Modelling and Performability Evaluation of Wireless Sensor Networks

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This thesis is submitted for the Degrees of Doctor of Philosophy

December 18, 2015
I would like to dedicate this thesis to my Father, Pst. Noah E. A. Onyiego and my late Mother, Mrs. Janet Achieng Adero
Acknowledgements

I would like to express my special gratitude to my director of studies, Professor Orhan Gemikonakli; you have been a tremendous mentor for me. I would like to thank you for encouraging my research and for allowing me to grow as a research scientist. I would also like to thank my supervisors, Dr. Enver Ever and Dr. Purav Shah, for walking me through this journey each step at a time even during hard times. I lack words to describe the tenderness and humility with which you supported me socially and emotionally. I also want to thank you for letting my defense be an enjoyable moment, and for your brilliant comments and suggestions.

I am deeply indebted to my office friends and work colleagues who were always there with me throughout the research period. I thank Ammar Zayouna, Arindam Gosh, Krishna Doddapaneni, Nallini Selvaraj, Joshua Nwokeji, Rand Hussein, and Ali Hussein.

Special thanks to my loving parents, my late Mother Janet Achieng Adero, who did all she could to shape my life, my step mother who has continued to be there for my siblings and more importantly providing my dad with the hope for living. To my dad, a very special gratitude I give to you for the best upbringing a father can offer his children. Thanks a great deal for standing firm always as you lovingly spelled out the moral, social and spiritual guidance to your cherished children, a huge beneficiary I truly am. To my adored wife and children, thanks a lot for bearing with me the pains that we have had to undergo because of this journey. We have indeed missed out on numerous enjoyable moments. Thanks a lot for standing by me all this long. A new down has finally come, the hope that we have all been waiting for has eventually come, and to God be the glory. To my
beloved brothers and sisters, I am indeed humbled by the way; you have continued to support me and my family. The journey has been very long but finally here I am with lots of gratitude to you all. This journey would not have been successful without your social, physical, emotional and spiritual support. For a moment, I sadly remember our brother Clifford, Mother Janet and Sister Schofield, all whom we have lost during this journey. However, I am hopeful that not very long now; we shall surely re-unite with these loved ones at the second advent of Christ.
Abstract

This thesis presents generic analytical models of homogeneous clustered Wireless Sensor Networks (WSNs) with a centrally located Cluster Head (CH) coordinating cluster communication with the sink directly or through other intermediate nodes. The focus is to integrate performance and availability studies of WSNs in the presence of sensor nodes and channel failures and repair/replacement. The main purpose is to enhance improvement of WSN Quality of Service (QoS). Other research works also considered in this thesis include modelling of packet arrival distribution at the CH and intermediate nodes, and modelling of energy consumption at the sensor nodes.

An investigation and critical analysis of wireless sensor network architectures, energy conservation techniques and QoS requirements are performed in order to improve performance and availability of the network. Existing techniques used for performance evaluation of single and multi-server systems with several operative states are investigated and analysed in details. To begin with, existing approaches for independent (pure) performance modelling are critically analysed with highlights on merits and drawbacks. Similarly, pure availability modelling approaches are also analysed. Considering that pure performance models tend to be too optimistic and pure availability models are too conservative, performability, which is the integration of performance and availability studies is used for the evaluation of the WSN models developed in this study. Two-dimensional Markov state space representations of the systems are used for performability modelling. Following critical analysis of the existing solution techniques, spectral expansion method and system of simultaneous linear
equations are developed and used to solving the proposed models. To validate the results obtained with the two techniques, a discrete event simulation tool is explored.

