Abstract—In this paper, we investigate different practical automatic repeat request (ARQ) retransmission protocols for two-way wireless relay networks based on network coding (NC). The idea of NC is applied to improve the achievable throughput for the exchange of information between two terminals through one relay. Using NC, throughput efficiency is significantly improved due to the reduction of the number of retransmissions. Particularly, two improved NC-based ARQ schemes are designed based on go-back-N and selective-repeat (SR) protocols. The analysis of throughput efficiency is then carried out to find the best retransmission strategy for different scenarios. It is shown that the combination of improved NC-based SR ARQ scheme in the broadcast phase and the traditional SR ARQ scheme in the multiple access phase achieves the highest throughput efficiency compared to the other combinations of ARQ schemes. Finally, simulation results are provided to verify the theoretical analysis.

Index Terms—Two-way relay network, network coding, ARQ protocol.

I. INTRODUCTION

Recently, there has been a growing interest in relaying techniques for wireless systems with the aim of throughput enhancement and quality improvement by exploiting the spatial diversity gains [2]–[5]. Relays can be used not only to improve the service quality and link capacity for local users which are located near the source but also to enhance the coverage and throughput for remote users [6], [7]. Motivated by these benefits, the relay-assisted communications are incorporated in some wireless standard bodies such as cellular [8], ad hoc [9], sensor [10], ultra-wideband body area [11], and storage [12] networks. Generally, we can classify a data transmission in relay networks into two categories: one-way or two-way relay channel. One-way relay channel considers unidirectional communication where a source terminal sends data to a destination through the relay. Several cooperation strategies were proposed for one-way relay channel such as discussed in [2], [4]. On the other hand, two-way relay channel refers to a scenario where two parties want to exchange information with each other. Hence, the relay can be used to improve the performance of both transmissions simultaneously. The two-way communication channel was first investigated by Shannon with the derivation of inner and outer bounds on the capacity region [13].

Network coding (NC) [14], [15] has emerged as a new coding paradigm to increase the system throughput for communication networks. NC allows intermediate nodes (relays) to mix signals received from multiple links for subsequent transmissions, e.g., using XOR operator to mix two signals from two terminals. Many NC-based schemes have been proposed to improve dramatically the network throughput of some particular relay topologies, such as relay-assisted bidirectional channels [16], [17], broadcast channels [18], [19], multicast channels [20], [21], and unicast channels [22], [23]. Here, we investigate the application of NC to two-way relay channels which have recently interested many studies, e.g., [24]–[29]. An end node in two-way relay channels can decode the signals from another node by XORing its known signal with the combined signals received from the relay.

In addition to the improvement of throughput of relay communications using NC techniques, the reliability of data transmission should be taken into consideration, especially in wireless environment with deep fade and background noise. Basically, reliable information transmission over error-prone channels such as wireless medium employs a retransmission mechanism via a standard automatic repeat request (ARQ) protocol [30], where, if a packet cannot be decoded, it is discarded and retransmitted again. The theoretical limits of ARQ systems in two-way communication were first investigated by Shannon [13], who demonstrated that the feedback could improve the reliability of a memoryless channel at all rates below its capacity although it could not increase the channel capacity. The most common basic schemes of ARQ include stop-and-wait (SW), go-back-N (GBN), and selective repeat (SR). Here, we consider the retransmission schemes for two-way wireless relay networks based on NC. In fact, ARQ protocols with NC were generally studied in [31] and applied to other system models such as broadcast channel in [18], multicast transmission in [21]. The principle of NC applied to ARQ schemes is that the sender may XOR the disjoint corrupted packets of different receivers together and retransmit them to all the involving receivers.

