Cross-Layer Optimisation for Topology Design of Wireless Multicast Networks Via Network Coding

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***Abstract*—One of the main challenges towards reliable multicast transmissions over wireless networks is the dynamics of the wireless links (e.g. wireless errors, fading, interference, collisions, etc.) that can cause retransmissions overhead over the limited available bandwidth. To this end this paper considers the scenario of wireless multicast networks where network coding is applied to improve network throughput. We first propose a novel cross-layer optimisation framework for network topology design in order to optimise the wireless multicast rate, data flow of the wireless links, energy supply and node lifetime. The performance of the proposed solution is evaluated and compared against other solutions from the literature in terms of system throughput, total energy, and network lifetime. The results show that the proposed cross-layer design outperforms the other schemes involved, reaching up to 50% increase in the system throughput.**

I. Introduction

Multicasting has attracted an increasing interest in wireless communications with extensive investigations [1]–[3]. With the advances of wireless communications, the demand of high-throughput multicasting is crucial, especially in services which require high multicasting traffic, e.g. teleconferencing [4] and multimedia streaming [5]. Recently, network coding (NC), which was initially proposed to increase the system throughput in lossless networks [6], has been applied at intermediate nodes to dramatically improve the throughput of wireless networks [7] by performing algebraic linear/logic operations on received packets at the intermediate nodes. In NC-based wireless multicast networks (WMNs), topology design has significant impacts on the system throughput [8]. The topology may affect the efficiency of NC in improving the system throughput since destination nodes may not receive enough linearly independent NC-based data packets to recover the original data packets. To the best of our knowledge, existent proposals in the literature do not entirely tackle the topology design under the constraints of quality-of-service (QoS) including multicasting rate, capacity of wireless channels, energy supply, and node/network lifetime.

To this extent this paper proposes a novel cross-layer optimisation framework for network topology design in order to maximise system throughput over design variables including wireless multicasting rate of source nodes, amount of wireless data flows, energy supply at nodes and lifetime of nodes subject to various QoS constraints (e.g. flow conservation, wireless link capacity, wireless multicast traffic rate, node energy, total energy and network lifetime). We model the optimisation problem as an NP-hard problem and propose a heuristic NC-based algorithm, namely NC-based link-controlled routing tree (LCRT) algorithm. The proposed NC-based LCRT is used to construct a multicasting tree and to reduce the number of relay nodes involved in the transmission process while guaranteeing the decoding capabilities at destination nodes. When the lifetime of nodes is fixed, the optimisation problem can be simplified to a linear programming problem and the network lifetime constraint can be relaxed as the lifetime of nodes approaches the network lifetime.

The proposed solution is evaluated and compared against other schemes from the literature in terms of system throughput, total energy and network lifetime. It is shown that the system throughput increases as either total energy available for network increases or network lifetime decreases. Additionally, the proposed NC-based LCRT algorithm achieves a significant increase in the system throughput when compared to the non-based LCRT protocol, especially with a large set of wireless multicasting nodes.

II. Network Model & Cross-Layer Optimisation

A. Network Model

A wireless network with a set of N nodes (denoted as $\mathcal{N}$), is considered, where each node can be a source, a relay, or a destination node. Let the distance and the capacity of the wireless link between two adjacent nodes, say the i-th node and the j-th node, be $d_{i,j}$ and $C_{i,j}$, respectively, where $\{i,j\} \in \mathcal{N}$. Suppose there are M wireless multicasting groups in our system. We use $\mathcal{S}_m = \{n_{m,0}, n_{m,1}, ..., n_{m,|\mathcal{S}_m|-1}\}$,$m \in \mathcal{M} \triangleq \{1,2,\dots,M\}$, to represent a subset of nodes in $\mathcal{N}$ that requires to join in the m-th group, where $n_{m,0}$ is the source of this group and $n_{m,l}$, $l \in \mathcal{L}_m \triangleq \{1,2,\dots,|\mathcal{S}_m|-1\}$, is a destination of this group. Also, we use K and B to denote the number of packets and the size of packets multicasted in the m-th group.

