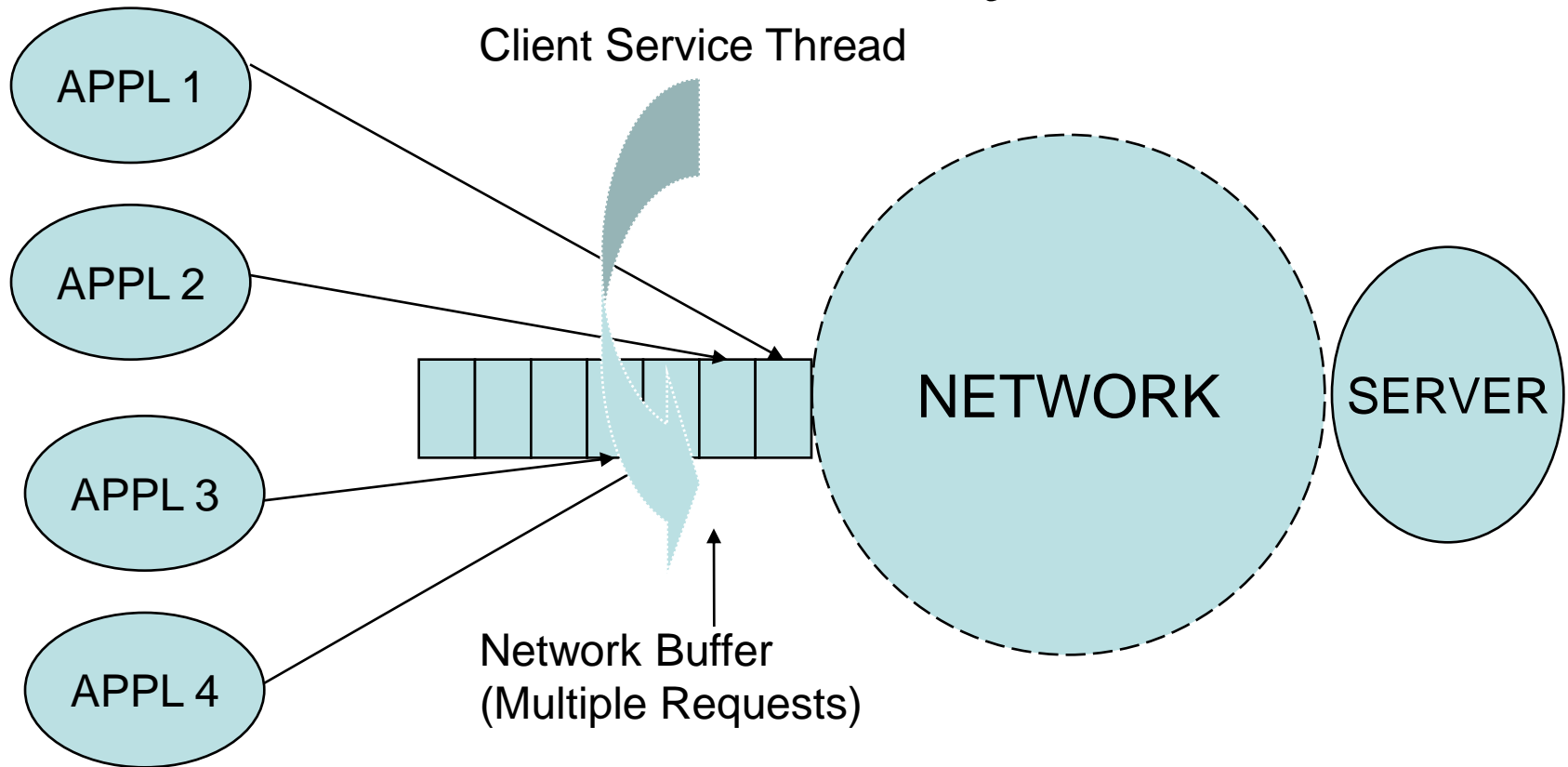
# Limited Service Derivatives

- Exhaustive-Limited: The server serves a maximum of  $k$  customers. Other customers are serviced on subsequent visits.
- Gated-Limited: The server only serves a maximum of  $k$  customers that it finds in the queue when it arrives. Other customers are serviced in the same manner but on subsequent visits

# Application

- Different applications may have different service models
- We focus on gated-limited systems for a number of reasons
  - Transport:
    - Large systems such as buses can be modelled as gated-limited servers.
  - Network Services can also be modelled as gated-limited service

# Simple Example of a Network-Based System





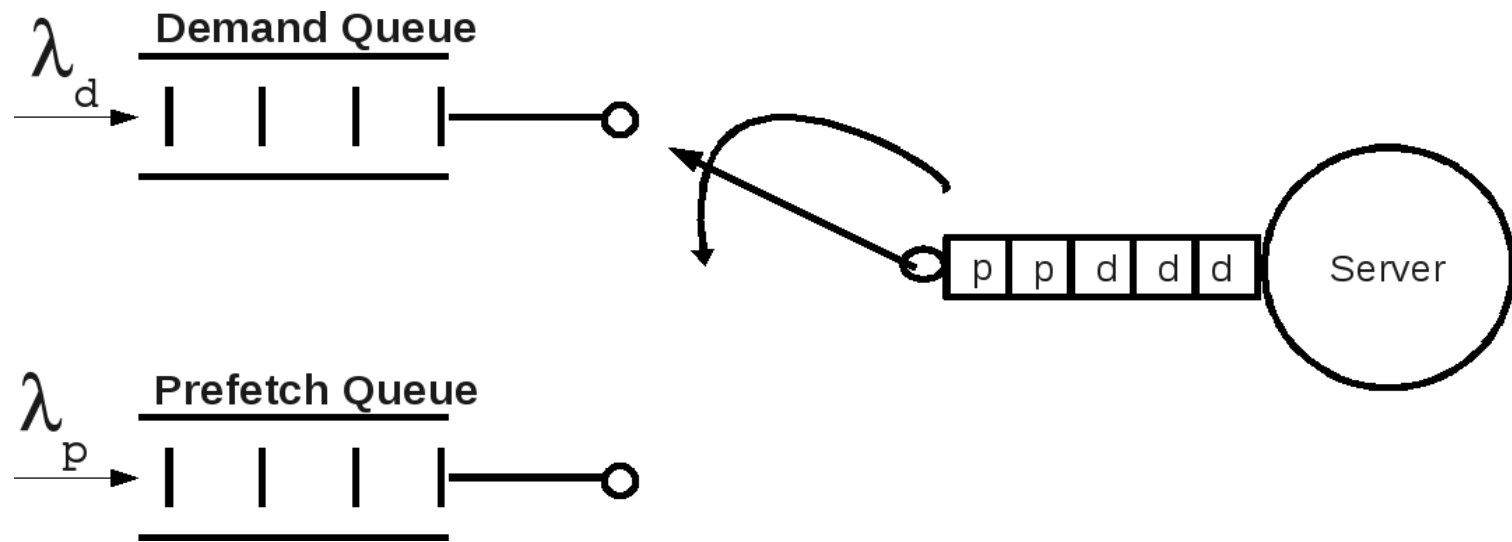
# Key Observations

- Once the network buffer is sent off to the server, applications must wait before putting new requests in the buffer
- The buffer is finite, so there is a maximum number of requests,  $K$ , that can be serviced at the same time
- Gated-Limited Service