The existing ARQ strategies for NC-based wireless system do not lend themselves to practical applications since they were developed with some ideal assumptions. For example, the ARQ scheme for broadcasting in [18] can be viewed as a SR ARQ scheme with infinite buffer at the sender and receivers.
In this paper, we consider the retransmission protocol of two-way relay networks including two end nodes and a relay node. Different ARQ strategies are thoroughly presented for the comparison between the non-NC based schemes and NC-based schemes. The applicability of traditional ARQ protocols such as SW, GBN, and SR is discussed in the context of NC-based two-way relay channel. The information exchange between two end nodes can be divided into two distinct phases: multiple access (MA) phase when two nodes send their data to relay, and broadcast (BC) phase when relay broadcasts the combined packet to both nodes. The traditional ARQ approaches can be easily applied to the MA phase because the relay receives signals from end nodes independently. In BC phase, the relay XORs the erroneous packets and broadcasts the combined packets to both end nodes. In this phase, we firstly study the NC-based SW ARQ scheme as the simplest form of ARQ retransmission strategies. Then, in order to obtain the benefits of NC, two improved schemes, namely NC-based GBN ARQ and NC-based SR ARQ, are designed and explained in details. For comparison purpose, we also analyse the performance of ARQ systems on NC-based two-way wireless relay networks in terms of throughput efficiency. The simulation results show that the better performance is achieved with our proposed schemes comparing with the traditional non-NC retransmission schemes and the theoretical analysis is consistent with numerical throughput efficiency.

The rest of this paper is organized as follows: In Section II, we describe the system model and ARQ strategies of the two-way wireless relay networks. The throughput efficiency is analyzed in Section III for each ARQ retransmission scheme. We present the numerical results in Section IV and Section V concludes this paper.

II. SYSTEM MODEL AND ARQ STRATEGIES

We consider a topology with two terminal nodes $T_i$, $i = 1, 2$, and one relay node $R$ as shown in Fig. 1. The exchange of information between $T_1$ and $T_2$ is assisted by $R$ with the assumption that $T_1$ cannot communicate directly with $T_2$. The nodes are assumed to operate in half-duplex mode, i.e., they cannot transmit and receive simultaneously. In two-way relay networks, the data exchange consists of two consecutive phases: multiple access (MA) phase and broadcast (BC) phase.

First, in the MA phase, $T_1$ and $T_2$ send their own packets to $R$ until the packets are received successfully (at $R$). When $R$ detects errors in a packet, $R$ requests the corresponding node to retransmit the erroneous packet. In reality, several ARQ retransmission mechanisms were developed for lossy channels such as SW, GBN, and SR. The choice of an ARQ scheme depends on the deployment scenario and application. In order to apply NC, the relay has to wait until it receives two correct packets from both nodes $T_1$ and $T_2$. Without loss of generality, we assume that $T_1$ and $T_2$ wish to exchange $K$ packets, denoted by $s_1[i]$ and $s_2[i]$, $i = 0, ..., K - 1$, respectively. The two-way relay communications with NC is illustrated in Fig. 2. When $R$ receives and decodes two packets from $T_1$ and $T_2$ successfully, it forms a new packet by XORing the bits of the received packets, which is given by

$$r[i] = s_1[i] \oplus s_2[i], \quad i = 0, ..., K - 1,$$

where $\oplus$ denotes the XOR operator.

Then, in the BC phase, the XOR packets $r[i]$, $i = 0, ..., K - 1$, are broadcasted to both terminal nodes. Assuming successful transmissions in BC phase, each node extracts its interested packets by XORing $r[i]$ with its own packets

$$s_2[i] = r[i] \oplus s_1[i], \quad i = 0, ..., K - 1,$$

$$s_1[i] = r[i] \oplus s_2[i], \quad i = 0, ..., K - 1.$$
In the retransmission phase, the combined packets for retransmission are received by the relay, and the terminal can recover the lost packet by XORing this packet with the XOR packet. If the combined packets have been received, then the packet is retransmitted until two terminals receive this packet with no error. Fig. 3(c) shows an example of this scheme. The lost packets of $T_1$ and $T_2$ are $\{r[0], r[4], r[6], r[7]\}$ and $\{r[1], r[2], r[6]\}$, respectively. In the retransmission phase, the combined packets for retransmission are $r[0] \oplus r[1], r[2] \oplus r[4], r[6]$, and $r[7]$. Hence, we only need to retransmit 4 packets, compared to 5 retransmissions without NC.

