SIC-Based Detection With List and Lattice
Reduction for MIMO Channels

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Abstract—To derive low-complexity multiple-input–multiple-output (MIMO) detectors, we combine two complementary approaches, i.e., lattice reduction (LR) and list within the framework of the successive interference cancellation (SIC)-based detection. It is shown that the performance of the proposed detector, which is called the SIC-based detector with list and LR, can approach that of the maximum-likelihood (ML) detector with a short list length. For example, the signal-to-noise ratio (SNR) loss of the proposed detector, compared with that of the ML detector, is less than 1 dB for a 4 × 4 MIMO system with 16-state quadrature amplitude modulation (QAM) at a bit error rate (BER) of 10⁻³ with a list length of 8.

Index Terms—Lattice reduction (LR)-based detection, list detection, successive interference cancellation (SIC), multiple-input–multiple-output (MIMO) detection.

I. INTRODUCTION

In wireless communications, it is well known that the channel capacity can linearly increase with the number of antennas (provided that the numbers of transmit and receive antennas are the same) [1], [21], [22]. Thus, to increase the channel capacity, the transmitter and receiver can be equipped with multiple antennas, and the resulting channel becomes a multiple-input–multiple-output (MIMO) channel. Various space-time architectures for signal transmission over MIMO channels are proposed to effectively exploit spatial and temporal diversity gain in [3] and [4].

In general, since more symbols are transmitted in MIMO systems, the detection complexity can be high. For example, the complexity of maximum-likelihood (ML) detection exponentially increases with the number of transmit antennas. Thus, various approaches are devised to reduce the complexity. The successive interference cancellation (SIC) approach is employed in [4]. The relation between SIC-based MIMO detection and the decision feedback equalizer (DFE) is exploited in [5].

In [6], the partial maximum a posteriori probability (MAP) principle is derived to discuss the optimality of SIC-based detection. List detectors are also considered for MIMO detection to obtain a soft decision in [7] and [8] based on [9].

In [10], a lattice reduction (LR)-based MIMO detector used as a low-complexity MIMO detector is first discussed. In [11], more LR-based MIMO detectors are proposed. It is shown that the performance of LR-based MIMO detectors using minimum mean square error (MMSE)-SIC approaches ML performance. An overview of LR-based detection can be found in [12]. In [13] and [14], it is shown that LR-based detection can achieve full diversity. This is an important observance as most low-complexity suboptimal MIMO detectors could not exploit full diversity. It is noteworthy that a soft decision can also be obtained from the LR-based detection [15].

Although the Lenstra–Lenstra–Lovasz (LLL) algorithm, which is one of the LR algorithms, has a polynomial (average) complexity (for a certain class of random channel matrices) [16], [17], the complexity increases relatively rapidly with the number of basis vectors (or the number of transmit antennas). Thus, for a large MIMO system, the computational complexity of the LR-based detection would still be high. To further reduce the complexity, we can decompose a large MIMO detection problem into multiple small MIMO subdetection problems with SIC, as in [6]. Due to SIC, this approach would suffer from error propagation. To mitigate error propagation, the list detection approach can be adopted. The resulting detector has low complexity as the number of basis vectors in the subdetection problem is small. Due to list detection, the proposed detector can enjoy the tradeoff between complexity and performance, i.e., it has better mitigation against error propagation as the list length increases at the expense of increasing complexity.

II. SYSTEM MODEL

Suppose that there are K transmit antennas and N receive antennas. The N × 1 received signal vector r is given by

\[ r = Hs + n \]  

where \( H, s, \) and \( n \) are the N × K channel matrix, K × 1 transmitted signal vector, and N × 1 noise vector, respectively. We assume that \( n \) is a zero-mean circular complex Gaussian random vector with \( E[|n|^2] = N_0 I \). Let \( S \) denote the signal alphabet for symbols, i.e., \( s_k \in S \), where \( s_k \) is the kth element of \( s \), and its size is denoted by \( M \), i.e., \( M = |S| \).

