Vien, Quoc-Tuan; Stewart, Brian G.; Tianfield, Huaglory; Nguyen, Huan X; Choi, Jinho, 2012. An efficient network coded ARQ for multisource multidestination relay networks over mixed flat fading channels. Available from Middlesex University’s Research Repository.

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An Efficient Network Coded ARQ for Multisource Multidestination Relay Networks over Mixed Flat Fading Channels

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Abstract

This paper proposes a new reliable automatic repeat request (ARQ) transmission protocol for wireless multisource multidestination relay networks over mixed fading channels. Conventional application of ARQ protocols to retransmit lost or erroneous packets in relay networks can cause considerable delay latency with a significant increase in the number of retransmissions when networks consist of multiple sources and multiple destinations. To address this issue, a new ARQ protocol based on network coding (NC) is proposed where the relay detects packets from different transmission sources, then uses NC to combine and forward lost packets to their
destinations. An efficient means for the retransmission of all lost packets is proposed through two packet-combination algorithms for retransmissions at the relay and sources. The paper derives mathematical formulation of transmission bandwidth for this new NC-based ARQ protocol and compares analytical and simulation results with some other ARQ protocols over both mixed Rayleigh and Rician flat fading channel. The mixed fading model permits investigation of two typical fading scenarios where the relay is located in the neighbourhood of either the sources or the destinations. The transmission bandwidth results show that the proposed NC-based ARQ protocol demonstrates superior performance over other existing ARQ schemes.

Keywords: Network coding, ARQ protocols, Rayleigh fading, Rician fading, multisource multidestination relay network.

1. Introduction

Relay techniques are normally deployed to increase coverage between remote transmission and reception nodes as well as improve service quality and link capacity for local users [1, 2]. Recently spatial diversity gain methods have been considered in an attempt to extend relay transmission coverage and further improve transmission integrity [3, 4].

Basically, relays transmit packets through a store-and-forward mechanism, and thus do not increase the network throughput. In an attempt to improve throughput, network coding (NC) techniques have been investigated at the relays [5–7]. The basic concept of NC is that the relays perform algebraic linear/logic operations on received packets from multiple transmission sources in order to create a new combined packet, which is then forwarded to
the destination nodes in the subsequent transmission. Various NC-based protocols have recently been proposed for some particular relay channel topologies such as relay-assisted bidirectional channels [8], broadcast channels [9], multicast channels [10] and unicast channels [11].

Automatic repeat request (ARQ) techniques permit information to be reliably delivered over multicast or broadcast networks. However, lost packets require to be retransmitted with ARQ protocols which may introduce significant packet latency since each packet is retransmitted individually. In addition, for ARQ, retransmissions are repeated until all packets are received correctly at each reception node. For multisource multistation relay networks (MMRN), in [12], the beamforming matrix was designed to minimize the sum transmit power at the relays subject to signal-to-interference constraints at the destinations to reliably support multiple parallel data streams. Also, in [13], stop-and-wait ARQ, go-back-N ARQ and the selective-repeat ARQ were investigated and compared to evaluate the maximum achievable throughput and the steady-state throughput of butterfly networks, a specific model of the MMRNs. However, the design of reliable transmissions over MMRNs that can achieve high network throughput efficiency and reduced retransmission packet latency has received little attention in the previous literature.

As an improved solution to these issues, we propose a new ARQ protocol based on NC for MMRNs. In this new protocol, the relay detects packets, combines information through NC, and transmits the lost packets from different sources to the destinations. Additionally, to achieve an optimal performance, multi-user detection (MUD) techniques [14] are implemented
at both the relay and destinations. Thus along with MUD, lost packets can be combined and retransmitted to achieve an improved ARQ mechanism. The representation of lost packets in MMRNs may be categorised into two classification types: Type-I - packets that are successfully received at the relay but lost at the destinations, and, Type-II - packets that are lost at both the relay and destinations. Retransmission of Type-II packets is undertaken by the source, but the issue of how the relay retransmits Type-I packets with the lowest number of retransmissions requires to be addressed. To solve this retransmission problem, we propose a relay algorithm and also a source algorithm to enable retransmission of Type-I and Type-II packets, respectively. As an example of the protocol implementation, a two source, relay, two destination configuration is considered. Specifically, for this scenario, the proposed algorithm employed for retransmission at the relay is based on a combination of NC and packet detection from the two different sources.

A further contribution of this paper involves a performance comparison between our proposed NC-based ARQ protocol and other typical ARQ protocols for MMRNs. The other typical ARQ protocols considered are the direct transmission (DT)\(^1\) and the relaying transmission (RT)\(^2\) protocol. The performance comparison is achieved through deriving principally the complex analytical expressions of the transmission bandwidth for the new NC-based ARQ protocol and comparing it with the general analytical formulations for

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\(^1\)The DT protocol refers to the model in which multiple sources simultaneously transmit information to the destinations without using the relaying technique [15].