# Motivation

- Everything is going to be network-based
  - The network is the computer
- New streaming applications are being used that require better than best-effort service.
  - You-Tube; BBC iPlayer, etc
- Can use pre-fetching techniques
  - but we have to see about demand misses

# Network-Based Storage Service to support pre-fetching



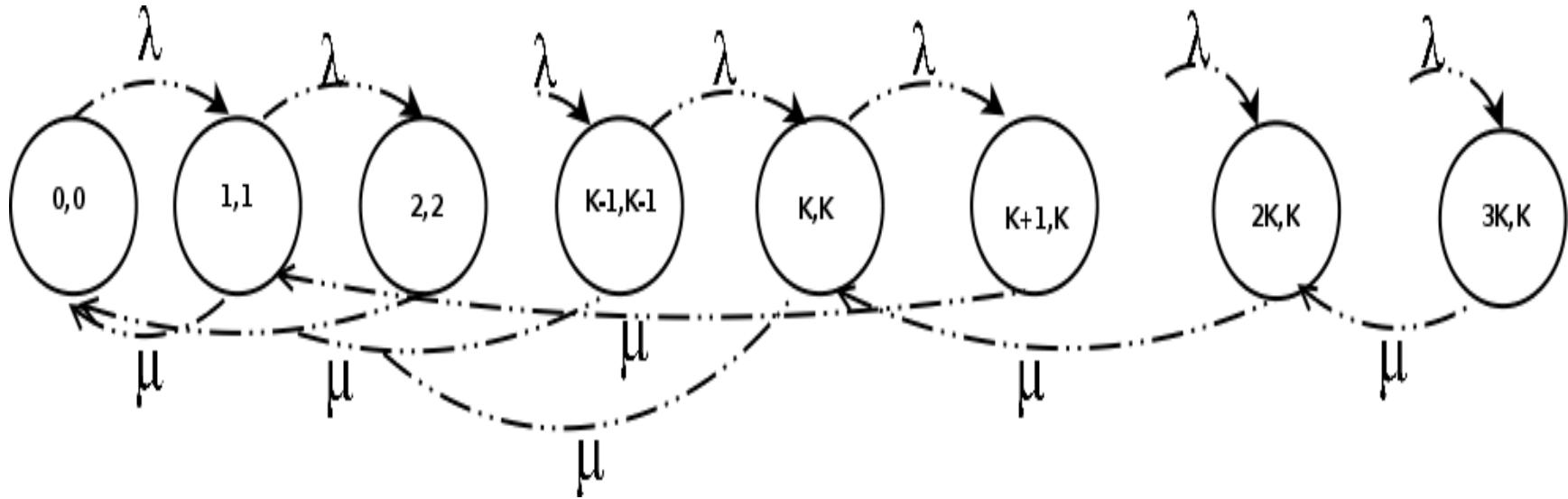
# What's already been done

- If you look at the literature, a lot of work has been done on exhaustive or exhaustive-limited systems
- Gated service also studied
- Gate-limited solutions exist but are generally too hard to calculate
  - Not back-of-the envelope stuff

# Standard Solution: The Partial Bulk Service Model (PBM)

- Found in standard text-books:
- Each Markov state is defined by 2 parameters:
  - $n$  = the total number of customers in the system
  - $s$  = the number of customers currently being served
- Note: the maximum number of customer that can be served at the same time is given by  $K$ .

# PBM Con't



The PBM is exhaustive-limited service, but we need to understand the solution. If  $K = 1$ , we have normal MM1. The general solution is quite similar to the MM1 solution

The balance equations for PBM:

$$0 = -(\lambda + \mu)p_n + \mu p_{n+k} + \lambda p_{n-1}$$

$$0 = -\lambda p_0 + \mu p_1 + \mu p_2 + \mu p_{K-1} + \mu p_K$$

For  $k = 1$ ;

$$0 = -(\lambda + \mu)p_n + \mu p_{n+1} + \lambda p_{n-1}$$

$$0 = -\lambda p_0 + \mu p_1$$

This are the basic equations for the  
MM1 queue!

So we can say solution for PBM is the same as the MM1 so:

$$p_n = p_0 r^n$$

Where  $r$  between 0 and 1. For the MM1 queue;  $r = \rho$  The mean queue length ( $L$ ):

$$L = \frac{r}{1 - r}$$

The average waiting time ( $W$ ):