We assume that \( N \geq K \) and consider the QR factorization of the channel matrix as \( H = QR \), where \( Q \) is unitary, and \( R \) is upper triangular. We have

\[ x = Q^H r = Rs + Q^H n. \]

Since the statistical properties of \( Q^H n \) are identical to those of \( n \), \( Q^H n \) will be denoted by \( n \). If \( N = K \), there are no zero rows in \( R \); otherwise, the last \( N - K \) rows become zero. Thus, the last \( N - K \) elements of \( x \) would be ignored for the detection if \( N > K \). If there is no risk of confusion, hereinafter, we assume that the sizes of \( x, R, n \), and \( n \) are \( K \times 1, K \times K, \) and \( K \times 1 \), respectively.

III. SIC-LIST-LR BASED DETECTION

The LR-based detectors in [10] and [11] have near-ML performance with relatively low complexity. It is shown that those LR-based detectors can achieve full diversity gain, just like the ML detector in [13] and [14]. Unfortunately, however, the complexity of LR can rapidly increase with the number of basis vectors, which implies that the complexity of the LR-based detectors may not be reasonably low for a large MIMO system. To avoid this problem, in this section, we propose an SIC-list-LR-based detection method within the framework of the partial MAP detection [7]. The main idea of this method is to break a high-dimensional MIMO detection problem into multiple lower-dimensional MIMO subdetection problems so that the complexity
associated with LR can be reduced. The notion of the partial MAP detection [6] is applied to include multiple lower dimensional MIMO subdetection problems, together with the list detection approach.

To perform the proposed LR and list-based detection, we consider the partition of \( x \) as follows:

\[
x_1 x_2 = \begin{bmatrix} R_1 & R_2 \\ 0 & R_2 \end{bmatrix} s_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}
\]

where \( x_i, s_i, \) and \( n_i \) are the \( K_i \times 1 \) \( i \)th subvectors of \( x, s, \) and \( n, \) respectively. Note that \( K_1 + K_2 = K. \) From (3), we can have two lower dimensional MIMO subdetection problems to detect \( s_1 \) and \( s_2. \) It is straightforward to extend the partition into more than two groups. However, for the sake of simplicity, we only consider the partition into two groups, as in (3).

### 105 A. Algorithm Description

In the proposed SIC-list-LR-based detection, the subdetection of \( s_2 \) is carried out first using the LR-based detector. Then, a list of candidate vectors of \( s_2 \) is generated. With the list of \( s_2, \) the subdetection of \( s_1 \) is performed with the LR-based detector. The candidate vector in the list is used for the SIC to mitigate the interference from \( s_2. \) The proposed SIC-list-LR-based detection is summarized here.

#### S1) The LR-based detection of \( s_2 \) is performed with the received signal \( x_2, \) i.e.,

\[
\tilde{c}_2 = \text{LRDet}(x_2)
\]

where LRDet is the function of the LR detection operation, which will be discussed in Section III-B, and \( \tilde{c}_2 \) is the estimated vector of \( s_2 \) in the corresponding LR domain. Note that there is no interference from \( s_1 \) in detecting \( s_2. \)

#### S2) A list of candidate vectors in the LR domain is generated by

\[
C_2 = \text{List}(\tilde{c}_2)
\]

where List is a function that chooses the \( Q \) closest vectors to \( \tilde{c}_2 \) in the LR domain. We will discuss the list generation in Section III-C.

#### S3) The list of candidates of \( s_2, \) which is denoted by \( S_2, \) can be converted from \( C_2. \) For convenience, denote \( S_2 = \{s_2^{(1)}, s_2^{(2)}, \ldots, s_2^{(Q)} \}. \)

#### S4) Once \( S_2 \) is available, the LR-based detection of \( s_1 \) can be carried out with SIC, i.e.,

\[
\tilde{c}_1^{(q)} = \text{LRDet} \left( x_1 - R_3 s_2^{(q)} \right)
\]

where \( s_2^{(q)} \) is the \( q \)th decision vector of \( s_2 \) from list \( S_2. \)