**Improved NC-based ARQ:** A dynamic change of the combined packets based on the correctly received packets at the terminals is considered to improve the throughput efficiency of NC-based ARQ scheme due to the retransmission of the same combined packets. Let us consider Fig. 3(d) with the same lost packets as Fig. 3(c) for NC-based ARQ retransmission. Suppose that the combined packets $r[0] \oplus r[1]$ and $r[2] \oplus r[4]$ are lost at $T_1$ and $T_2$ in the first retransmission, respectively. Thus, $T_1$ and $T_2$ cannot recover packet $r[0]$ and $r[2]$, respectively. Instead of retransmitting these two lost combined packets as NC-based ARQ scheme, $R$ can transmit only $r[0] \oplus r[2]$, i.e., 1 retransmission is reduced.

Theoretically, we can apply the above ARQ protocols in the BC phase. However, the latency in the transmission should be considered in practice. In what follows, we describe different practical ARQ retransmission mechanisms in the BC phase in the context of NC.

1) **Scheme A - NC-based SW ARQ:** This is the simplest form of NC-based ARQ retransmission strategies. The relay sends one XOR packet at a time and waits for ACKs from both terminals. Scheme A produces a low bandwidth efficiency since the relay does not send any further packets in the waiting period.

2) **Scheme B - Improved NC-based GBN ARQ:** In this scheme, $R$ transmits $N$ packets (window size) that can be sent continuously without receiving ACKs from $T_1$ and $T_2$. With traditional GBN ARQ, the nodes only accept packets in order. If a packet is erroneous at $T_1$, $T_1$ will send a non-acknowledgement (NACK) signal to $R$ and reject all subsequent packets until the missing packet is received. However, in order to get the benefit of NC, we develop a modified GBN ARQ in our improved NC-based GBN ARQ scheme where a window is required at both $T_1$ and $T_2$. The idea of this window design for both terminals means that if a packet is erroneous at $T_1$, $T_1$ will try to receive some more following packets from $R$ until it receives a second erroneous packet. When the second error happens, $T_1$ rejects all subsequent packets. Upon receiving NACKs from $T_1$ and/or $T_2$, the relay creates new packets by XORing the erroneous packets and broadcasts these XOR packets. We illustrate the operation of NC-Based GBN ARQ in Fig. 4(a) with a window size of 8 packets. In this example, suppose the packets $\{r[1], r[3], r[6], r[7]\}$ and $\{r[2], r[5], r[6]\}$ are not successfully decoded at $T_1$ and $T_2$, respectively. At first, $T_1$ receives packet $r[1]$ with error, but $T_2$ receives this packet with no error. Next, $T_1$ successfully receives $r[2]$ and saves this in its buffer. Since $T_1$ receives $r[3]$ with error, it ignores all the subsequent packets. At $T_2$, it receives erroneous $r[2]$. It also tries to receive $r[3]$ and $r[4]$. After that, it ignores all following packets received from $R$ since $r[5]$ has error. Thus, in the retransmission phase, the sequence of retransmitted packets are $\{a = r[1] \oplus r[2], r[3], r[4], r[5], ...\}$.