$$W = \frac{r}{\lambda(1 - r)}$$



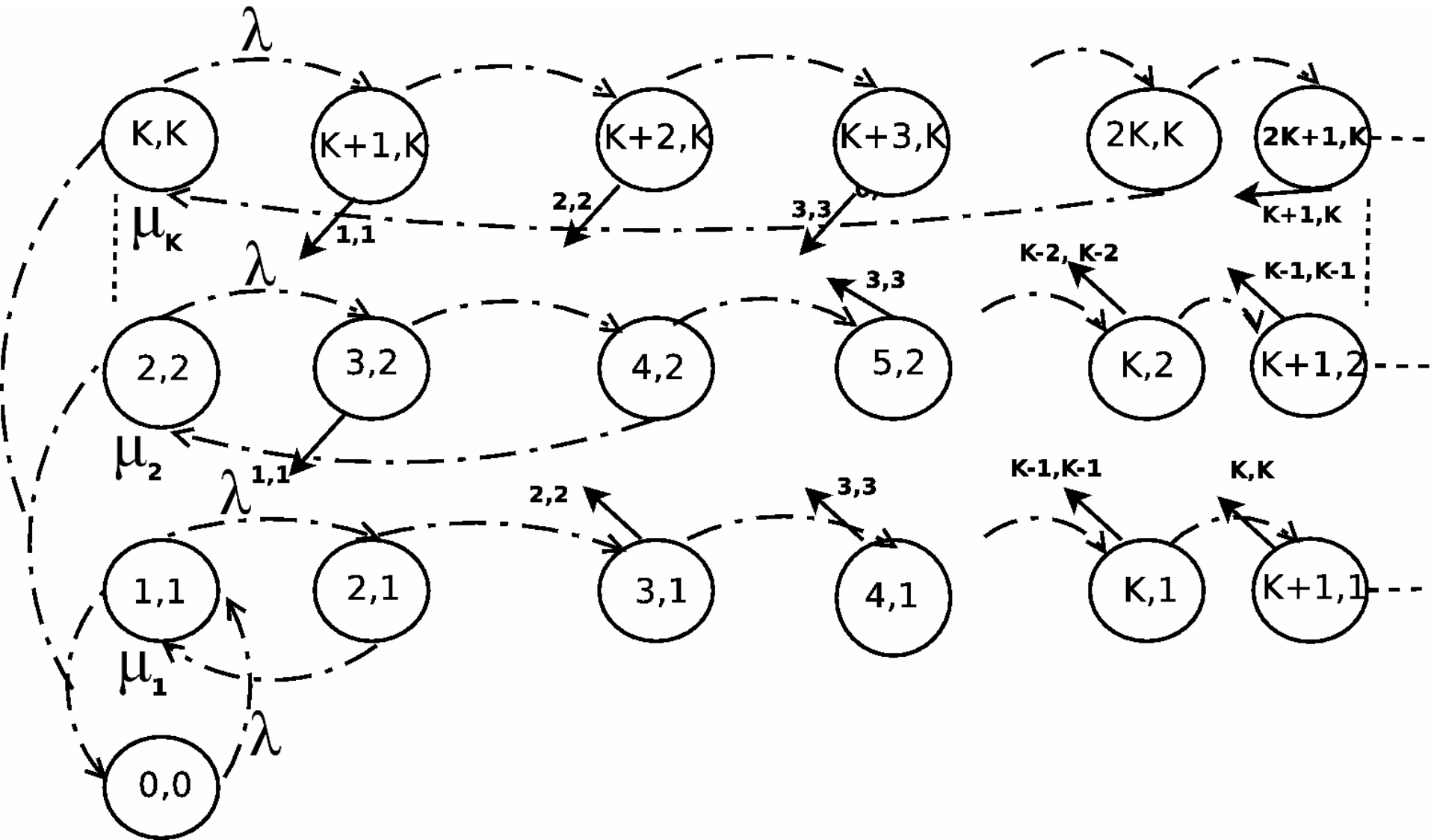
# Evaluation of PBM

- Exhaustive-limited not gated-limited
- However, at extremely high loads, the gated-limited waiting times will approach the exhaustive waiting times because there will almost always be  $K$  or more customers in the queue.
- We need to remember this!

# Our approach

- Use Markov Models
  - Previous approaches have been very mathematical from the word go!
- Look at arrival and departure moments
- For the gated-limited model, the number of customers served in the next cycle is evaluated at the end of the current cycle.

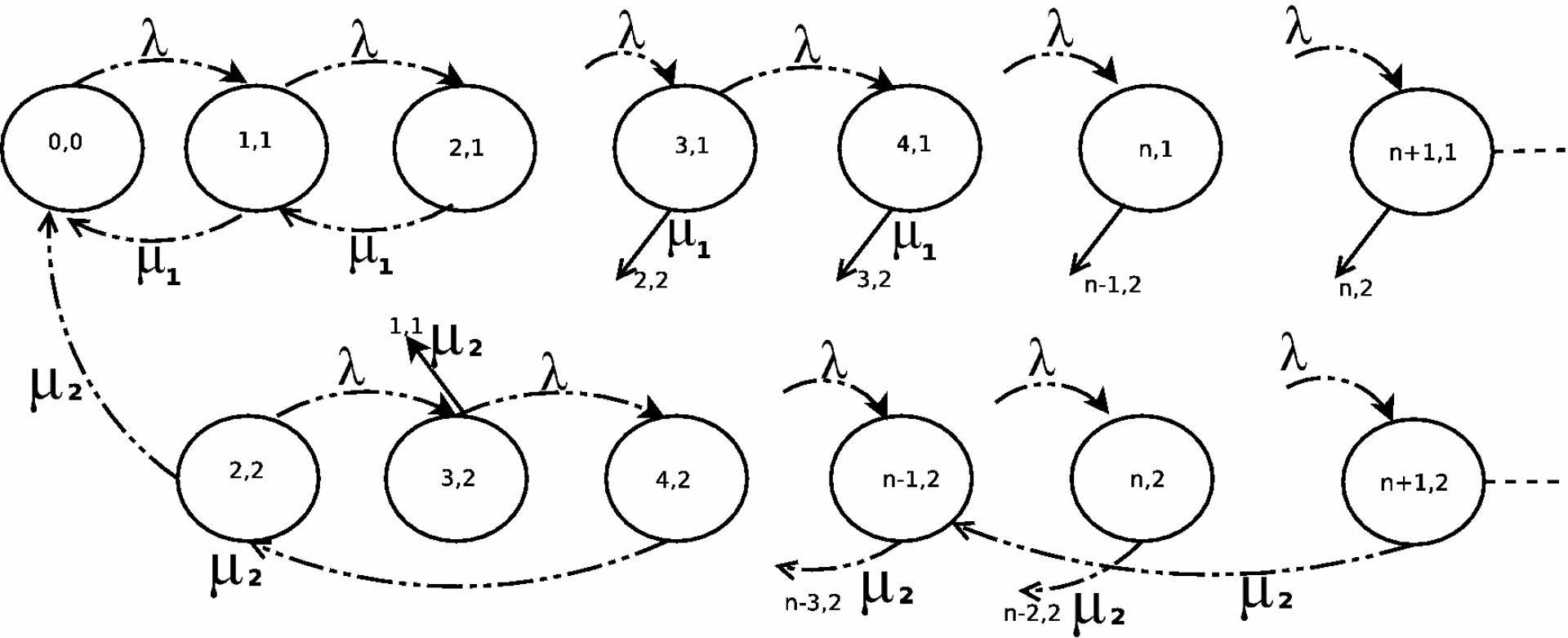
# Our Markov Model for Gate-Limited



# Model is complicated because..

- There are several chains where each chain represents the number of customers currently being served.
  - So Chain 1, represents 1 customer being served, etc.
- You can jump over several chains in one go depending on how many people are left in your queue at the end of the current service time.