\(^2\)The RT protocol refers to the model in which the relay participates in the transmission but NC is not employed at the relay [4, 15].
the other two protocols. The paper also extends the analytical performance analysis to include channel fading for the situations when the sources and destinations are located near to, or distant, from the relay. In these scenarios, it is approximated that the links from the sources to the relay, or the links from the relay to the destinations are line-of-sight (LOS) transmissions (close by, Rician fading), or non-line-of-sight (NLOS) transmissions (distant, Rayleigh fading)\(^3\). Accordingly, the considered fading channels are modelled as a mix of both Rayleigh and Rician fading, or are both Rayleigh or Rician fading.

It is shown through appropriate analytical and simulation examples, that our proposed ARQ protocol when applied to two-source two-destination single-relay network, significantly reduces the number of retransmissions for all fading situations, when compared with the DT and RT protocols.

The paper is organized as follows: Section 2 describes the system model and the different retransmission protocols of MMRNs; Section 3 derives the transmission bandwidths; Section 4 presents the numerical evaluation results and Section 5 concludes the paper.

2. MMRN System Model and Transmission Protocols

Consider the MMRN displayed in Fig. 1 where data multicast from two sources \(S_1\) and \(S_2\) to two destinations \(D_1\) and \(D_2\) is assisted by one relay \(R\).

\(^3\)Rayleigh fading is used to model the communication channels where there are many scatters caused by objects between the source and the destination. However, there is no dominant propagation along a line of sight from the source to the destination. If there is a dominant line of sight, the communication channels are modelled by Rician fading [16].
Increasing the number of sources and destinations to the model is straightforward. The sources are able to send data packets which must be received without error after a number of transmissions and retransmissions. Basic ARQ technique is considered, where the sender simply waits for a positive or negative acknowledgement message from the receiver for every data transmission then retransmits the lost or erroneous packets. It is also assumed that the channel link $A \rightarrow B$ (where $A \in \{S_1, S_2, R\}$, $B \in \{R, D_1, D_2\}$, $A \neq B$) is characterized by either Rayleigh or Rician flat fading with a channel gain of $h_{AB}$. Here, the statistics for $h_{AB}$ can be represented by the probability density function of channel fading amplitude $\alpha_{AB}$ as [16]

$$f\text{^{(Rayleigh)}}(\alpha_{AB}) = \frac{2\alpha_{AB}}{\nu_{AB}} \exp\left(-\frac{\alpha_{AB}^2}{\nu_{AB}}\right),$$

(1)
\[ f^{(\text{Rician})}(\alpha_{AB}) = \frac{2(1 + K_{AB})e^{-K_{AB} \alpha_{AB}}}{\nu_{AB}} \exp\left( -\frac{(1 + K_{AB})\alpha_{AB}^2}{\nu_{AB}} \right) \times I_0 \left( 2\alpha_{AB} \sqrt{K_{AB}(1 + K_{AB})} \right), \]

where \( \nu_{AB} \) is mean-square value of \( \alpha_{AB} \), \( K_{AB} \) is the Rician fading parameter and \( I_0(\cdot) \) is the zeroth-order modified Bessel function of the first kind.

\( R \) receives data packets from \( S_1 \) and \( S_2 \) in addition to feedback from \( D_1 \) and \( D_2 \), thus \( R \) has knowledge of the destinations still waiting for retransmission of lost packets. \( R \) then decides how to combine and forward the data to the intended destinations. The purpose of any retransmission protocol is to facilitate \( R \) in resending the lost packets to \( D_1 \) and \( D_2 \).

The three retransmission protocols considered in this paper will now be described.

2.1. DT Protocol

In the DT protocol, \( S_1 \) and \( S_2 \) transmit data directly to \( D_1 \) and \( D_2 \). The transmission employs ARQ and is completed when both \( D_1 \) and \( D_2 \) receive correctly the data packets from both \( S_1 \) and \( S_2 \).

2.2. RT Protocol

The RT protocol differs from the DT protocol because \( R \) now participates in the transmission process. When \( D_j \) \((j = 1, 2)\) does not receive a packet from \( S_i \) \((i = 1, 2)\) but \( R \) successfully receives the packet, \( R \) can assist \( S_i \) by forwarding the correctly received packet to \( D_j \) in the next transmission time slot. Using ARQ, retransmissions at \( R \) continue until the transmitted packet is correctly received by \( D_j \). If \( D_j \) and \( R \) do not receive the same packet from \( S_i \), then \( S_i \) resends the lost packet.
that \( R \) packets. The distinctiveness and novelty in the proposed ARQ protocol is retransmits packets of Type-I, and \( R \) are received from two different sources. To improve network throughput, (2.3. NC-based Protocol

\[ S_i \rightarrow R \]
\[ S_i \rightarrow D_i \]
\[ S_i \rightarrow D_i \]
\[ S_i \rightarrow D_i \]
\[ S_i \rightarrow D_i \]

Retransmission phase

\[ RT \text{ Protocol} \]
\[ \begin{array}{cccccccccc}
1 & 1 & 2 & 3 & 5 & 6 & 7 & 8 & 10 \\
1 & 1 & 2 & 3 & 5 & 6 & 7 & 8 & 9 & 10,
\end{array} \]

\[ \begin{array}{cccccccccc}
1 & 1 & 2 & 3 & 5 & 6 & 7 & 8 & 10 \\
1 & 1 & 2 & 3 & 5 & 6 & 7 & 8 & 9 & 10.
\end{array} \]

Proposed Protocol

\[ \begin{array}{cccccccccc}
1 & 1 & 2 & 3 & 5 & 6 & 7 & 8 & 9 & 10 \\
1 & 1 & 2 & 3 & 5 & 6 & 7 & 8 & 9 & 10.
\end{array} \]

Figure 2: Retransmission packets with RT and the new NC-based ARQ protocol.