B. Cross-Layer Optimisation Framework

Design network topology to maximise system throughput:

$$\sum_{m=1}^{M} R_m \tag{1}$$

Over design variables:

1) Wireless multicast rate: $\{R_m\}$, $m \in \mathcal{M}$
2) Amount of flow on wireless links: $\{f_{i,j}^{(n_{m,0},n_{m,l})}\}$, $m \in \mathcal{M}$, $l \in \mathcal{L}_m$, $\{i,j\} \in \mathcal{N}$, $i \neq j$
3) Energy supply at nodes: $\{E_n\}$, $n \in \mathcal{N}$

4) Lifetime of nodes: $\{T_n\}$, $n \in \mathcal{N}$

Subject to:

1) *Flow conservation constraint*: For intermediate nodes assisting the wireless multicast in the network, the amount of total outgoing wireless multicast traffic should be equal to the amount of total incoming wireless multicast traffic. In the case of source or destination nodes, the amounts of incoming traffic and outgoing traffic are different and the difference should be the amount of traffic generated at sources. We thus have

$$\sum_{\substack{a\in\mathcal{N}\\a\neq n}} f_{n,a}^{(n_{m,0},n_{m,l})} - \sum_{\substack{b\in\mathcal{N}\\b\neq n}} f_{b,n}^{(n_{m,0},n_{m,l})} = \begin{cases} -R_m & \text{if } n=n_{m,l}, \\ R_m & \text{if } n=n_{m,0}, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where $n \in \mathcal{N}, m \in \mathcal{M}, l \in \mathcal{L}_m$.

2) *Wireless link capacity constraint*: The total amount of wireless multicasting traffic through the wireless link from the i-th node to the j-th node, $\{i,j\} \in \mathcal{N}$, should not exceed the capacity limitation of the wireless link, i.e. $C_{i,j}$. Here, for simplicity, let us consider additive white Gaussian noise (AWGN) channels. Thus, $C_{i,j}$ can be determined by $C_{i,j} = W \log_2(1+\gamma_{i,j})$ [bits/sec], where W and $\gamma_{i,j}$ denote the channel bandwidth and the signal-to-noise ratio (SNR) of the wireless link $i \rightarrow j$, respectively. With linear NC technique, the traffic amount of the wireless link $i \rightarrow j$ is determined by $\sum_{m=1}^{M} \max_{l\in\mathcal{L}_m} f_{i,j}^{(n_{m,0},n_{m,l})}$. Therefore, we have

$$\sum_{m=1}^{M} \max_{l\in\mathcal{L}_m} f_{i,j}^{(n_{m,0},n_{m,l})} \leqslant W \log_2(1+\gamma_{i,j}), \{i,j\} \in \mathcal{N} \quad (3)$$

3) *Multicast traffic rate constraint*: The multicasting performance is acceptable if $R_m \geq \delta_m$ can always be guaranteed during the communication, where δ_m is the rate for the basic-layer performance rate, i.e.

$$R_m \geqslant \delta_m, m \in \mathcal{M} \quad (4)$$

4) *Node energy constraint*: Let α_n and β_n be the energy consumed to transmit and receive a unit of data at the n-th node respectively and suppose the lifetime of the n-th node, $n \in \mathcal{N}$, is T_n. The energy consumption at the n-th node is defined as the energy needed to transmit and receive data throughout the lifetime of this node. Let E_n denote the energy supply at the n-th node and $t_{i,j}$, $\{i,j\} \in \mathcal{N}$, denote the transmission delay from the i-th node to the j-th node. The node energy constraint can be written as

$$\begin{aligned} &[\max_{\substack{a\in\mathcal{N}\\a\neq n}} \alpha_n t_{n,a} \sum_{m=1}^{M} \max_{l\in\mathcal{L}_m} f_{n,a}^{(n_{m,0},n_{m,l})} + \\ &\max_{\substack{b\in\mathcal{N}\\b\neq n}} \beta_n t_{b,n} \sum_{m=1}^{M} \max_{l\in\mathcal{L}_m} f_{b,n}^{(n_{m,0},n_{m,l})}] T_n \leqslant E_n \end{aligned} \quad (5)$$

5) *Total energy constraint*: It is observed that the equal energy allocation for all nodes in the network may not be optimal since the energy allocation at each node depends on many factors, such as the role of node (source and/or relay and/or destination), the location of node and the connections of node with the other nodes in the topology. Let ξ denote the total energy available for the whole network. We then have the following constraint

$$\sum_{n=1}^{N} E_n \leqslant \xi \quad (6)$$

6) *Network lifetime constraint*: Let us denote τ as network lifetime, which is defined as the minimum duration for the survival of all nodes in network satisfying the wireless multicast rate constraints. This means $\tau = \min_{n\in\mathcal{N}} T_n$. In other words, we have

$$T_n \geqslant \tau, n \in \mathcal{N} \quad (7)$$

It can be observed that, if we consider a wired network with no constraints on the energy and network lifetime (i.e. with invariant link capacity and without constraints (5), (6) and (7)), the proposed problem can be regarded as the topology design problems in previous work for unicast and multicast wired networks, which were shown to be NP-hard. Accordingly, the proposed topology design problem contains these problems as special cases and thus is also NP-hard.