3) **Scheme C - Improved NC-based SR ARQ:** The relay in scheme B re-sends multiple packets when errors or losses occur. Thus, it shows a poor efficiency, especially when the packet error rate (PER) is small. In NC-based SR ARQ, $R$ continues to send a number of packets in its window even after a frame loss. Each node maintains a receive window of sequence numbers that can be accepted. Only the corrupted packets are repeated. Then, the relay continues the transmission sequence where it left off instead of repeating any subsequent correctly received packets. Let
where $p_r$ denotes the bit error rate (BER) of the link $T_i \rightarrow R$ and $N$ is the number of bits in a packet. Thus, the average number of transmissions is computed as

$$n_i^{MA} = \frac{1}{1 - P_r}.$$  

(5)

Let us define $R_{t_i}$ as the transmission rate in bit per second (bps) of each terminal. The required time to transmit one packet of $N$ bits from terminal $T_i$ to $R$ using SW ARQ is given by

$$t_{p_i}^{MA} = \frac{N (1 + 2(t_{prop} + t_{emis}) R_{t_i})}{R_{t_i}},$$

(6)

where $t_{prop}$ and $t_{emis}$ are defined as propagation and emission delay in seconds, respectively. Thus, the required transmission time of terminal $T_i$ is

$$t_{r_i}^{MA} = t_{p_i}^{MA} n_i^{MA}.$$  

(7)

In order to apply NC, the relay should wait until it receives successfully data from both terminals, i.e., after $\max\{t_{r_1}^{MA}, t_{r_2}^{MA}\}$. Therefore, the number of bits received at $R$ is

$$N_R = \max\{R_{t_1} t_{r_1}^{MA}, R_{t_2} t_{r_2}^{MA}\}.$$  

(8)

In BC phase using scheme A, the number of required transmissions that both terminals simultaneously receive a packet with no error follows a geometric distribution with parameter $(1 - P_{t_i})(1 - P_{t_2})$, where $P_{t_i}$ is defined as PER of the link $R \rightarrow T_i$ and is calculated by

$$P_{t_i} = 1 - (1 - p_t)^N,$$

(9)

where $p_t$ denotes the BER of the link $R \rightarrow T_i$. Thus, the average number of required transmissions to transmit a correct packet from $R$ to both $T_1$ and $T_2$ is given by

$$n_i^{BC} = \frac{1}{(1 - P_{t_1})(1 - P_{t_2})}.$$  

(10)

With SW ARQ protocol, the required time to transmit one packet from $R$ to $T_i$ is 1, 2, is equally given by

$$t_{p_i}^{BC} = \frac{N (1 + 2(t_{prop} + t_{emis}) R_e)}{R_e},$$

(11)

where $R_e$ [bps] is the transmission rate of relay. Thus, the required transmission time of $R$ is

$$t_{r_i}^{BC} = t_{p_i}^{BC} n_i^{BC},$$  

(12)

and the number of transmitted bits at $R$ is

$$N_T = R_t t_{r_i}^{BC}.$$  

(13)

Therefore, the throughput efficiency when using SW ARQ in MA phase and scheme A in BC phase is computed as Eq. (14) (see below), where $M$ is the number of data bits each terminal wants to exchange.

From now on, we use (14) as a general formula of throughput efficiency for other following schemes. We observe that $M$, $R_{t_i}$, $n_i^{MA}$, $i = 1, 2$, and $R_e$ are unchanged. Thus, the analysis is simplified to determine $t_{p_i}^{MA}$, $n_i^{BC}$, and $t_{r_i}^{BC}$ depending on the type of ARQ retransmission protocol $X$, where $X \in \{A, B, C\}$, in BC phase.
from both terminals is similarly given by (16).

\[
\eta_A = \frac{2M}{N_R + N_T} = \frac{2M}{\max\{R_{t_1} t_{p_1}^{MA}, R_{t_2} t_{p_2}^{MA}\} + R_{t} t_{p_A}^{BC}}.
\]

Remark 1. Scheme C achieves the highest throughput efficiency while that of scheme A is the lowest. For the comparison, let us assume that

\[
\frac{2(l_{\text{prop}} + t_{\text{emis}}) R_{t_1}}{N} \approx \frac{2(l_{\text{prop}} + t_{\text{emis}}) R_{t_2}}{N} \approx \frac{2(l_{\text{prop}} + t_{\text{emis}}) R_{t}}{N} \approx W_s - 1.
\]