2.3. NC-based Protocol

Rather than resending the lost packet when \( D_j \ (j = 1, 2) \) fails to receive it, the retransmission in our proposed NC-based ARQ protocol will retransmit after \( N \) received packets. A buffer length of \( N \) packets is necessary at \( S_i \ (i = 1, 2) \) whilst buffers of size \( 2N \) are required at \( R \) and \( D_j \) since packets are received from two different sources. To improve network throughput, \( R \) retransmits packets of Type-I, and \( S_i \) organises retransmission of Type-II packets. The distinctiveness and novelty in the proposed ARQ protocol is that \( R \) can mix information from packets received through the two network
data flows.

The following packet transmission example outlines the principles of the protocol (see Fig. 2). \( S \) delivers \( N = 10 \) packets \( \{s_i[1], s_i[2], \ldots, s_i[10]\} \) to both \( D_1 \) and \( D_2 \). In Fig. 2, the packets which are crossed through are considered lost or erroneously received packets. For data flow from \( S_1 \), consider the received packets in error at \( R, D_1, \) and \( D_2 \) as \( \{s_1[4], s_1[6], s_1[9]\}, \{s_1[1], s_1[2], s_1[4], s_1[8]\}, \) and \( \{s_1[3], s_1[5], s_1[7], s_1[9], s_1[10]\} \), respectively. Similarly, assume that the received packets which are in error at \( R, D_1, \) and \( D_2 \) arriving from \( S_2 \) are \( \{s_2[2], s_2[3], s_2[5], s_2[7]\}, \{s_2[3], s_2[6], s_2[7], s_2[9], s_2[10]\} \), and \( \{s_2[1], s_2[2], s_2[5], s_2[8]\} \), respectively.

As shown in Fig. 2, \( R \) will retransmit 12 packets using the RT protocol. For packets lost at \( R \) and also lost at \( D_1 \) and/or \( D_2 \), i.e., \( \{s_1[4], s_1[9], s_2[2], s_2[3], s_2[5], s_2[7]\} \), \( S_1 \) and \( S_2 \) will retransmit \( \{s_1[4], s_1[9]\} \) and \( \{s_2[2], s_2[3], s_2[5], s_2[7]\} \), respectively. In total 18 retransmissions are required for the RT protocol.

Now let us compare the NC-based ARQ protocol. In this case, a significant reduction in retransmission of lost packets is possible. For example, packets \( \{s_1[1], s_2[1], s_1[2], s_1[3], s_1[5], s_2[6], s_1[7], s_1[8], s_2[8], s_2[9], s_1[10], s_2[10]\} \) are classified as Type-I packets and \( \{s_1[4], s_1[9], s_2[2], s_2[3], s_2[5], s_2[7]\} \) are Type-II packets. In this scheme, to improve network throughput, in the retransmission phase, \( R \) forwards \( \{s_1[1] \oplus s_2[1], s_1[2] \oplus s_1[3], s_1[5] \oplus s_2[6], s_1[7] \oplus s_1[8], s_2[8] \oplus s_2[9], s_1[10] \oplus s_2[10]\} \), whilst, \( S_1 \) and \( S_2 \) retransmit \( \{s_1[4] \oplus s_1[9]\} \) and \( \{s_2[2] \oplus s_2[3], s_2[5] \oplus s_2[7]\} \), respectively, where \( \oplus \) denotes the bitwise XOR operator. A summary of the NC combination algorithms at \( R \) and \( S_i \), \( (i = 1, 2) \) are described in Algorithms 1 and 2,
respectively.

In total, the proposed NC-based ARQ scheme requires only 9 retransmissions, compared to 18 when deploying the RT scheme. \( R, S_1, \) and \( S_2 \) will retransmit these 9 packets until all are successfully received at both \( D_1 \) and \( D_2 \). The lost packets at \( D_j \) \((j = 1, 2)\) may be recovered through the standard method of XORing the correctly received packets located at \( D_j \) with the XORed packets received from either \( R \) or \( S_i \).

3. Transmission Bandwidth Analysis

In this section, the transmission bandwidths\(^4\) of the three protocols discussed above are derived for the scenarios of mixed Rayleigh and Rician flat fading channels for the MMRNs as described in Fig. 1.