In this research, open queuing networks are used to model the behaviour of the CH when subjected to streams of traffic from cluster nodes in addition to dynamics of operating in the various states. The research begins with a model of a CH with an infinite queue capacity subject to failures and repair/replacement. The model is developed progressively to consider bounded queue capacity systems, channel failures and sleep scheduling mechanisms for performability evaluation of WSNs. Using the developed models, various performance measures of the considered system including mean queue length, throughput, response time and blocking probability are evaluated. Finally, energy models considering mean power consumption in each of the possible operative states is developed. The resulting models are in turn employed for the evaluation of energy saving for the proposed case study model. Numerical solutions and discussions are presented for all the queuing models developed. Simulation is also performed in order to validate the accuracy of the results obtained.

In order to address issues of performance and availability of WSNs, current research present independent performance and availability studies. The concerns resulting from such studies have therefore remained unresolved over the years hence persistence poor system performance. The novelty of this research is a proposed integrated performance and availability modelling approach for WSNs meant to address challenges of independent studies. In addition, a novel methodology for modelling and evaluation of power consumption is also offered.

Proposed model results provide remarkable improvement on system performance and availability in addition to providing tools for further optimisation studies. A significant power saving is also observed from the proposed model results. In order to improve QoS for WSN, it
is possible to improve the proposed models by incorporating priority queuing in a mixed traffic environment. A model of multi-server system is also appropriate for addressing traffic routing. It is also possible to extend the proposed energy model to consider other sleep scheduling mechanisms other than On-demand proposed herein. Analysis and classification of possible arrival distribution of WSN packets for various application environments would be a great idea for enabling robust scientific research.
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Nomenclature

**Roman Symbols**

- **MQL**: Mean Queue Length
- **ACQ**: Asymmetric Cyclic Quorum
- **ADC**: Analogue to Digital Converter
- **AFECA**: Adaptive Fidelity Energy Conserving Algorithm
- **APTEEN**: Adaptive Periodic Threshold-sensitive Energy Efficient Network
- **ASCENT**: Adaptive Self-Configuring sEnsor Networks Topology
- **AWP**: Asynchronous Wakeup Protocol
- **BCH**: Back-Up Cluster Head
- **BDD**: Binary Decision Diagrams
- **BLE**: Bluetooth Low Energy
- **CH**: Cluster Head
- **CMOS**: Complementary Metal-Oxide Semiconductor
- **CSMA/CA**: Carrier Sense Multiple Access with Collision Avoidance
- **CTMC**: Continuous Time Markov Chain
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>DWEHC</td>
<td>Distributed Weight-based Hierarchical Clustering Protocol</td>
</tr>
<tr>
<td>EECS</td>
<td>Energy Efficient Clustering Scheme</td>
</tr>
<tr>
<td>FCFS</td>
<td>First Come First Served</td>
</tr>
<tr>
<td>FFD</td>
<td>Full Function Device</td>
</tr>
<tr>
<td>FIFO</td>
<td>First In First Out</td>
</tr>
<tr>
<td>FPS</td>
<td>Flexible Power Scheduling</td>
</tr>
<tr>
<td>GAF</td>
<td>Geographical Adaptive Fidelity</td>
</tr>
<tr>
<td>GeRaF</td>
<td>Geographic Random Forwarding</td>
</tr>
<tr>
<td>HEED</td>
<td>Hybrid Energy-Efficient Distributed clustering</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>KHZ</td>
<td>Kilohertz</td>
</tr>
<tr>
<td>KSTS</td>
<td>Kolmogorov-Smirnov Test Statistic</td>
</tr>
<tr>
<td>LCL</td>
<td>Lower Confidence Level</td>
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<tr>
<td>LEACH</td>
<td>Low-Energy Adaptive Clustering Hierarchy</td>
</tr>
<tr>
<td>MAC</td>
<td>Medium Access Control protocol</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro-Electromechanical Sensing devices</td>
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<tr>
<td>MHz</td>
<td>Megahertz</td>
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<tr>
<td>MRM</td>
<td>Markov Reward Model</td>
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<tr>
<td>MTTF</td>
<td>Mean Time To Failure</td>
</tr>
<tr>
<td>MTTR</td>
<td>Mean Time To Repair</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>NAG</td>
<td>Numerical Algorithm Group</td>
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<tr>
<td>OQN</td>
<td>Open Queing Networks</td>
</tr>
<tr>
<td>PANEL</td>
<td>Position-based Aggregation Node Election protocol</td>
</tr>
<tr>
<td>PEGASIS</td>
<td>Power-Efficient Gathering Sensor Information System</td>
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<tr>
<td>PJL</td>
<td>Percentage of Jobs Lost</td>
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<td>PON</td>
<td>Passive Optical Network</td>
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<tr>
<td>PTW</td>
<td>Pipelined Tone Wakeup</td>
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<tr>
<td>QBDs</td>
<td>Quasy Birth and Death process</td>
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<td>QoS</td>
<td>Quality of Service</td>
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<td>QPS</td>
<td>Quorum based Power Saving</td>
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<td>RAM</td>
<td>Random Access Memory</td>
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<tr>
<td>RBD</td>
<td>Reliability Block Diagram</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>RFD</td>
<td>Reduced Function Device</td>
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<tr>
<td>S-MAC</td>
<td>Sensor-MAC</td>
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<tr>
<td>SD card</td>
<td>Secure Digital Non-Volatile Memory Card</td>
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<tr>
<td>SDP</td>
<td>Sum of Disjointed Products</td>
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<tr>
<td>SHARPE</td>
<td>Symbolic Hierarchical Automated Reliability/Performance Evaluator</td>
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<tr>
<td>STEM</td>
<td>Sparse Topology and Energy Management</td>
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<tr>
<td>T-MAC</td>
<td>Timeout-MAC</td>
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Nomenclature