Let \( s_1^{(q)} \) denote the signal vector corresponding to \( \tilde{c}_1^{(q)} \) in the LR domain and \( \tilde{s} = [ \tilde{s}_1^{(1)} \tilde{s}_1^{(2)} \ldots \tilde{s}_1^{(Q)} ]^T; \) the final decision of \( s \) is found as

\[
\hat{s} = \arg \min_{q=1,2, \ldots, Q} \| x - R s_1^{(q)} \|^2.
\]

Note that a soft decision is also available from the list generated in S5. There are \( Q \) candidate vectors for \( s, \) and they can be used to approximate the log-likelihood ratio as a soft decision, as in [18]. In the succeeding sections, we will explain the proposed detection in detail.

### 105 B. LR-Based Detection

In this section, we describe the LR-based detection used in steps S1) and S4).

### Table I

For the LR-based detection in steps S1) and S4) or S2) and S3) (see Section III-D), we will explain the proposed detection in detail.

![Fig. 1. List in different domains. (Left) \( C_2 \) in the LR domain, which is orthogonal and where the black dot represents \( \tilde{c}_2. \) (Right) \( S_2 \) in the original domain, where the black dot represents \( s_2. \)

Let \( C \) denote the set of complex integers or Gaussian integers \( \mathbb{C} = \mathbb{Z} + j\mathbb{Z}, \) where \( \mathbb{Z} \) is the set of integers, and \( j = \sqrt{-1}. \) We assume that \( \{\alpha s + \beta \mid s \in S\} \subseteq C, \) where \( \alpha \) and \( \beta \) are the scaling and shifting coefficients, respectively. For example, for \( M\)-QAM, if \( M = 2^{2m}, \) we have

\[
S = \{s = a + jb \mid a, b \in \{\pm A, \pm 3A, \ldots, \pm (2m - 1)A\}\}
\]

where \( A = \sqrt{(3E_s/2(M - 1))}, \) and \( E_s = E[|s|^2] \) is the symbol energy. Thus, \( \alpha = 1/(2A), \) and \( \beta = ((2m - 1)/2)(1 + j). \) Note that 144 the pair of \( \alpha \) and \( \beta \) is not uniquely decided.

Consider the MIMO detection with the following signal:

\[
y = Ax + v
\]

where \( A \) is a MIMO channel matrix, \( x \in \mathbb{C}^{K_1}, \) is the signal vector, and \( v \) is a zero-mean Gaussian noise with \( E[|v|^2] = N_0. \) We scale and 148 shift \( y \) as

\[
d = \alpha y + \beta A 1
\]

\[
= A(\alpha z + \beta 1) + \alpha v
\]

\[
= Ab + \alpha v
\]

where \( 1 = [1 \ 1 \ \ldots \ 1]^T, \) and \( b = \alpha z + \beta 1 \in \mathbb{C}^{K_1}. \) Let

\[
\bar{A} = AU
\]

where \( U \) is a unimodular matrix. Using any LR algorithm, including 151 the LLL algorithm [16], we can find the value of \( U \) that makes the 152 152 column vectors of \( \bar{A} \) shorter. It follows that

\[
d = \bar{A}U^{-1}b + \alpha v
\]

where \( \bar{c} = U^{-1}b. \) Note that, as the basis vectors are complex, 154 we can use complex LR algorithms [19] or convert a complex matrix into a 155 real matrix, as in [11]. The MMSE filter for estimating \( c \) is given by

\[
W = \min \|W(d - \bar{d}) - (c - \bar{c})\|^2
\]

\[
= (\bar{A}\bar{c} + \bar{c} + \bar{c})\bar{c} + E_s
\]

\[
= (\bar{A}\bar{c}^H + |\bar{c}|^2 N_0 I)^{-1} \bar{c}
\]

\[
= (\bar{A}\bar{c}^H + |\bar{c}|^2 N_0 I)^{-1} \bar{A}U^{-1}c
\]

\[
\bar{A} \text{ unimodular is a square integer matrix with determinant } \pm 1.\]
where \( \tilde{d} = E[d] = \beta A_1 \), \( \bar{c} = E[\bar{c}] = U^{-1} \beta I \), and \( \text{cov}(c) = \alpha^2 U^{-1} C U^H \) since
\[
\text{cov}(c) = \text{cov}(U^{-1} \bar{b}) = \alpha^2 E[I].
\]