When a channel is affected by fading, the signal \( y_{AB} \) received at any node \( B \) when transmitted from any node \( A \), where \( \{A, B\} \in \{S_1, S_2, R, D_1, D_2\} \), \((A \neq B)\), can be expressed through

\[
y_{AB} = \sqrt{\Gamma_{AB}} h_{AB} x_{AB} + n_{AB},
\]

where \( \Gamma_{AB} \) describes the long-term fading (i.e., path loss and shadowing) within the transmission link \( A \rightarrow B \), \( h_{AB} \) is the fading channel, \( x_{AB} \) is the binary phase shift keying (BPSK) modulated signal of the transmitted packet\(^5\), and \( n_{AB} \) is the channel noise. This noise can be considered as an

\(^4\)Transmission bandwidth is defined as the average number of transmissions to successfully transmit two packets from two sources to two destinations.

\(^5\)Uncoded BPSK is considered in this paper for simple analysis. The proposed scheme is applicable for any coded modulation schemes.
Algorithm 1 Combination algorithm at $\mathcal{R}$ to retransmit Type-I packets

1: Let $\mathcal{G}_1$ and $\mathcal{G}_2$ denote the ordered sets of correctly received packets at $\mathcal{R}$ transmitted from $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively: $\mathcal{G}_1 = \{s_1[i_1], s_1[i_2], \ldots, s_1[i_m]\}$, where $i_1 < i_2 < \cdots < i_m \in \{1, 2, \ldots, N\}$, $\mathcal{G}_2 = \{s_2[j_1], s_2[j_2], \ldots, s_2[j_n]\}$, where $j_1 < j_2 < \cdots < j_n \in \{1, 2, \ldots, N\}$.

Define $\Omega = \mathcal{G}_1 \cup \mathcal{G}_2$ and divide $\Omega$ into 3 groups as follows:

- Group $\Omega_1$ includes packets that $\mathcal{R}$ receives successfully from both $\mathcal{S}_1$ and $\mathcal{S}_2$.

- Group $\Omega_2$ includes packets that $\mathcal{R}$ receives successfully from $\mathcal{S}_1$ but fails to receive from $\mathcal{S}_2$.

- Group $\Omega_3$ includes packets that $\mathcal{R}$ receives successfully from $\mathcal{S}_2$ but fails to receive from $\mathcal{S}_1$.

2: For packets in $\Omega_1$, if one packet is received correctly at $\mathcal{D}_1$ but lost at $\mathcal{D}_2$, while another packet is received correctly at $\mathcal{D}_2$ but lost at $\mathcal{D}_1$, we can combine these two packets. Start from left to right in the group of packets in $\Omega_1$ and choose the suitable XOR combination of packets in one of three ways as follows: $s_1[k_1] \oplus s_2[k_2]$, $s_1[m_1] \oplus s_1[m_2]$ and $s_2[n_1] \oplus s_2[n_2]$ where $k_1, k_2, m_1, m_2, n_1, n_2 \in \{1, 2, \ldots, N\}$.

3: For packets in $\Omega_2$ and $\Omega_3$, similarly if one packet is received correctly at $\mathcal{D}_1$ but lost at $\mathcal{D}_2$, while another packet is received correctly at $\mathcal{D}_2$ but lost at $\mathcal{D}_1$, we can combine these two packets as $s_1[m_1] \oplus s_1[m_2]$ for $\Omega_2$ and $s_2[n_1] \oplus s_2[n_2]$ for $\Omega_3$.

4: For the remaining lost packets at $\mathcal{D}_1$ and $\mathcal{D}_2$ that $\mathcal{R}$ receives successfully but cannot perform the combination, these are normally resent without using NC.
**Algorithm 2** Combination algorithm at $S_i$ to retransmit Type-II packets

1: Through the feedback from $D_1$, $D_2$, and $R$, $S_i$ determines the number and the position of remaining lost packets at destinations that $R$ also fails in receiving them.

2: Combine the packets for retransmission by NC with the condition that only one packet in the combined packet should be received correctly by only one destination, similar to the combination performed for packets in $\Omega_2$ and $\Omega_3$ as explained in Algorithm 1.

3: For the remaining lost packets at $D_1$ and $D_2$ that $S_i$ cannot perform the combination, these are resent without NC.

Independent circularly symmetric complex Gaussian (CSCG) noise vector with each entry having zero mean and noise variance denoted by $N_0$. For the situation where Rayleigh fading is considered, the bit error probability (BEP) of the signal transmission through link $A \rightarrow B$ is expressed by [16]

$$P_b^{(\text{Rayleigh})}(E_{AB}) = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma_{AB}}{1 + \gamma_{AB}}} \right),$$  \hspace{1cm} (4)

where $\gamma_{AB}$ is the average signal-to-noise ratio (SNR) defined through $\gamma_{AB} = \Gamma_{AB}/N_0$.

For the case of Rician fading channels with Rician fading parameter $\mathcal{K}_{AB}$, the BEP of the transmission through link $A \rightarrow B$ is expressed through [16]

$$P_b^{(\text{Rician})}(E_{AB}) = \frac{1}{\pi} \int_0^\pi \frac{(1 + \mathcal{K}_{AB}) \sin^2 \theta}{(1 + \mathcal{K}_{AB}) \sin^2 \theta + \gamma_{AB}} \times \exp\left( -\frac{\mathcal{K}_{AB}\gamma_{AB}}{(1 + \mathcal{K}_{AB}) \sin^2 \theta + \gamma_{AB}} \right) d\theta. \hspace{1cm} (5)$$

Thus, for any specified SNR, the packet loss of the transmission link
\( A \rightarrow B \) can be calculated by

\[
P_{AB} = 1 - [1 - P_b(E_{AB})]^{N_b},
\]

where \( N_b \) is the number of bits in a packet and \( P_b(E_{AB}) \) is denoted either by Eq. (4) or (5) depending on the fading channel model adopted.