TEEN  Threshold-sensitive Energy Efficient Network
TEG   Thermoelectric Energy Generators
UCL   Upper Confidence Level
UCS   Unequal Clustering Size
UHEED Unequal clustering algorithm based on HEED
Wi-Fi Wireless Local Area Network
WiMAX Worldwide Interoperability of Microwave Access
WSF   Wakeup Schedule Function
WSN   Wireless Sensor Network

Greek Symbols

\( \alpha \)  Used as Mean active time in Chapter 6
\( \beta \)   Used as Eigen-values in Chapter 3
\( \beta \)   Used as Mean sleep time in Chapter 6
\( \eta \)   System repair rate
\( \gamma \)  System throughput
\( \lambda \) Mean packet arrival time
\( \lambda \) Used as Eigen-Values in chapter 3
\( \mu \)   Mean Service Time
\( \phi \)   Left-Eigen Vectors
\( \psi \)   Left-Eigen Vectors
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\theta$</td>
<td>Channel failure rate</td>
</tr>
<tr>
<td>$\xi$</td>
<td>System failure rate</td>
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<tr>
<td>$\zeta$</td>
<td>Channel restoration rate</td>
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Chapter 1

Introduction

Wireless Sensor Networks (WSNs) are composed of a large number of sensing nodes equipped with limited power and radio communication capabilities. Once deployed the sensing nodes are used for monitoring, sensing and forwarding event occurrences in the habitat of interest to the sink for further processing. Due to their ability to support diverse applications and availability of low-cost end devices (sensors), WSNs have attracted much interest in both academia and industry. The diversity of use is enhanced by technological advancement and subsequent explosion of inexpensive wireless communication, computation and sensing devices Akyildiz et al. [2002]. The application areas include seismic, acoustic, chemical, and physiological sensing that enable various applications as; battlefield surveillance, home security, habitat monitoring, forecast systems, smart agriculture, health monitoring, industrial systems, traffic control and animal tracking among many others. Many application environments present varying Quality of Service (QoS) requirements hence the need for optimisation for better performance.

Performance and availability of WSNs have mainly been hindered by nodes, network and link failures caused by hardware and/or software malfunctions and environmental effects. Earlier research by [Kim et al., 2010] categorised these as network, cluster and node failures with a complete sensor network failure experienced when the sink or a percentage of cluster heads fail. Network failures
may also result from poor terrains and other environmental conditions leading to channel failures.