The estimate of \( \bar{c} \) is given by
\[
\tilde{c} = \hat{c} + W_{\text{MMSE}}^H (d - \hat{d}). \tag{13}
\]

In Table I, the signals and parameters for the LR-based MMSE de-
tection for each step are shown. Note that other approaches, including
the LR-based MMSE-SIC detector in [11] or non-LR-based detectors,
can also be used for subdetection.

### C. List Generation in the LR Domain

To avoid or mitigate the error propagation, the use of a list of
166 candidate vectors of \( s_2 \) in detecting \( s_1 \) is crucial. Using the ML metric,
we can find the candidate vectors for the list \( S_2 \). Let
\[
f(\bar{r} | s_2^{(1)}) \geq f(\bar{r} | s_2^{(2)}) \geq \cdots \geq f(\bar{r} | s_2^{(K)})
\]
where \( f(\bar{r} | s) \) is the likelihood function of \( s \) for a given \( r \), and \( s_2^{(q)} \) is the
symbol vector that corresponds to the \( q \)th largest likelihood. With log-
likelihood values, we can also find the candidate vectors as follows:
\[
\left\| r - R_2 s_2^{(1)} \right\|^2 \leq \left\| r - R_2 s_2^{(2)} \right\|^2 \leq \cdots \leq \left\| r - R_2 s_2^{(K)} \right\|^2.
\]

Therefore, the ML-based list becomes
\[
S_2 = \left\{ s_2^{(1)}, s_2^{(2)}, \ldots, s_2^{(Q)} \right\}. \tag{14}
\]

However, for each log-likelihood value, we need to perform a
matrix–vector multiplication. Thus, the resulting computational com-
plexity could be high.

To avoid high computational complexity in generating the list, we
176 can find a suboptimal list in the LR domain that can be obtained with
177 a low complexity. Consider (9). According to Table I, let \( A = R_2 \),
\[ d = \alpha x_2 + \beta A_1 \], and \( b = \alpha s_2 + \beta 1 \). Then, from (10), we have
\[
\left\| r - R_2 s_2 \right\| \propto \left\| d - A b \right\| = \left\| d - A c \right\|.
\]

\[
\text{16–QAM; } N = K = 4, \quad K_1 = K_2 = 2
\]

**Fig. 2.** Error probability with the list of \( c_2 \) for various list lengths.

\[
\text{List from } (17) \quad \text{List from } (18)
\]

**Fig. 3.** BER performance of a 4 \times 4 MIMO system with 16-QAM signaling.

It is noteworthy that the metric on the right-hand side of (15) 179 is defined in the LR domain. Let \( \bar{s}_2 \) be the signal vector in \( S^{K_2} \) corresponding to \( \bar{c}_2 \), and assume that \( \bar{s}_2 \) is sufficiently close to \( s_2^{(1)} \). Then, we can have \( d \approx \bar{A} \bar{c}_2 \). From this, the ML metric (ignoring a 182 scaling factor) for constructing the list in the LR domain becomes
\[
\left\| d - \bar{A} c \right\| \approx \left\| \bar{A} \bar{c}_2 - \bar{A} \bar{c} \right\| = \left\| \bar{c}_2 - c \right\| \bar{A}^H \bar{A}
\]
\[
\left\| r - R_2 s_2 \right\| \leq \left\| d - A b \right\| = \left\| d - A c \right\|. \tag{15}
\]
\[
\left\{ c_2 \left| \left\| \bar{c}_2 - c \right\| \bar{A}^H \bar{A} < r_A \left( Q \right) \right. \right\}
\]
\[
C_2 = \left\{ c_2 \left| \left\| \bar{c}_2 - c \right\| \bar{A}^H \bar{A} < r_A \left( Q \right) \right. \right\}
\]