# Gated Limited Model for $K=2$



# How do you solve this model?

- There are  $K$  chains; so if  $K$  is 2; there are 2 chains
- Our approach
  - Try to express the probability of being in each state of each chain in terms of the probability of lowest element in the chain. For  $K = 2$ ; we need to express Chain 1 in terms of  $P_{11}$  and  $P_{22}$  for Chain 2.
  - Then we concentrate on solving each chain

For CHAIN 1:

$$\lambda p_{n-1,1} = (\lambda + \mu_1) p_{n,1}$$

For any  $n > 1$ , in Chain 1:

$$p_{n,1} = \frac{\lambda}{\lambda + \mu_1} p_{n-1,1}$$

So we can write:

$$p_{n,1} = \left( \frac{\lambda}{\lambda + \mu_1} \right)^{n-1} p_{1,1}$$

And for  $n = s = 1$ , we have:

$$(\lambda + \mu_1) p_{1,1} = \lambda p_{0,0} + \mu_1 p_{2,1} + \mu_2 p_{3,2}$$

Finally, for  $n = s = 0$ :

$$\lambda p_{0,0} = \mu_1 p_{1,1} + \mu_2 p_{2,2}$$

For CHAIN 2:

$$(\lambda + \mu_2)p_{n,2} = \lambda p_{n-1,2} + \mu_2 p_{n+2,2} + \mu_1 p_{n+1,1}$$

And for  $n = s$  :

$$(\lambda + \mu_2)p_{2,2} = \mu_2 p_{4,2} + \mu_1 p_{3,1}$$

$$(\lambda + \mu_2)p_{3,2} = \lambda p_{2,2} + \mu_2 p_{5,2} + \mu_1 p_{4,1}$$

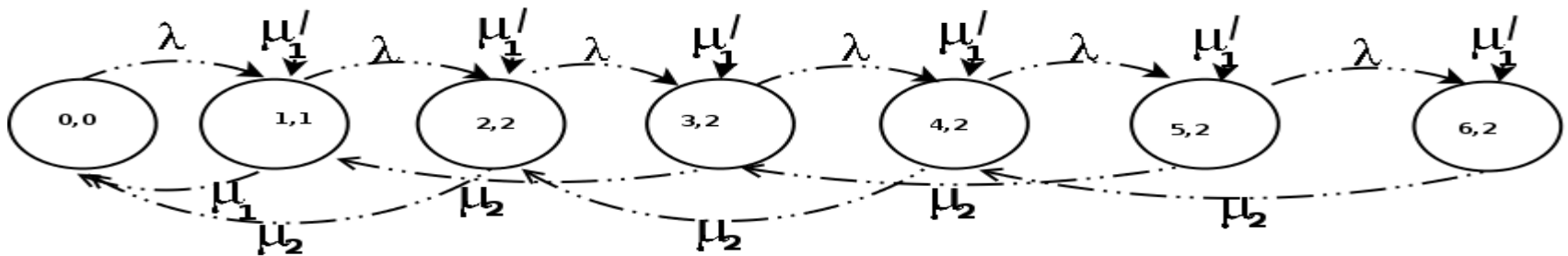
$$\mu_1 p_{4,1} = (\lambda p_{0,0} - \mu_2 p_{2,2}) \left( \frac{\lambda}{\lambda + \mu_1} \right)^3$$

$$0 = -(\lambda + \mu_2)p_{3,2} + \lambda p_{2,2} + \mu_2 p_{5,2} \\ + (\lambda p_{0,0} - \mu_2 p_{2,2}) \left( \frac{\lambda}{\lambda + \mu_1} \right)^3$$



# The Effect of this Approach

By only expressing Markov state equations for Chain 2 in terms of other Markov states for Chain 2, we are saying that we can imagine that the states for Chain 2 in the gated-limited model to be part of an IMAGINARY PBM chain. So we can solve for root  $r$ , between 0 and 1, just like in the PBM approach.



$$p_{n,2} = p_{2,2} r^{n-2}$$

So we can write:  $p_{3,2} = r p_{2,2}$

$$p_{1,1} = C_{1,1} p_{2,2}; \quad p_{0,0} = C_{0,0} p_{2,2}$$

$$C_{1,1} = \mu_2(1+r) \left( \frac{\lambda + \mu_1}{\lambda^2} \right)$$

$$C_{0,0} = \frac{\mu_1 C_{1,1} + \mu_2}{\lambda}$$

$$S_1 = \sum_{n=1}^{\infty} p_{n,1}; \quad S_2 = \sum_{n=2}^{\infty} p_{n,2}$$

Then we can say that:

$$p_{0,0} + S_1 + S_2 = 1$$

$$p_{2,2} = \frac{1}{C_{0,0} + \frac{\lambda + \mu_1}{\mu_1} C_{1,1} + \frac{1}{1-r}}$$