The transmission bandwidths will now be evaluated for each of the three protocols.

3.1. DT Protocol

When \( R \) is omitted from the network, and NC not considered, the DT protocol transmission bandwidth, \( n_{DT} \), may be expressed by

\[
n_{DT} = \max\{n_{DT}^{(S_1)}, n_{DT}^{(S_2)}\},
\]

where \( n_{DT}^{(S_i)} (i = 1, 2) \) denotes the transmission bandwidth required for \( S_i \) to send a packet to both \( D_1 \) and \( D_2 \), and is easily evaluated as

\[
n_{DT}^{(S_i)} = \frac{1}{1 - P_{S_iD_1}} + \frac{1}{1 - P_{S_iD_2}} - \frac{1}{1 - P_{S_iD_1}P_{S_iD_2}}.
\]

3.2. RT Protocol

Including \( R \) in the network and still omitting NC, transmission bandwidth for successfully transmitting two packets from \( S_i \) and \( S_2 \) to \( D_i \) \((i = 1, 2)\)
is given by

\[ n_{RT}^{(D_i)} = \frac{1}{1 - P_{S_iR}P_{S_jR}P_{S_jD_i}P_{S_iD_i}} \left[ 1 + P_{S_jR}P_{S_iD_i}(1 - P_{S_jD_i})n_{RT}^{(S_i,D_i)} \right. \]
\[ + P_{S_jR}(1 - P_{S_jD_i})P_{S_iD_i}n_{RT}^{(S_i,D_i)} \]
\[ + (1 - P_{S_iR})P_{S_jD_i}(1 - P_{S_jD_i})n_{RD_i} \]
\[ + (1 - P_{S_jR})(1 - P_{S_iD_i})P_{S_iD_i}n_{RD_i} \]
\[ + 2(1 - P_{S_iR})(1 - P_{S_jR})P_{S_iD_i}P_{S_jD_i}n_{RD_i} \]
\[ + (1 - P_{S_jR})P_{S_jR}P_{S_iD_i}P_{S_jD_i}(n_{RD_i} + n_{RT}^{(S_i,D_i)}) \]
\[ + P_{S_jR}(1 - P_{S_jR})P_{S_iD_i}P_{S_jD_i}n_{RD_i} \]
\[ + n_{RT}^{(S_i,D_i)} \right] \]

where \( n_{RD_i} \) and \( n_{RT}^{(S_i,D_i)} \) denote the transmission bandwidths of a packet from \( R \) to \( D_i \) and from \( S_i \) to \( D_j \) with the assistance of \( R \), respectively. Thus, \( n_{RD_i} \) and \( n_{RT}^{(S_i,D_i)} \) may be computed respectively through

\[ n_{RD_i} = \frac{1}{1 - P_{RD_i}}, \]  \hspace{1cm} (10)
\[ n_{RT}^{(S_i,D_j)} = \frac{1 + P_{RD_j} + P_{S_jD_j}(1 - P_{S_jR})}{(1 - P_{S_jR}P_{S_jD_j})(1 - P_{RD_j})}. \]  \hspace{1cm} (11)

The transmission bandwidth of the RT protocol is therefore given by

\[ n_{RT} = \max\{n_{RT}^{(D_1)}, n_{RT}^{(D_2)}\}. \]  \hspace{1cm} (12)

### 3.3. Proposed NC Based Protocol

In the proposed NC-based protocol, \( R \) combines lost packets from the two different packet flows. Since a total of \( 2N \) packets are transmitted from \( S_1 \) and \( S_2 \), the transmission bandwidth \( n_{NC} \) is expressed as

\[ n_{NC} = \frac{n^{(1)} + n^{(2)} + n^{(3)}}{2N}, \]  \hspace{1cm} (13)
where $n^{(i)}$ $(i = 1, 2, 3)$ denotes the transmission bandwidth in the $i$-th step of the proposed protocol. These steps include the following:

- **Step 1.** Both $S_1$ and $S_2$ transmit $N$ packets.
- **Step 2.** $R$ retransmits Type-I packets.
- **Step 3.** $S_1$ and/or $S_2$ retransmit Type-II packets.