where \( r_A \left( Q \right) > 0 \) is the radius of an ellipsoid centered at \( \bar{c}_2 \), which 186 contains \( Q \) elements in the LR domain. If the column vectors of \( A \) 187 or the basis vectors in the LR domain are orthogonal, \( \bar{A}^H \bar{A} \) becomes 188 diagonal. Furthermore, if they have the same norm, \( \bar{A}^H \bar{A} \approx I \). Thus, 189 for nearly orthogonal basis vectors of almost equal norm, the list of \( c_2 \) 190 can be approximated as
\[
C_2 \approx \left\{ c_2 \left| \left\| \bar{c}_2 - c \right\| \bar{A}^H \bar{A} < r \left( Q \right) \right. \right\}
\]

**Fig. 4.** BER performance of a 4 \times 4 MIMO system with 64-QAM signaling.
204 where $r(Q) > 0$ is the radius of a sphere centered at $e_2$, which contains $Q$ elements. Since the LR provides a set of nearly orthogonal basis vectors for the LR-based detection, we can see that the column vectors in $\mathbf{A}$ are nearly orthogonal, as shown in Fig. 1, with a two-basis system. Let $\mathcal{S}_2$ denote the list in the original domain obtained from $C_2$ as in step S3). Since no matrix–vector multiplications are required to generate $C_2$ or $\mathcal{S}_2$, we can use $\mathcal{S}_2$ as the list in the 209 proposed detector to reduce computational complexity.

### IV. Simulation Result

210 In this section, we present simulation results. We mainly focus 211 on the case of $K = 4$, particularly the case of $K_1 = K_2 = 2$. The 212 elements of $\mathbf{H}$ are independent zero-mean circular complex Gaussian random variables with unit variance. This case is particularly interesting as the Gaussian reduction, which can find the two shortest vectors in two-basis systems [10], [20], can be used for LR.

213 In the proposed SIC-list-LR-based detection, list length $Q$ plays a key role in the tradeoff between complexity and performance. In general, it is desirable that the list has the true transmitted vector of $e_2$. If not, the proposed detector will have an incorrect decision. If $Q$ increases, the error probability that $\mathcal{S}_2(C_2)$ does not have the correct vector of $e_2$, which is denoted by $P_e(C_2)$, decreases. Error probability $P_e(C_2)$ is considered for the MIMO system with 16-state quadrature amplitude modulation (16-QAM), and $N = K = 4$. Similar results are shown in Fig. 2, where the error probabilities are 216 shown with two different lists in (17) and (18). As the list in (18) 217 is suboptimal, the performance is worse. However, this performance 218 degradation is not significant as the column vectors of $\mathbf{A}$ are nearly orthogonal.

219 The bit error rate (BER) performance of a $4 \times 4$ MIMO system 220 with 16-QAM signaling is shown in Fig. 3. In this case, a near-ML 221 performance can be achieved when $Q \geq 8$. For example, the signal-
to-noise ratio (SNR) loss of the proposed detector, compared with that of the ML detector, is less than 1 dB at a BER of $10^{-3}$ when $Q = 8$. 222

Fig. 4 shows the simulation results with 64-state quadrature amplitude modulation (16-QAM). This result again confirms that the proposed SIC-list-LR-based detector can provide a near-ML performance with low complexity. At a BER of $10^{-3}$, the SNR loss is less than 1 225 dB, compared with that of the ML detector when $Q = 12$. As the SNR or $E_b/N_0$ increases, the SNR loss increases. However, by increasing 226 list length $Q$, this loss can be reduced as the list length can exploit the tradeoff between performance and complexity. Note that a full diversity may not be achieved by the proposed detector with a fixed list 227 length, as shown in Figs. 3 and 4. The relationship between diversity order and list length needs to be investigated in the near future.