On the other hand, the production of small-sized sensor nodes as a result of technological advancement has introduced the demand to make the sensor components that fit the physical size and operation requirements of the sensors. This in turn has resulted in constrained sensor resources, including energy source, computation power, and offered memory sizes for both operating system and temporary storage. Limited storage can significantly cause data loss in WSNs, especially where a Cluster Head (CH) is used to coordinate communication within a clustered network. Data intensive applications are particularly restricted due to a large amount of storage space required for temporary data storage. The resource constraints eventually degrade the system performance hence the need to optimise their usage for better performance.

In order to extend the life time of sensor nodes, two main approaches include developing mechanisms for prolonging battery life and replacing failing nodes. In [Chiasserini and Garetto, 2006], [Shin and Sun, 2011], [Dash et al., 2012], [Li, 2011], it is shown that node lifespan can be extended by limiting battery use during active operations only. This is achieved by alternating sensor operation modes between sleep and active states. Other methods include the use of Back-Up Cluster Heads (BCH) for temporarily fall back when a CH begins to fail [Hashmi et al., 2010], dynamically reducing transmission and data aggregation power [Gao et al., 2010] and the use of energy aware protocols [Dash et al., 2012]. In other studies, use of mobile nodes for repairing failing nodes and network holes has been proposed [Dini et al., 2007], [Almasaeid and Kamal, 2009], [Liu et al., 2011], [Jun et al., 2012]. Performance and availability measures such as end to end delay, network capacity, mean time to failure, etc, are considered separately in [Chiasserini and Garetto, 2006], [Hashmi et al., 2010], [Munir and Gordon-Ross, 2011], [Bruneo et al., 2010].

Continuous demand for use of WSN Technology in the presence of complexities of various deployment environments calls for the need to improve performance and availability of WSNs. Though several issues outlined in [Chiasserini and Garetto, 2006], [Almazydeh et al., 2010] [Bagula, 2010] make design and modelling of
WSNs a very challenging task, nodes and network failures remain a major reliability concern \cite{Chiasserini and Garetto, 2006}, \cite{Kim et al., 2010}, \cite{Hashmi et al., 2010}. Once deployed in hazardous environments, it becomes difficult to replace faulty units or depleted batteries thereby degrading network performance and finally reducing the network lifespan. To prolong the lifespan of WSNs, it is possible to use redundant nodes deployed during the initial setup, but they are activated only when need arises to replace a failing node. A few research presented in this area tended to separately consider the performance and availability-related issues as noted in \cite{Chiasserini and Garetto, 2006}, \cite{Munir and Gordon-Ross, 2011}, \cite{Hashmi et al., 2010}, \cite{Bruneo et al., 2010}. To address concerns resulted from these independent studies, the need for integrating performance and availability modelling for WSNs is highly recommended. These studies will enable understanding of the general system behaviour and allow modification of the system to meet the required QoS in terms of performance and availability in the presence of application and environmental complexities along with failures and repairs.

Existing approaches used for performance evaluation of computer and communication systems are benchmarking, simulation and analytical modelling. Benchmarking involves actual measurements taken with the input workloads used as benchmarks. Though it is very accurate, the results cannot be extrapolated to suit changes in the system. It is also very costly in terms of personnel, equipment and time. Building a model of the system using existing software tools is known as Simulation. These models are normally validated against existing systems then altered using proposed modifications. The approach is typically preferred since it is flexible and the results are fair and acceptable. For better results, however, simulation models usually require long runs, which ultimately consume higher computation time and resources. Like benchmarking, simulation is an experimental approach.