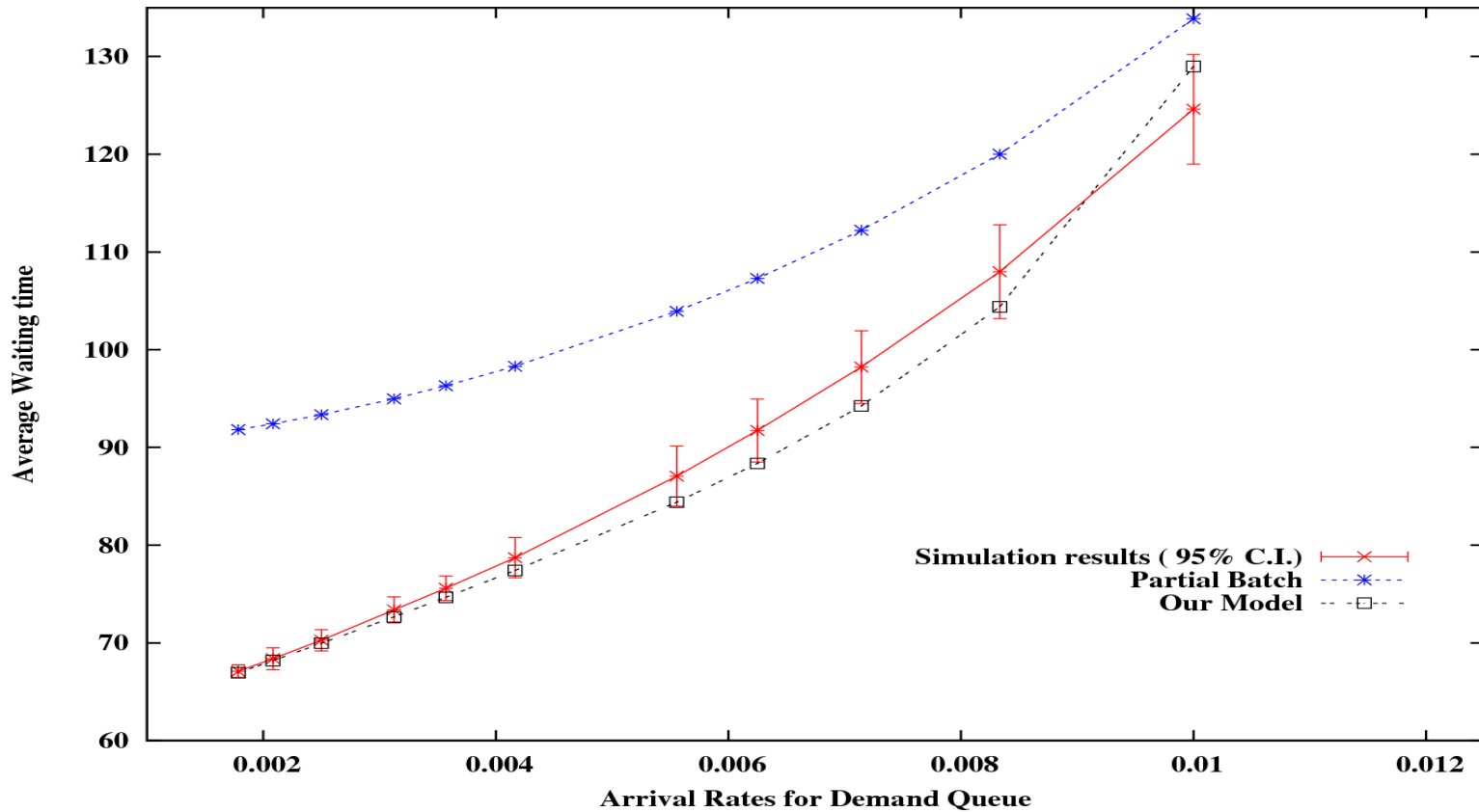
The average queue length,  $L$  is:

$$L = \sum_{n=1}^{\infty} n \left( \frac{\lambda}{\lambda + \mu_1} \right)^{n-1} p_{1,1} + \sum_{n=2}^{\infty} n r^{n-2} p_{2,2}$$

$$L = \left( \frac{\lambda + \mu_1}{\mu_1} \right)^2 p_{1,1} + \frac{2-r}{(1-r)^2} p_{2,2}$$

Average waiting time:  $W_d = \frac{L}{\lambda_d}$

# Results for $K = 2$



# Towards a general solution

1

If we express  $L$  for each chain in terms of its first element and then sum we get:

$$L = \sum_{n=1}^k \frac{n - (n - 1)r_n}{(1 - r_n)^2} p_{n,n}$$

For  $n < k$ ,

$$r_n = \frac{\lambda}{\lambda + \mu_n}$$

For  $n = k$ , we use the imaginary PBM technique

# Towards a general solution

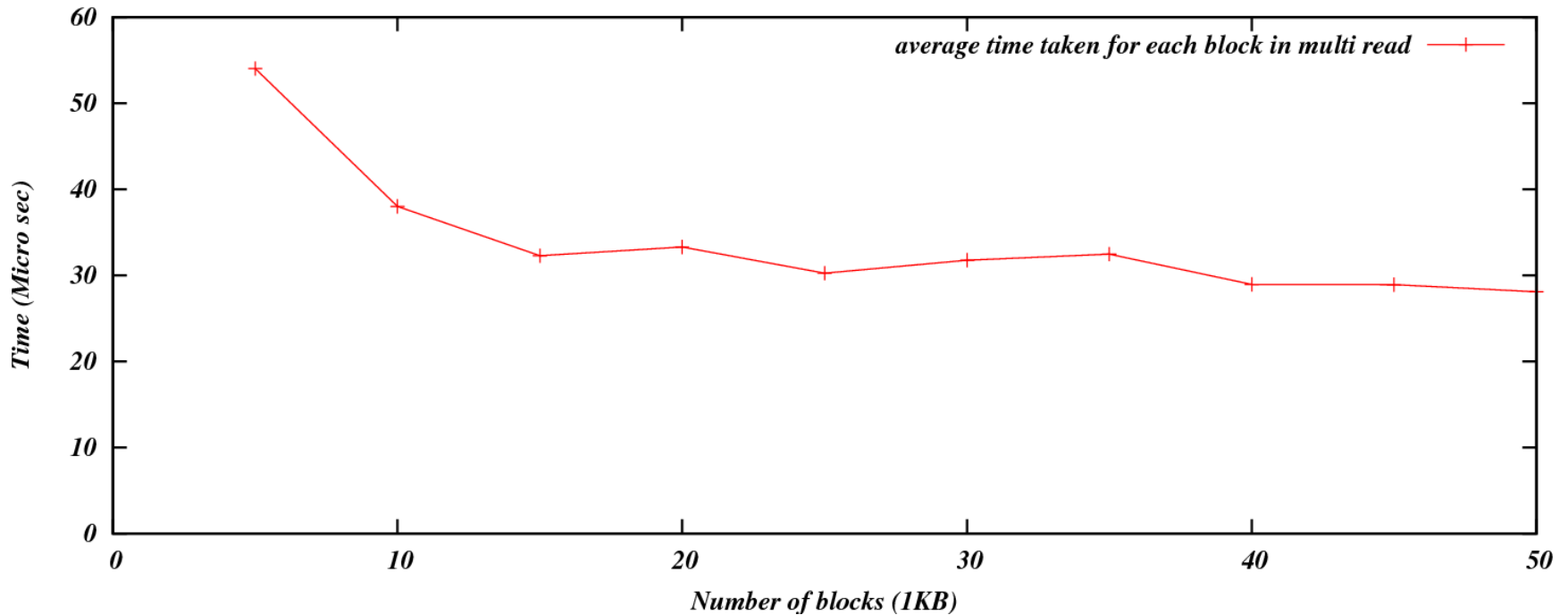
- The most difficult part is to calculate  $p_{n,n}$
- Use the equations for  $p_{n,n}$  and express these variables in terms of  $p_{k,k}$ .
- Mathematics is complicated
  - Need to look at using matrix techniques to solve these equations.
- Still a lot of work to be done but the approach looks promising.