From (14) and (19), it can be seen that \( \eta_B > \eta_A \) since \( (W_s - 1) P_{t_1} < \frac{2(l_{\text{prop}} + t_{\text{emis}}) R_{t_1}}{N} i = 1, 2, (W_s - 1) \max\{P_{t_1}, P_{t_2}\} < W_s - 1 = \frac{2(l_{\text{prop}} + t_{\text{emis}}) R_{t}}{N} \) and \( 1 - \max\{P_{t_1}, P_{t_2}\} > (1 - P_{t_1})(1 - P_{t_2}) \). Similarly, from (23) and (19), we can also observe that \( \eta_C > \eta_B \) since \( W_s > 1 \). Thus,

\[
\eta_C > \eta_B > \eta_A.
\]

Remark 2. While schemes B and C require a higher computational complexity at the relay node due to the NC operation in the retransmission and a buffer window of size \( W_s \) at all nodes, scheme A is the simplest ARQ strategy with no computation at the relay and only one packet in the buffer of all nodes. Specifically, \( N \) additional XOR operations are required for each combination in schemes B and C since each packet includes \( N \) bits.

IV. NUMERICAL RESULTS

In this section, Monte Carlo simulation results of throughput efficiency for different ARQ schemes in different phases - MA phase, BC phase, and whole system - are shown in Figs. 5, 6, and 7, respectively. The packet size is assumed to be 1024 bits, including 1000 data bits and 24 error checking bits, i.e., \( N = 1024 \) and \( M = 1000 \). The fraction \( \frac{[2(l_{\text{prop}} + t_{\text{emis}}) R_{t_1}]}{N} \), \( i = 1, 2 \), is assumed to be \( W_s - 1 \). The window size \( W_s \) is set to be 10.

Fig. 5 shows the simulation and theoretical results of throughput efficiency as a function of BER in the MA phase with SW, GBN, and SR ARQ retransmission schemes. In this phase, NC is not applied in the retransmission. The BERs of the MA links from the source to the relay are set to be equal to each other, i.e., \( p_{r_1} = p_{r_2} = \) BER. We observe that with the same parameters in good channel condition (i.e., low BER), the throughput efficiency of the SR ARQ protocol is always higher than that of the GBN ARQ protocol, and both protocols outperform the SW ARQ protocol. Additionally, the simulation results show that they are consistent with the theoretical analysis.

Fig. 6 shows the results of throughput efficiency in the BC phase of proposed schemes and traditional non-NC schemes.
The BERs of two terminal nodes are set to be equal to each other, i.e. $p_{t_1} = p_{t_2} = \text{BER}$. We observe that the throughput efficiency of improved NC-based SR ARQ is higher than that of improved NC-based GBN ARQ, and again these two protocols outperform the SW ARQ protocol. With NC, the proposed schemes are shown to be better than the traditional non-NC schemes for all ARQ retransmission protocols in terms of throughput efficiency thanks to the reduction in the number of retransmissions. The simulation and theoretical results are again proved to be consistently matched.

In summary, we observe through simulation results that the higher throughput efficiency is achieved with improved techniques based on NC. However, there is a trade-off between the throughput efficiency and the complexity due to the requirement of buffer size and NC process.

V. CONCLUSION

In this paper, we design ARQ retransmission strategies for network coding based two-way wireless relay networks. Different retransmission techniques with three basic ARQ protocols are studied through the comparison of throughput efficiency. We found that the best strategy in the sense of throughput efficiency is the scheme where the improved NC-based SR ARQ is applied in BC phase and the SR ARQ protocol is used in MA phase. With this combination, better throughput efficiency is obtained when compared with the traditional non-NC schemes, however, at the expense of higher complexity due to additional signal processing of using NC and the requirement of buffer at the transmitter/receiver.

REFERENCES


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