It is obvious that $n^{(1)} = 2N$. Following the proposed Algorithms 1 and 2 for the retransmissions at $R$ and $S_i$ $(i = 1, 2)$, $n^{(2)}$ and $n^{(3)}$ can be computed by

\begin{align}
    n^{(2)} &= \sum_{k=0}^{N} \left\{ C^N_k P^{N-k}_{S_1R} (1 - P_{S_1R})^k P^{N-k}_{S_2R} (1 - P_{S_2R})^k E[n^{(2)}|K = k] \\
    &+ \sum_{l=0}^{N-k} \left\{ C^{N-k}_l P^{N-k-l}_{S_1R} (1 - P_{S_1R})^l P^{l}_{S_2R} (1 - P_{S_2R})^{N-k-l} E[n^{(2)}|L = l] \\
    &+ \sum_{m=0}^{N-k-l} \left\{ C^{N-k-l-m}_m P^{m}_{S_1R} (1 - P_{S_1R})^{N-k-l} \times P^{N-k-l-m}_{S_2R} (1 - P_{S_2R})^m E[n^{(2)}|M = m] \right\} \right\},
\end{align}

\begin{align}
    n^{(3)} &= \sum_{k=0}^{N} \left\{ C^N_k P^{N-k}_{S_1R} (1 - P_{S_1R})^k P^{N-k}_{S_2R} (1 - P_{S_2R})^k E[n^{(3)}|K = k] \\
    &+ \sum_{l=0}^{N-k} \left\{ C^{N-k}_l P^{N-k-l}_{S_1R} (1 - P_{S_1R})^l P^{l}_{S_2R} (1 - P_{S_2R})^{N-k-l} E[n^{(3)}|L = l] \\
    &+ \sum_{m=0}^{N-k-l} \left\{ C^{N-k-l-m}_m P^{m}_{S_1R} (1 - P_{S_1R})^{N-k-l} \times P^{N-k-l-m}_{S_2R} (1 - P_{S_2R})^m E[n^{(3)}|M = m] \right\} \right\},
\end{align}

\[ (14) \]

\[ (15) \]
where $E[.]$ denotes the expectation value and $C_k^N = N!/k!/(N-k)!$ represents the total number of subsets consisting of $k$ elements in a set of $N$ elements. Here, $K$, $L$, and $M$ denote three random variables used to represent the numbers of packets that $R$ successfully receives in groups $\Omega_1$, $\Omega_2$, and $\Omega_3$, respectively.

Given that $K = k$ packets are received successfully at $R$ in $\Omega_1$, the average number of transmissions at $R$ based on the proposed algorithm (i.e., Algorithm 1) in the second step can be computed through

$$E[n(2)|K = k] = \sum_{i=0}^{k} \sum_{j=0}^{k} \sum_{u=0}^{k} \sum_{v=0}^{k} C_i^k P_{S_1 D_1}^i (1 - P_{S_1 D_1})^{k-i} C_j^k P_{S_2 D_1}^j (1 - P_{S_2 D_1})^{k-j} \times C_u^k P_{S_1 D_2}^u (1 - P_{S_1 D_2})^{k-u} C_v^k P_{S_2 D_2}^v (1 - P_{S_2 D_2})^{k-v} \times \min\{i + j, u + v\} n_{DT}^{(R)} + |(i + j) - (u + v)| n_{RD_a}^{(R)},$$

(16)

where $n_{DT}^{(R)}$ is the transmission bandwidth required at $R$ to send a packet to both $D_1$ and $D_2$, and $n_{RD_a}$ is given by (10) with $a = 1$ if $i + j > u + v$, and $a = 2$ otherwise. Here, $n_{DT}^{(R)}$ can be similarly obtained as (8), i.e.,

$$n_{DT}^{(R)} = \frac{1}{1 - P_{RD_1}} + \frac{1}{1 - P_{RD_2}} - \frac{1}{1 - P_{RD_1} P_{RD_2}}.$$

(17)

For packets in groups $\Omega_2$ and $\Omega_3$ within the second step of the retransmission at $R$, the average number of transmissions may be calculated by

$$E[n(2)|L = l] = \sum_{i=0}^{l} \sum_{j=0}^{l} C_i^l P_{S_1 D_1}^i (1 - P_{S_1 D_1})^{l-i} C_j^l P_{S_2 D_2}^j (1 - P_{S_2 D_2})^{l-j} \times \min\{i, j\} n_{DT}^{(R)} + |i - j| n_{RD_a}^{(R)},$$

(18)
\[ E[n^{(2)}|M = m] = \sum_{i=0}^{m} \sum_{j=0}^{m} C_i^m P_{S_2D_1}(1 - P_{S_2D_1})^{m-i} C_j^m P_{S_2D_2}(1 - P_{S_2D_2})^{m-j} \]
\[ \times [\min\{i, j\} n^{(R)} + |i - j| n_{PD}] , \]

(19)

where \( a = 1 \) if \( i > j \), and \( a = 2 \) otherwise.