# Application – Global Storage Server Project

- Aim
  - To develop a high performance globally accessible storage server
    - Eliminate dependency on local storage
      - More green: less power, noise
  - Network Memory Server (NMS)
    - Uses the memory of another machine
    - Appears as a hard-disk to the client OS

# Clustering in the NMS

*Plotting multi blocks against time*



• Time to Fetch  $p$  blocks =  $L + Cp$



# Approach

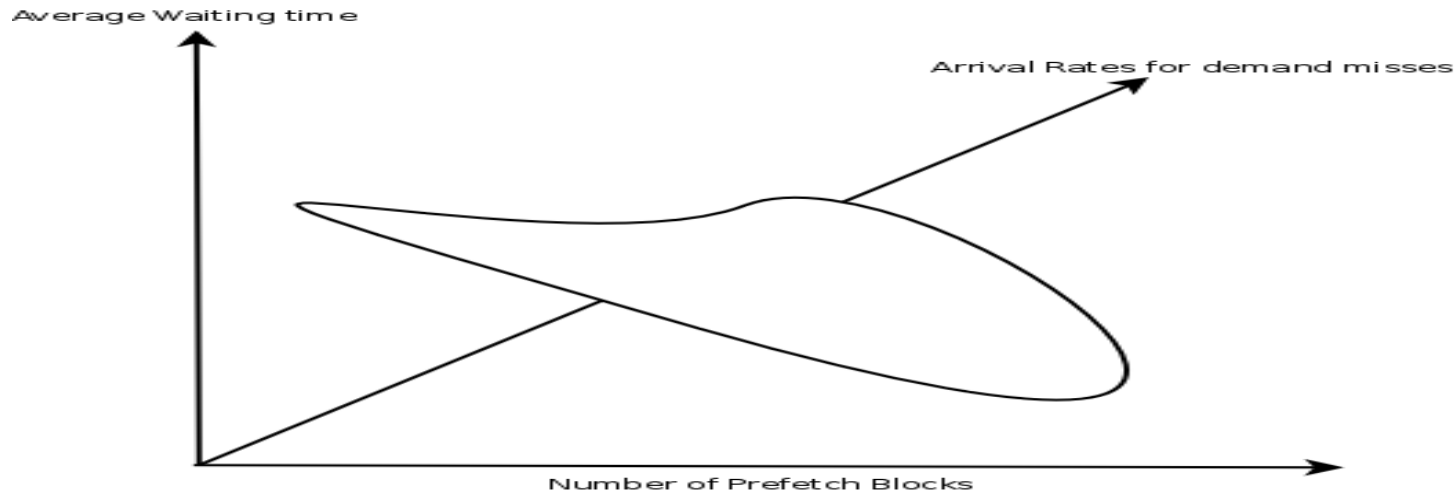
- To allow stream to run without jitter
  - Time to fetch < Time to process
    - $L + C_p < T_{cpu} * p$
- Average waiting time experience to satisfy Demand misses < the average waiting time on disk ( $T_{disk}$ )
  - $L + (d * C) + T_{wait} < T_{disk}$

# Looking at Conservative Prefetching (PonD)

- Prefetch only when there is a demand miss
- Therefore, using PonD:
  - $L + (p+d) * C < (T_{cpu} * p)$
- For demand misses, the waiting time  $< T_{disk}$ :
  - Time to fetch +  $T_{wait} < T_{disk}$ 
    - $L + (p+d) * C + T_{wait} < T_{disk}$