When used to model systems, analytical approach results into efficient formulæ and numerical procedures. For successful use of analytical methods, one requires high level of mathematical skills \cite{Banks and Nicol, 2005}, \cite{Ever, 2007}. This approach has successfully been used in modelling and evaluation of performance, availability, and reliability of complex computer and communication
systems. Once validated the approach is known to provide quick and accurate results [Trivedi, 2002a], [Banks and Nicol, 2005], [Ever, 2007]. Given that analytical models are abstractions of the real-world problems, their model predictions ought to be validated against actual measurements collected from the real phenomena [Trivedi, 2002a], [Ever, 2007]. In cases where the derived solutions are not exact and certain assumptions are made, benchmarking and/or simulation results are recommended for validation of the analytical model [Chakka, 1995], [Ever, 2007].

In this research existing analytical approaches used for modelling and performance evaluation of open queuing networks are used to model the complex states of WSNs. In order to achieve some degree of mathematical tractability, the methods were employed in consideration to certain assumptions. A two-dimensional representation of the system state space is used and spectral expansion method together with the system of linear simultaneous equations are employed for computation of the steady-state solution of the systems. The models are then validated using an event-based simulation program for the system developed using Visual C++.

1.1 Motivation

Considering the complexities of WSN applications that are dependent upon deployment environments, this thesis is motivated by the need to provide integrated performance and availability/reliability models for WSNs in the face of varying application demands and frequent node and/or network failures and recovery/replacement. In order to address these requirements, this work will focus on the following key sub-questions:

1. How possible is it to integrate performance and availability studies of WSNs?
2. How does the implied infinite queue capacity of sensor nodes affect overall sensor network performance and availability?
3. How can node and channel failures in WSNs be used to optimise system performance?
4. How does the use of sleep and active mechanisms for extending lifetime of WSN affect system performance and availability?

WSNs have continued to find use in diverse application environments with varying QoS demands Chiasserini and Garetto [2006]. The demands, when considered together with deployment and operation complexities, make modelling WSNs networks complicated. However, in order to improve the QoS in these networks, existing studies have concentrated on pure availability and performance studies hence avoiding related concerns. Therefore, integrating WSN performance and availability studies is important in addressing overestimation of the system’s ability to perform or being too conservative as known with independent performance and availability studies respectively.

Considering the limited node memory and data centric applications that require more temporary storage space [Chang et al., 2007], the assumption that sensors have infinite capacity memory does not hold and becomes a major source of data loss. Therefore, queueing management is crucial in the provision of better performance subject to the desired QoS.

A common phenomena in WSNs is the recurrent node and link failures that adversely degrade network performance and availability Chiasserini and Garetto [2006], Munir and Gordon-Ross [2011]. When fault recovery and node replacements mechanisms are considered, it is important to evaluate their impact on QoS and overall network performance. Relevant studies are therefore necessary for optimising WSN systems.

Limited energy available for sensor nodes remains a major challenge in sustaining operations in WSN. The normal practice is to implement a sleep/wakeup scheme to ensure the nodes are switched on when required to capture event occurrences. The implementation of the schemes proposed may cause variations in nodes and network operations and as a result influence the overall network performance. In some cases such as periodic and on demand sleep scheduling schemes, high traffic intensity may cause negative impacts hence negating the primary purpose for their implementation. Therefore, it is essential to investigate the operation levels to ensure desired QoS are met.
1.2 Research Objectives and Scope

The aim of this study is to develop general analytical models for evaluating performance and availability measures for WSNs with a considerable focus on obtaining models that provide most effective and accurate solutions for improving WSN performance and power conservation in the presence of failures and recovery/replacement. Using analytical modelling techniques, and employing some assumptions in order to achieve a certain degree of mathematical tractability, different complex operative states of WSNs are modelled. Using probabilistic analysis, Markov processes, and queuing theoretic modelling technique, various models of the system under study are developed.