Since this is an NP-hard problem, in the next section, we develop an heuristic algorithm to construct an NC supported wireless multicasting tree with the minimum number of intermediate nodes and wireless links.

III. Heuristic NC-based LCRT Algorithm

In the proposed NC-based LCRT algorithm, in order to be able to recover original packets from the linear NC-based combined packets, the destination nodes should try to receive at least K combined packets from the other nodes. A LCRT multicast tree employs a minimal number of on-tree forwarding nodes that can reliably cover a group of receivers. Additionally, LCRT metric [2, Sect. 4.3.1, eq. (3)] is utilised in the NC-based LCRT algorithm to evaluate the availability of nodes and measure the weight of nodes with respect to interference from the other nodes. Here, the availability of the n-th node, $n \in \mathcal{N}$, is defined as

$$\theta_n = \frac{C_n}{\sum_{\substack{i\in\mathcal{N}\\i\neq n}} \sum_{m=1}^{M} \max_{l\in\mathcal{L}_m} f_{n,i}^{(n_{m,0},n_{m,l})}}, \quad (8)$$

where C_n denotes the total capacity available at the n-th node. Taking into account the benefit of randomised linear NC in exploiting various data streams, the weight of the n-th node, $n \in \mathcal{N}$, is defined as

$$\eta_n = \vartheta_n \theta_n, \quad (9)$$

where ϑ_n denotes the number of flows coming from the n-th node. Let us define the level of a node as the least number of wireless hops from a node to the source node. By using LCRT, source node $n_{m,0}$, $m \in \mathcal{M}$, assigns a node level to a multicast node according to its hop distance to $n_{m,0}$. Destination nodes $n_{m,l}$, $l \in \mathcal{L}_m$, of the m-th wireless multicast are assigned the levels of Q_m. Also, let $u_{q,m}$, $c_{q,m}$ and $v_{q,m}$ denote the number of uncovered nodes, covered nodes and fully covered nodes, respectively, at the (q_m+1)-th level of the m-th wireless multicast. Here, node at the (q_m+1)-th level is fully covered if it is covered by at least K nodes at the q_m-th level. This condition is helpful in assisting the recovery of original packets at destinations. The NC-based LCRT algorithm is summarised in Algorithm 1. Protocols can be designed to enable nodes to cooperatively set up the NC-based multicast topology by using the proposed algorithm in a distributed manner.

Algorithm 1 NC-based LCRT algorithm

for $m = 1$ to M **do**
  $q_m = Q_m - 1$
  while $q_m > 1$ **do**
    $c_{q,m} = 0$
    while $u_{q,m} > 0$ **do**
      ▪ $n_{m,0}$ selects a maximum of K q_m-th nodes based on 2 ordered criteria: i) covering a maximum number of the $(q_m + 1)$-th nodes and ii) having maximum weight values (see (9)) $\Rightarrow$ Determine $v_{q,m}$
      if $v_{q,m} > 0$ **then**
        $u_{q,m} = u_{q,m} - v_{q,m}$
        $c_{q,m} = c_{q,m} + v_{q,m}$
      else
        $u_{q,m} = u_{q,m} - 1$
        $c_{q,m} = c_{q,m} + 1$
      end if
    end while
    $q_m = q_m - 1$
  end while
  if $u_{0,m} \geqslant K$ **then**
    ▪ $n_{m,0}$ selects a minimum number of the 1st nodes ($\geqslant K$) to cover all the 2nd nodes
    $\Rightarrow$ Determine $c_{0,m}$, $c_{1,m}$
  else
    ▪ $n_{m,0}$ selects all the 1st nodes, i.e. $c_{0,m} = u_{0,m} \Rightarrow$ Determine $c_{1,m}$
  end if
end for

Lemma 1. WMN topology designed with the proposed NC-based LCRT algorithm allows a lower number of nodes involved than the total number of available nodes in the network system, especially when the number of data packets is small.