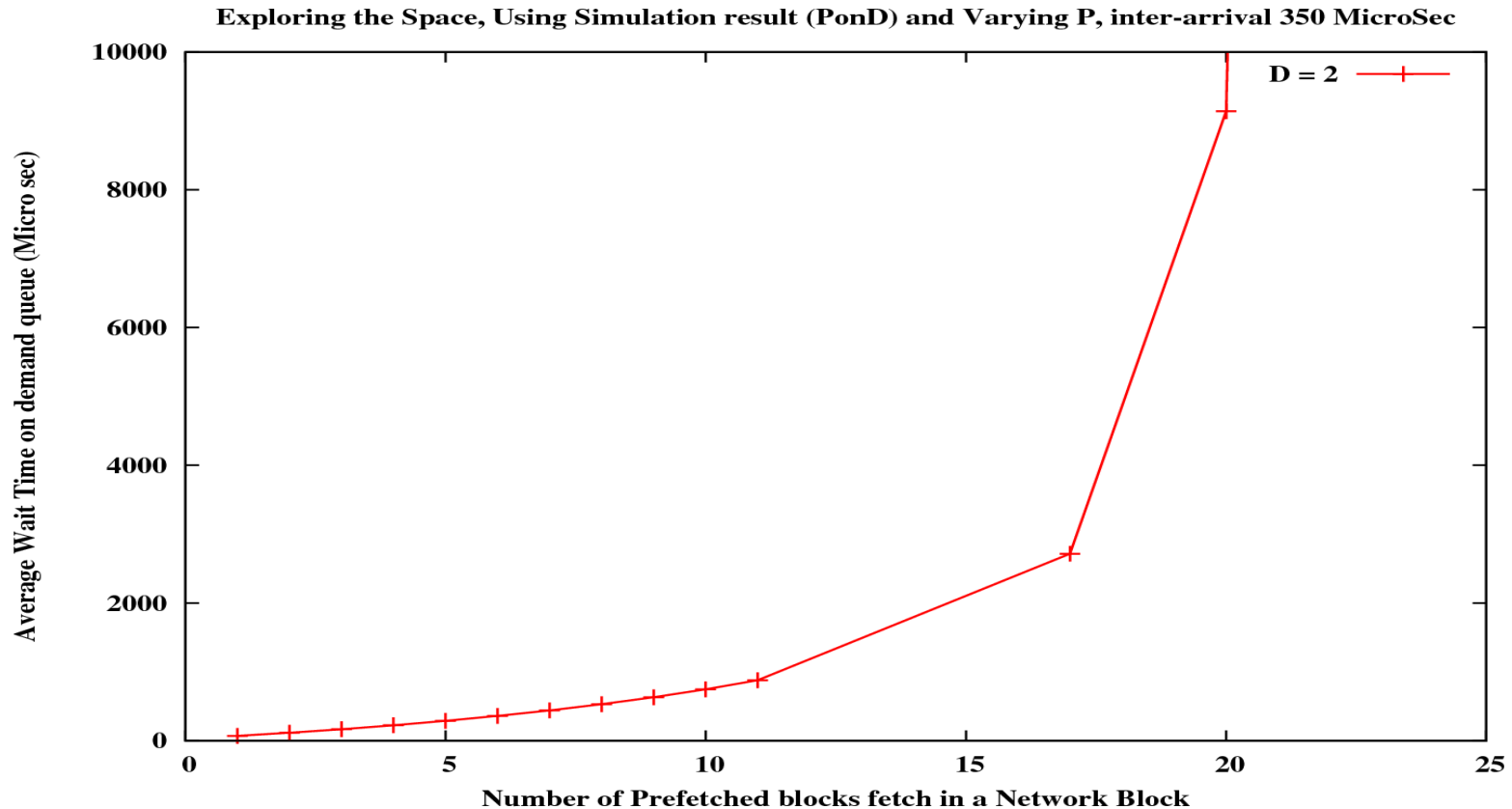
# Developing an Operational Space

- Define an operational space consisting of three axes

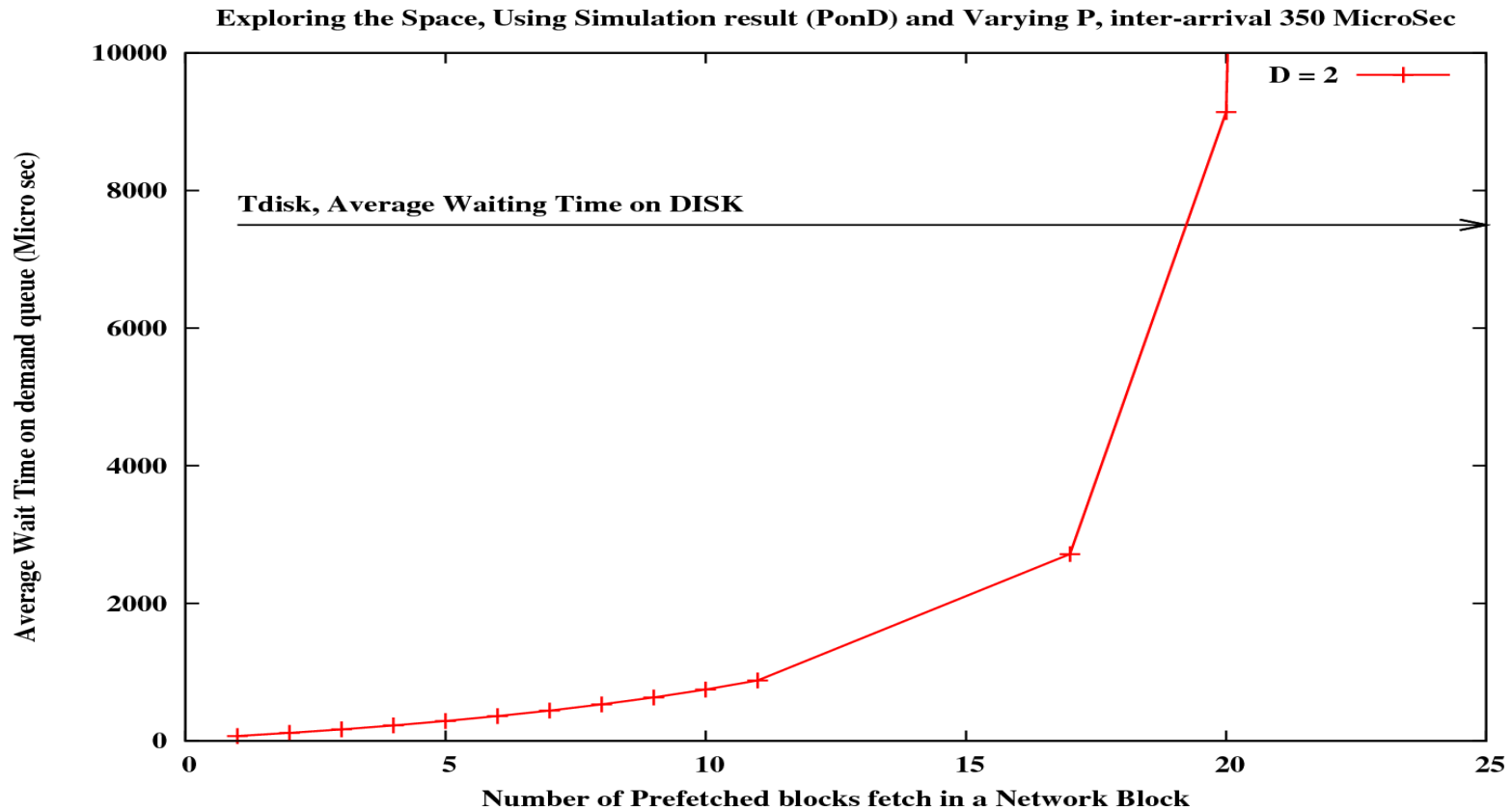


- $T_{wait, demand\ misses} < T_{disk}$
- $T_{net}(p+d) < P^* T_{cpu}$

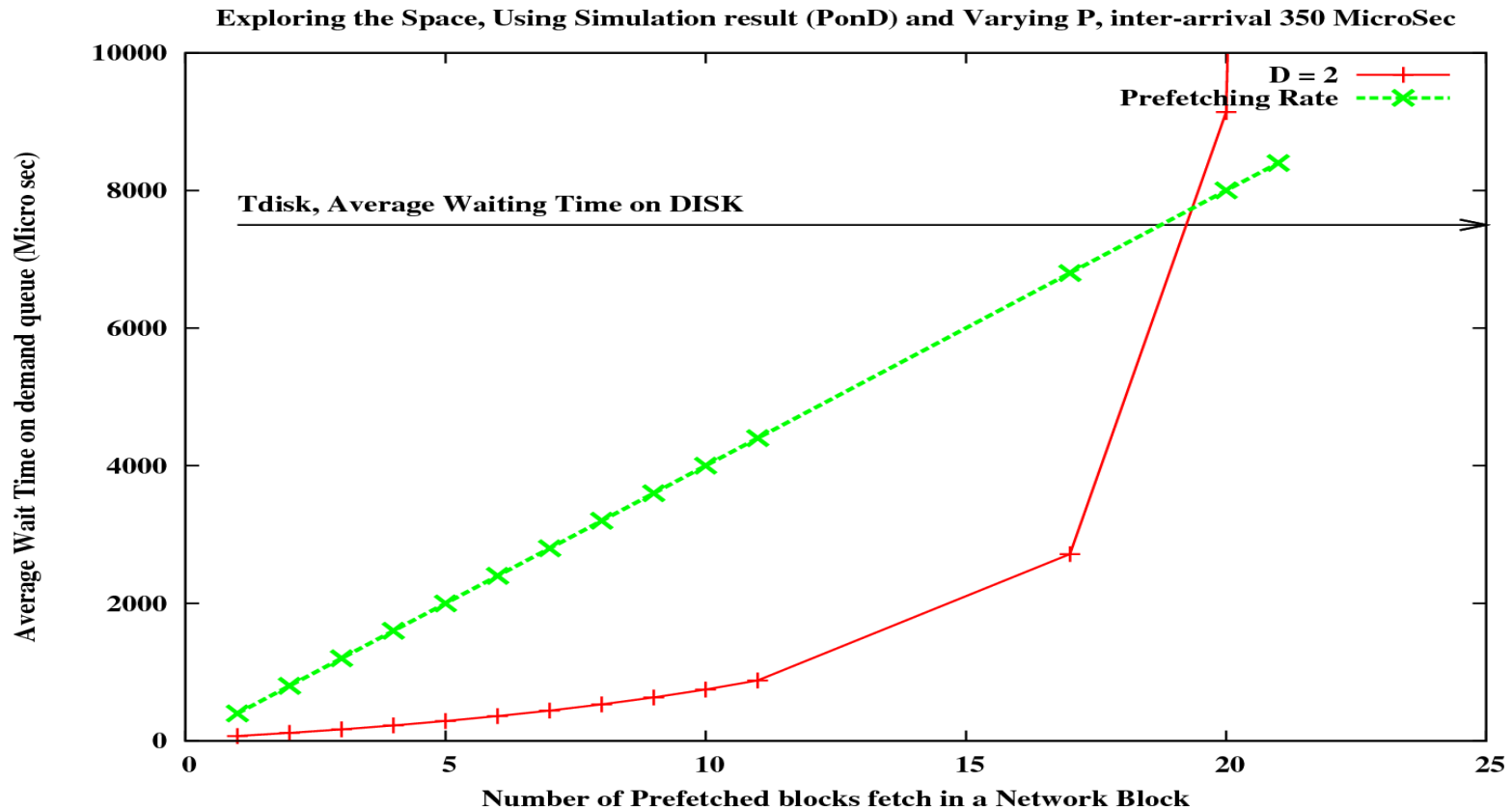
# Exploring the Space for a given inter-arrival time, 350 microseconds



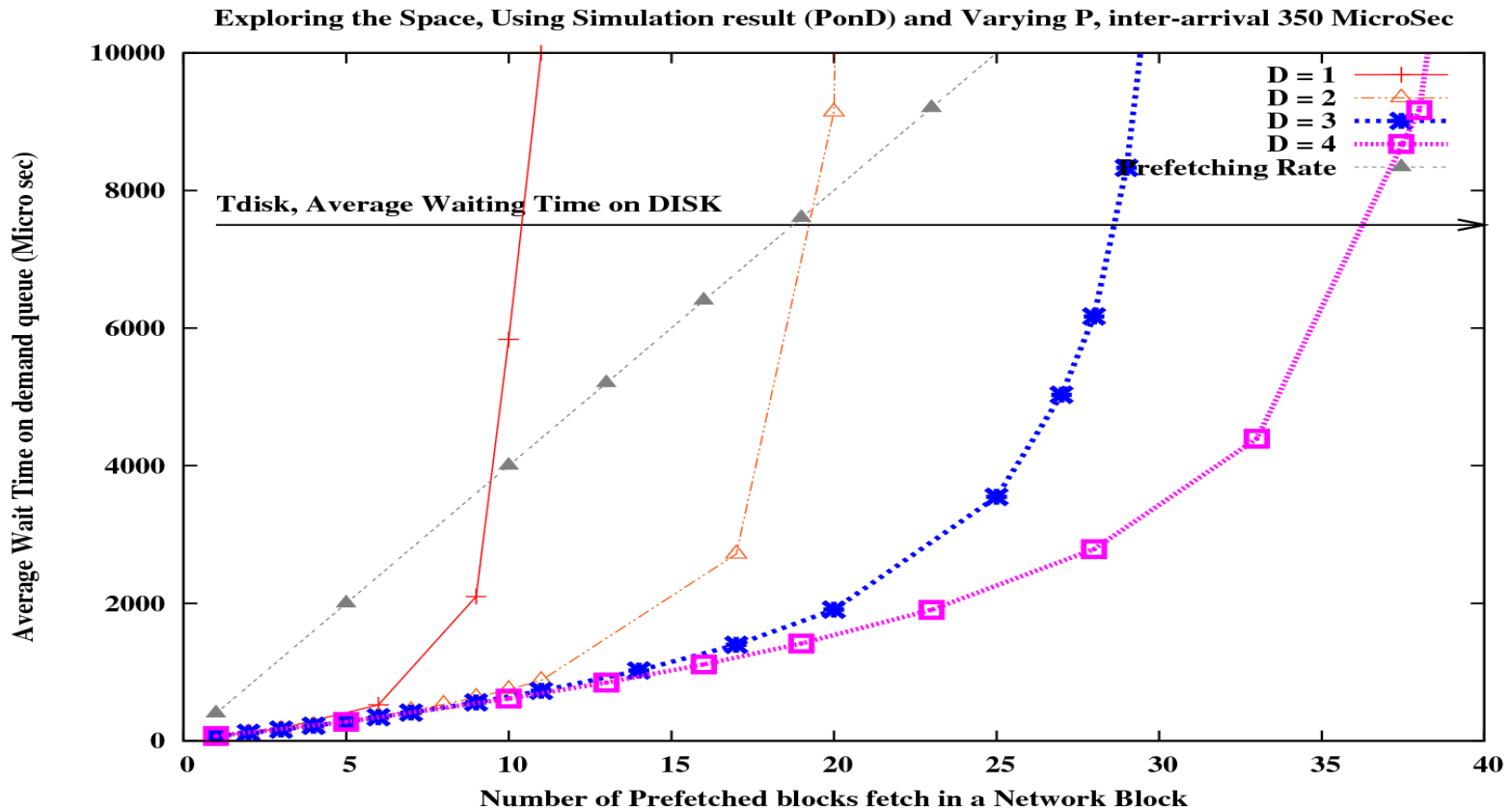
# Exploring the Space, 350 microseconds



# Exploring the Space, 350 microseconds



# Exploring the Space, 350 microseconds



# Future Work

- Further explore analytical model
- Investigate the effect of different network loads
- Develop a practical algorithm that can be part of an autonomous system that can balance pre-fetching and demand paging.
- Develop a file server that uses the algorithm and measure its performance.



Thank You

QUESTIONS?