In the third step where \( R \) fails to receive packets of the first group in the first step, \( S_1 \) and \( S_2 \) are required to retransmit the remaining lost packets with the average number of transmissions given by

\[ E[n^{(3)}|K = k] = \sum_{i=0}^{N-k} \sum_{j=0}^{N-k} \sum_{u=0}^{N-k} \sum_{v=0}^{N-k} C_i^{N-k} P_{S_1D_1}^i (1 - P_{S_2D_1})^{N-k-i} \]
\[ \times C_j^{N-k} P_{S_2D_2}^j (1 - P_{S_2D_2})^{N-k-j} \]
\[ \times C_u^{N-k} P_{S_1D_2}^u (1 - P_{S_1D_2})^{N-k-u} \]
\[ \times C_v^{N-k} P_{S_2D_2}^v (1 - P_{S_2D_2})^{N-k-v} \]
\[ \times [\min\{i + j, u + v\} n_{RT} + |(i + j) - (u + v)| n_{PD}] , \]

(20)

where \( a = 1 \) if \( i + j > u + v \), and \( a = 2 \) otherwise. For the second group and the third group in the third step, the average numbers of transmissions are computed, respectively, through

\[ E[n^{(3)}|L = l] = \sum_{i=0}^{N-k-l} \sum_{j=0}^{N-k-l} C_i^{N-k-l} P_{S_2D_1}^i (1 - P_{S_1D_1})^{N-k-l-i} \]
\[ \times C_j^{N-k-l} P_{S_2D_2}^j (1 - P_{S_1D_2})^{N-k-l-j} \]
\[ \times [\min\{i, j\} n_{RT} + |i - j| n_{PD}] , \]

(21)
\[ E[n^{(3)}|M = m] = \sum_{i=0}^{N-k-l-m} \sum_{j=0}^{N-k-l-m} C_i^{N-k-l-m} P^i_{S_2D_1} (1 - P_{S_2D_1})^{N-k-l-m-i} \times C_j^{N-k-l-m} P^j_{S_2D_2} (1 - P_{S_2D_2})^{N-k-l-m-j} \times [\min\{i, j\} n_{RT}^{(S_i)} + |i-j| n_{RT}^{(S_i, D_d)}], \]

where \( a = 1 \) if \( i > j \), and \( a = 2 \) otherwise. In Eqs. (21) and (22), \( n_{RT}^{(S_i)} \), \( i = 1, 2 \) denotes the average number of transmissions to transmit packets from \( S_i \) to both \( D_1 \) and \( D_2 \) through \( R \) that can be computed by

\[
n_{RT}^{(S_i)} = \frac{1}{1 - P_{S,R} P_{S,D_1} P_{S,D_2}} [1 + P_{S,R} P_{S,D_1} (1 - P_{S,D_2}) n_{RT}^{(S_i, D_1)} + P_{S,R} (1 - P_{S,D_1}) P_{S,D_2} n_{RT}^{(S_i, D_2)} + (1 - P_{S,R}) P_{S,D_1} (1 - P_{S,D_2}) n_{RD_1} + (1 - P_{S,R}) (1 - P_{S,D_1}) P_{S,D_2} n_{RD_2} + (1 - P_{S,R}) P_{S,D_1} P_{S,D_2} n_{RT}^{(R)}].
\]

4. Numerical and Simulation Results

In this section, the transmission bandwidths of the different protocols are evaluated both from the analytical formulations above and also simulation models over mixed Rayleigh and Rician flat fading channels. Rayleigh flat fading channels are considered NLOS transmissions reflecting more distant locations, whilst Rician flat fading channels are considered LOS transmissions representing closer proximities. Four scenarios representing typical fading situations are now considered.

4.1. Scenario (a): \( S_i \to R \) and \( R \to D_i \) \( i = 1, 2 \) are both NLOS

In this case the channels \( S_i \to R \) and \( R \to D_i \) \( i = 1, 2 \) are both Rayleigh fading channels, i.e., \( K_{S,R} = K_{SR} = 0 \) and \( K_{RD_i} = K_{RD} = 0. \)
The range of $\gamma_{S_1,R}$ was selected to cover 0 to 20 dB in order to characterize the performance over a wide range of SNR conditions. Fig. 3 shows the transmission bandwidth of the three ARQ protocols as a function of $\gamma_{S_1,R}$, i.e., the SNR of the wireless link $S_1 \rightarrow R$.

In order to evaluate the influence on the transmission bandwidth performance of the channels between the sources and relay, we initially assume $\gamma_{S_1,R} = \gamma_{S_2,R}$. The other channel SNRs may be arbitrarily set to $\gamma_{S_1,D_1} = \gamma_{S_2,D_2} = 5$ dB, $\gamma_{S_1,D_2} = \gamma_{S_2,D_1} = 0$ dB, and $\gamma_{R,D_1} = \gamma_{R,D_2} = 10$ dB. It is also assumed that the packet size (i.e., $N_b$) is 10 bits and the buffer length at the sources (i.e., $N$) is 10 packets. Fig. 3 demonstrates that the proposed NC-based ARQ protocol outperforms the other two ARQ schemes as it is capable of combining the lost packets from different transmission flows.

Figure 3: Transmission bandwidth of different protocols over Rayleigh fading channels with various SNR$_{S_1,R}$. 

![Figure 3](image-url)
within the retransmission phase. It may also be observed that the proposed NC scheme shows significant transmission bandwidth gain over the other ARQ methods. For packets in the $\Omega_1$ grouping, the proposed scheme significantly reduces the number of retransmissions simply through the process of mixing packets from the two different flows. Importantly, the simulation results match exactly the analytical results demonstrating the validity of the derived analytical expressions.