Proof: Let us denote N' as the total of nodes required in the proposed NC-based LCRT algorithm and $Q_{\max}$ as the maximum node level for all M wireless multicasts, i.e. $Q_{\max} = \max_{m \in \mathcal{M}} Q_m$. Also, let χ_q and ψ_q, $q \in \{1, 2, \ldots, Q_{\max}\}$, denote the total number of nodes at the q-th level and the total number of nodes required at the q-th level in the NC-based LCRT algorithm, respectively. We have $M + \sum_{q=1}^{Q_{\max}} \chi_q = N$ and $M + \sum_{q=1}^{Q_{\max}} \psi_q = N'$. Here, χ_q can be given by $\chi_q = \bigcup_{m=1}^{M} \chi_{q,m} = \bigcup_{m=1}^{M} (c_{q-1,m} + u_{q-1,m})$, where $\chi_{q,m}$, $m \in \mathcal{M}$, denotes the number of nodes at the q_m-th level of the m-th wireless multicast. From Algorithm 1, we also have $\psi_q = \bigcup_{m=1}^{M} \psi_{q,m} = \bigcup_{m=1}^{M} c_{q-1,m}$, where $\psi_{q,m}$, $m \in \mathcal{M}$, denotes the number of nodes required at the q_m-th level of the m-th multicast using Algorithm 1.

It can be easily observed that $c_{q,m} + u_{q,m} \geqslant c_{q,m} \geqslant v_{q,m}$ $\forall q \in \{1, 2, \ldots, Q_{\max}\}$, $m \in \mathcal{M}$ and thus $\psi_q \leqslant \chi_q$. Accordingly, we obtain $N' \leqslant N$.

Furthermore, when K is small, as shown in Algorithm1, we can easily find K q_m-th nodes to cover the $(q_m + 1)$-th node. So, it can be approximated that $c_{q,m} \approx v_{q,m} \ll c_{q,m} + u_{q,m}$, which means $N' \ll N$. ■

Remark 1. *Improved system throughput with the proposed NC-based LCRT algorithm for WMN.* From (6), if the energy supply at each node is fixed and N decreases to N', then $\sum_{n=1}^{N} E_n$ decreases to $\sum_{n=1}^{N'} E_n$. This means more energy can be allocated for N' nodes while still satisfying the total energy constraint. Thus, from (5), higher data flow can be allocated for wireless links for a higher system throughput.

Although the NC operation at nodes causes higher computation complexity, a much higher throughput is achieved with significantly reduced data transmission. This increased complexity would not have much impact on the node lifetime with the development of integrated circuits for high-complexity computations.

IV. Cross-Layer Design

In the previous section, we have obtained the WMN topology tree using the proposed NC-based LCRT algorithm. In this section, we prove the tractability of the optimisation problem over the design variables in the proposed topology tree. Let us assume that our topology design for NC-based multicast requires a total of N' nodes ($N' < N$) in a node set $\mathcal{N}'$.

Lemma 2. Given a fixed topology tree and fixed lifetime of nodes, the optimisation problem for topology design of NC-based WMN is a linear programming (LP) problem.

Proof: From (1), it can be seen that the system throughput function is linear over multicast rate variables of wireless multicast requirements R_m, $m \in \mathcal{M}$. With fixed topology tree and fixed $T_{n'}$, $n' \in \mathcal{N}'$, we can easily prove that the constraints (2), (3), (4), (5), (6) and (7) are linear over R_m, $f_{i,j}^{(n'_{m,0}, n'_{m,l})}$, $E_{n'}$, $n' \in \mathcal{N}'$, $m \in \mathcal{M}$, $l \in \mathcal{L}_m$. Therefore, the system throughput optimisation problem is an LP problem. ■

Remark 2. *Given a fixed topology tree, the system throughput increases as lifetime of nodes approaches network lifetime.* In fact, this can be easily observed through the flow constraint in (2) and node energy constraint in (5).

Therefore, in order to maximise the system throughput, the lifetime of nodes should be set as the network lifetime (i.e. $T_{n'} \approx \tau$) and thus the network lifetime constraint (7) can be relaxed. As proved in Lemma 2, the optimisation problem in Section II-B is now a LP problem which is tractable.

Remark 3. *System throughput increases as either total energy available for network increases or network lifetime decreases.* This can be observed from (5) and (6).

Remark 4. *A significantly improved throughput is achieved with NC technique, especially with a large multicast node set.*

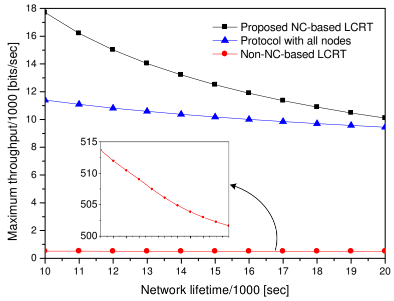

Fig. 1: Maximum throughput versus network lifetime.