In most WSN deployment, cluster-tree topology is widely used because it provides a compromise for better performance compared to other topologies. Like in other communication networks, WSNs are subject to failures that may result from software or hardware errors. In some application environments, it is possible to reconfigure or replacing failing sensor nodes. For such systems, the consistency of the CH in handling the incoming data packets and its availability to perform operations at a given instant of time is significant in attaining desired QoS. Recent researches in performance modelling of WSNs indicate the need for more realistic performance, reliability and availability models. The existing studies have mainly proposed independent approaches to performance and availability [Chiasserini and Garetto, 2006], [Hashmi et al., 2010], [Munir and Gordon-Ross, 2011]. A more practical approach is to use performability analysis introduced in [Beaudry, 1978] and conceptualised by [Meyer, 1980]. This approach enables modelling a combined performance and availability/reliability concerns together, making the outcome be more realistic to the system under study.

Deployment of homogeneous sensors is a common practice in many application areas. The sensors, which are pre-configured, are assumed to have infinite memory capacity. In the traditional applications like battlefield surveillance, low data rates are common hence; the available memory is assumed adequate for such applications. However, the available memory resource is no longer sufficient for use in data intensive application environment. In most cases, limited memory
has resulted into data loss and severe performance degradation. In order to address memory issues in WSNs, attempts have been made to manage packet flow in the network [Qiu et al., 2011]. End-to-end delays have also been studied [Wang et al., 2012]. However, these studies do not consider node failures and recovery/replacement that are a common occurrence in WSNs. Additionally, considering the effects caused by bounded queue capacities is a good practice that may facilitate improving WSN performance. This will enable analysis of job losses due to the behaviour of bounded queues (blocking), the mean number of jobs in the queue (MQL) and system, and other performance measures such as response time and throughput for accurate and useful predictions.

Limited sensor node power has remained a major challenge in WSN service provision. In order to conserve power, alternating sensor operations between sleep and active periods is widely used [Chiasserini and Garetto, 2006]. The sleep scheduling schemes used are employed in MAC-layer and routing protocols [Heinzelman et al., 2000], [Ye et al., 2002], [Van Dam and Langendoen, 2003], [Li and Lazarou, 2004]. However, the dynamics of implementing sleep scheduling schemes introduce operation challenges [Chiasserini and Garetto, 2004]. Detailed performance studies on the available sleep/active implementation schemes are necessary to establish the impact they introduce into the system. This is significant bearing in mind that whilst the choice of a particular power saving scheme may be preferred in some application areas, the underlying performability trade-off’s require consideration for optimum performance. In this study, the available sleep implementation schemes are analysed for performance optimisation purposes in the presence of node and link failures and recovery/replacement.

Once the models are developed and necessary performance and availability solutions are obtained, they can be evaluated further for energy optimisation purposes. Such models can provide realistic system behaviour considering that they integrate performance and availability studies in addition to power saving schemes, which make them more robust and accurate in providing useful predictions.

In this research, considerable attention is given to traffic analysis within the cluster with the main focus on how the CH manages communication between
the cluster nodes and the sink. The novelty of this research work is to address concerns emanating from independent availability and performance studies by proposing a modelling approach that incorporate performance and availability (performability) studies. The resulting models are in turn used for evaluating system performance and availability measures. Furthermore, the approach is used to develop power consumption models employed for the evaluation of WSN energy conservation.

Open queuing network is used to model the behaviour of the CH and spectral expansion solution technique [Chakka, 1998] and system of linear equations employed for solving the system models. The obtained results are further validated using simulation results. In order to address different QoS requirements using the models developed, various performance measures are considered. The results show that it is feasible to integrate performance and availability studies to successfully optimise WSN services. Furthermore, the results provide a pleasant planning and deployment tool for use by WSN designers. Finally, the energy models also provide efficient tools for regulating sleep/active operation periods compare to tools developed using independent studies [Zhou et al., 2011], [Chia-asserini and Garetto, 2006]. The proposed tools regulate sleep based on traffic intensity hence eliminating energy wastage resulting from frequent sleep during high traffic loads.

1.3 List of Publications

The work presented in this thesis has given rise to the following publications.