In the LR-based detection, since the number of column swaps in the LR operation is not fixed, the complexity can vary from a channel matrix to another. Thus, in practice, the maximum number of column swaps can be fixed to limit the maximum complexity for two-basis systems. It is shown in [10] that the two shortest vectors can be found within two iterations for more than 99% of $2 \times 2$ random matrices [241] (of Rayleigh fading). However, when the number of column swaps 228 is limited, the basis vectors may not be properly reduced for some 243 channels, and the BER performance could be degraded because of it. 244 To see the impact of the maximum number of column swaps, a 245 simulation is considered with 16-QAM. Table II presents the BER 246 performance when the maximum number of column swaps $N_{cs}$ is limited. It is shown that the performance degradation is negligible, even though $N_{cs} = 1$.

For comparison purposes, we consider the BER performance of the 250 LR-based MMSE-SIC detector, which is the best LR-based detector 251 among the LR-based detectors proposed in [11]. The BER perfor-
mance results are shown in Fig. 5. It is shown that the proposed LR-based detector can provide a performance that is better by about 1 dB than 254 the LR-based MMSE-SIC detector at a BER of $10^{-2}$. Again, we 255

<table>
<thead>
<tr>
<th>$E_b/N_0$</th>
<th>$N_{cs} = 1$</th>
<th>$N_{cs} = 2$</th>
<th>$N_{cs} = 3$</th>
<th>$N_{cs} = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 dB</td>
<td>2.2472 $\times 10^{-2}$</td>
<td>2.2459 $\times 10^{-2}$</td>
<td>2.2459 $\times 10^{-2}$</td>
<td>2.2459 $\times 10^{-2}$</td>
</tr>
<tr>
<td>10 dB</td>
<td>1.9068 $\times 10^{-3}$</td>
<td>1.9062 $\times 10^{-3}$</td>
<td>1.9062 $\times 10^{-3}$</td>
<td>1.9062 $\times 10^{-3}$</td>
</tr>
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256 can confirm that the combination of the LR-based detection and list detection can improve the performance of the LR-based detector and is an effective means to approach the ML performance.

259 For complexity comparison, we can take the upper bound on the average number of LLL iterations in [17], which is given by $N_{\text{cs}} = K^2 \log K/(N - K + 1)$. (We ignore some minor terms to simplify the comparison.) For $4 \times 4$ MIMO channels, we have $N_{\text{cs}} = 263 K^2 \log K/(N - K + 1) = 16 \log 4$ for the LR-based MMSE-SIC detector and $N_{\text{cs}} = 2(K/2)^2 \log((K/2)/(N/2 - (K/2) + 1)) = 265 \log 2$ for the proposed detector. This shows complexity reduction by more than half in terms of LLL iterations. Note that the proposed detector has additional complexity to build a list, which may offset the complexity advantage of the proposed detector over conventional LR-based detectors [11]. To further see the complexity of each detector, simulations are considered under the same environment, as shown in Fig. 5. Fig. 6 shows the estimated flops using MATLAB execution time that was obtained over all operations for each detector through simulations. The execution time is averaged over hundreds of thousands of channel realizations. The Sphere Schnorr–Euchner algorithm [21] is used for the ML decoding, whereas the LLL-reduced algorithm with a reduction factor $\delta = 3/4$ [16] is chosen for the LR-based MMSE-SIC detector [11]. (This is the same as that in Fig. 5.) No limitation on the number of iterations is imposed for any LR algorithm. The proposed LR-based list detector clearly requires the lowest execution time. We can also see that the execution time of the proposed detector is slightly higher than half of the execution time of the LR-based MMSE-SIC detector where the LLL-reduced algorithm is used.

V. CONCLUDING REMARK

283 In this paper, we have derived an SIC-list-LR-based detector for MIMO detection using two complementary techniques, i.e., LR and list detection, within a framework of SIC-based detection. It was shown that the proposed detector has a near-ML performance with low complexity. The list length plays a key role in the tradeoff between performance and complexity. The performance is improved for a longer list length, whereas the complexity increases with list length $Q$.

REFERENCES