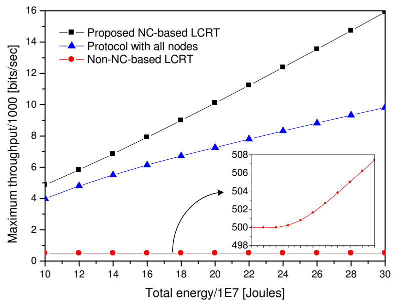

Fig. 2: Maximum throughput versus total energy.

Without linear NC technique, the wireless link utilisation and node energy constraints can be given by

$$\sum_{m=1}^{M}\sum_{l=1}^{|\mathcal{S}_m|-1} f_{i,j}^{(n'_{m,0},n'_{m,l})} \leqslant W\log_2(1+\gamma_{i,j}), \{i,j\}\in\mathcal{N}', \quad (10)$$

$$\begin{aligned}&[\max_{\substack{a\in\mathcal{N}'\\ a\neq n'}}\alpha_{n'}t_{n',a}\sum_{m=1}^{M}\sum_{l=1}^{|\mathcal{S}_m|-1} f_{n',a}^{(n'_{m,0},n'_{m,l})}+\\ &\quad \max_{\substack{b\in\mathcal{N}'\\ b\neq n'}}\beta_{n'}t_{b,n'}\sum_{m=1}^{M}\sum_{l=1}^{|\mathcal{S}_m|-1} f_{b,n'}^{(n'_{m,0},n'_{m,l})}]T_0\leqslant E_{n'},\end{aligned} \quad (11)$$

respectively. It can be observed from (10) and (11) that, subject to fixed SNR of the wireless link and limited node power, we cannot allocate high data flows for wireless links in the non-NC-based LCRT protocol. However, in the proposed NC-based LCRT protocol, the summation of flows in the wireless link capacity and node energy constraints is replaced by the maximum of flows (see (3) and (5)). This means that, as $|\mathcal{S}_m|$ is large, much higher data flows can be allocated for wireless links and thus the system throughput is significantly improved.

V. Numerical Results

The performance of the proposed solution is evaluated through numerical results in MATLAB and compared against other two solutions, namely: a non-NC-based LCRT algorithm and a basic protocol that is using all the nodes. A wireless multicast network consisting of 19 nodes is considered. These nodes are located in a scale of 100×100, where their coordinates are uniformly distributed in $(0,100)$. For simplicity, 1 multicast requirement is considered and the multicast node set is assumed to be $\mathcal{S}_1=\{1,16,17,18,19\}$. For comparison, the protocol using all nodes and the non-NC-based LCRT protocol are both considered.

Figs. 1 and 2 plot the maximum throughput of various protocols as a function of network lifetime and total energy, respectively. We assume that $K=3$ packets, $B=1000$ bits and node 1 need to transmit these packets to nodes $\{16,17,18,19\}$. The SNR of the adjacent wireless links is 10 dB and the channel bandwidth is 300 KHz. Also, $\delta_1=500$ bits/sec, $\alpha=1$ Joule and $\beta=0.1$ Joule. In Fig. 1, let us assume $\xi=200\times10^6$ Joules, while $\tau=20\times10^3$ sec in Fig. 2. First, it can be observed that, with respect to network lifetime and total energy, the proposed NC-based LCRT protocol achieves an improved performance up to 50% over the compared protocols in terms of maximum throughput. This improved performance again confirms the statements in Remarks 1 and 4 regarding the increased system throughput achieved with the proposed NC-based LCRT protocol when the number of nodes required for the wireless multicast decreases and linear NC technique is applied. Secondly, the maximum throughput of all protocols is shown to increase as either the network lifetime decreases or the total available energy increases, which verifies the observation in Remarks 2 and 3.

VI. Conclusions

In this paper, we have proposed a cross-layer optimisation framework for topology design of WMN. Given various constraints on QoS, we have developed an heuristic NC-based LCRT algorithm and optimised wireless multicast rate, data flows, energy supply and node lifetime. The numerical results show that the proposed NC-based LCRT algorithm can achieve up to 50% increase in the system throughput when compared against the all nodes and non-NC-based LCRT protocols. The future work would be the performance evaluation of the proposed algorithm for various network settings and channel conditions.

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