4.2. Scenario (b): $S_i \rightarrow R$ ($i = 1, 2$) is LOS and $R \rightarrow D_i$ ($i = 1, 2$) is NLOS

For this situation, $S_i \rightarrow R$ ($i = 1, 2$) is considered as a Rician channel and $R \rightarrow D_i$ ($i = 1, 2$) as a Rayleigh channel. Fig. 4 provides an example of the transmission bandwidth performance for all three protocols as a function of $\gamma_{S_i R}$. The fading parameters for the results in Fig. 4 are $K_{S_i R} = K_{SR} = 9$
and $K_{RD_i} = K_{RD} = 0$. The SNRs of the other links are set similar to those in Fig. 3.

As the performance of the DT protocol is clearly not as good as the other two, a further comparison specifically between the RT and the proposed NC protocol for scenario (b) is shown in Fig. 5 for the situations of $K_{SR} = \{0, 9, 25\}$ and $K_{RD} = 0$.

4.3. Scenario (c): $S_i \rightarrow R\ (i = 1, 2)$ is NLOS and $R \rightarrow D_i \ (i = 1, 2)$ is LOS

In a similar fashion, $S_i \rightarrow R\ (i = 1, 2)$ is considered now as a Rayleigh fading channel and $R \rightarrow D_i \ (i = 1, 2)$ as a Rician fading channel. The comparison of the transmission bandwidth of various protocols as a function of $\gamma_{S_iR}$ and the comparison between the RT and the proposed NC-based
protocol for scenario (c) with the same $K$ factors used in Fig. 5 can be similarly considered. Our additional results show the same behaviour, and thus they are omitted for brevity.

In scenarios (b) and (c), the results again demonstrate that the proposed NC-based ARQ protocol achieves better performance when compared with other two schemes for both scenarios of mixed fading channel models. Again, the analytical results in all Figs. are shown to match precisely with the simulation results. It can be observed that the transmission bandwidth curves show reduced transmission bandwidth performance as $K_{SR}$ increases. This can be explained as the influence of the LOS component on the BEP gain through all ranges of SNR, which accordingly results in the reduction of the transmission bandwidth.

4.4. Scenario (d): $S_i \rightarrow R$ and $R \rightarrow D_i$ ($i = 1, 2$) are both LOS

The final scenario is a general scenario where all fading channels $S_i \rightarrow R$ and $R \rightarrow D_i$ ($i = 1, 2$) are characterised by Rician fading alone. Fig. 6 shows the comparison of transmission bandwidths specifically between the RT protocol and the proposed NC-based ARQ protocol against $\gamma_{SR}$ with respect to various $K$ factor fading values and with the same assumptions of SNR values as in Fig. 3.

Specifically, in Fig. 6, three cases $\{K_{SR} = 9, K_{RD} = 9\}$, $\{K_{SR} = 9, K_{RD} = 25\}$, and $\{K_{SR} = 25, K_{RD} = 25\}$ have been considered. Similarly, it can be observed that a reduced transmission bandwidth performance is always achieved when either $K_{SR}$ or $K_{RD}$ increases. This again reflects the influence of the LOS components on the BEP gain which is helpful in reducing the transmission bandwidth. It is important to note that at small SNR levels the
RT - $K_{SR} = 9, K_{RD} = 9$ (analytical and simulation)
Proposed scheme - $K_{SR} = 9, K_{RD} = 9$ (analytical and simulation)
RT - $K_{SR} = 9, K_{RD} = 25$ (analytical and simulation)
Proposed scheme - $K_{SR} = 9, K_{RD} = 25$ (analytical and simulation)
RT - $K_{SR} = 25, K_{RD} = 25$ (analytical and simulation)
Proposed scheme - $K_{SR} = 25, K_{RD} = 25$ (analytical and simulation)

Figure 6: Transmission bandwidth of different protocols over Rician fading channels $S_i \rightarrow R$ and Rician fading channels $R \rightarrow D_i$ ($i = 1, 2$) with various $K$ factors as a function of $\text{SNR}_{S_i,R}$.

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proposed scheme has a much increased performance over the other protocols. As the SNR increases, the improvement in the new protocol is still evident, though as expected, the improvement is smaller due to the improved SNR.

In summary, the analytical and simulation results of transmission bandwidth in all Figs. above are shown to be consistent. This means that the transmission bandwidth of various ARQ protocols for MMRN over various fading channels can be evaluated using the derived expressions rather than simulation.

5. Conclusions

In this paper, a new improved and reliable retransmission scheme for multisource multidestination relay networks based on network coding has been proposed. It has been shown that the new protocol significantly reduces the number of retransmissions. The performance of the proposed retransmission scheme was investigated for the specific case with two sources and two destinations and shown to be superior in terms of transmission bandwidth improvement when compared with RT and DT protocols even over mixed Rayleigh and Rician flat fading channels. Specifically, two packet-combination algorithms have been developed to retransmit lost packets. The efficiency of retransmission is improved since the algorithms are able to differentiate between different types of retransmission situations. Further, simulation results of the transmission bandwidth for RT and DT protocols over different Rician and Rayleigh fading factors have validated the theoretically derived analytical expressions. This indicates that any evaluation assessment of transmission bandwidth for the topology presented in this paper can be
determined accurately without the requirement of a simulation model.

References