2. **Fredrick A. Omondi**, Enver Ever, Purav Shah, Orhan Gemikonakli and


Chapter 2 introduces the domain of this study by providing a critical review of related literature. A comprehensive discussion on sensor node and WSN deployment architectures providing details of WSN resources and communication channels is presented. Energy conservation mechanisms available for WSNs are analysed. Existing performance and availability models for WSNs are investigated. This chapter ends by an introduction of the system under study.

Modelling approaches and solution techniques are critically analysed and compared in Chapter 3. Existing performance and availability modelling techniques for communication systems are investigated and critically analysed. In addition, possible solution methods for two-dimensional state spaces are compared. A detailed explanation of System of Linear Equations and Spectral Expansion methods is given.

In chapter 4, an analytical model for a clustered WSN with unbounded queue capacity CH is presented. A queue model for packet arrival distribution at the CH is initially developed. An integrated performance and availability model with inputs taken from arriving data packet is then developed to mimic the system behaviour. The model is subsequently solved using Spectral Expansion solution technique and Poisson approximation solution approach. The results are further validated using results obtained from an event-driven simulation program.

In real-life situation, queuing systems do not have infinite queues.
cations for example, the available memory space is very small hence the amount of data that can be stored is restricted. In chapter 5, bounded queue capacities are introduced and incorporated in the model of chapter 4. The model is then used to study performance and availability measures, including Mean Queue Length (MQL), throughput, queuing delay, blocking probability and packet loss. Results are obtained using Spectral Expansion Solution technique and validated using an event-driven simulation program. The obtained results are critically analysed and compared with results attained from independent performance and availability studies.

Sleep scheduling is widely used in WSNs to conserve the limited node energy. While this may save energy significantly, it may also hamper system performance and increase energy consumption during heavy traffic load when some sleep scheduling schemes are used. In chapter 6, models that employ sleep operations are considered. First operation dynamics for sleep schedules in WSNs are critically analysed, and results used to model the system behaviour. The developed models are solved using two analytical approaches: Spectral Expansion and System of linear equations, which are further validated using a dedicated simulation program. Using the developed models, the effects of sleep scheduling on system performance are investigated in terms of MQL, throughput, response time, Failure probability and blocking probability and probability of operating in the various states. Finally, the obtained results are critically analysed and evaluated for purposes of improving system performance.

Like other wireless networks, WSNs are subject to link failures that may arise from environmental, interference and transceiver hardware malfunctions. These eventually degrade system performance hence require consideration. In order to make the models more realistic, chapter 7 presents a complete analytical model for a clustered WSN system with unreliable links in addition to node failures. The model considers a finite queue CH that also conserves energy by entering sleep mode. Both failures are considered repairable and operation restored once repair is complete. Similar to the previous cases, the model is solved using spectral expansion technique and the system of linear simultaneous equations and the results further validated using simulation results. The model is then used to
evaluate system performance in terms of MQL, throughput and response time. Furthermore, channel and node failure rates are varied in order to establish the worst case scenarios that can still allow the system to attain desired performability measures. Additionally, effects of limited queue capacity on system performance are evaluated to establish the blocking probability. Finally, the results obtained from the two solution approached are presented comparatively and validated using simulation results.

Chapter 8 presents a model for energy evaluation based on the models presented in chapter 7. More specifically, the composite model presented in chapter 8 is considered for energy evaluation as a case study. The energy model considers that power is consumed in the various operation states. For this study, mean energy consumed in each operation state is employed for computation of the overall CH energy consumption. The obtained results are further analysed to obtain optimum operation range that provides best power saving for the network.

Arrival distributions of packets at the CH in WSNs may vary depending on the application environment and pre-configurations. In chapter 9, a methodology for modelling data arrival distribution for WSNs at the intermediary nodes and the CHs is proposed based on experimental results. First, data delivery models for WSNs are critically analysed. Existing approaches for modelling arrival processes, Bernoulli and Poisson, are critically analysed. Inter-arrival times for data packets are generated using Castalia simulator that runs over OMNET++ platform. Kolmogorov-Smirnov Test is then used to identify if the empirical data follows any known probability distribution and numerical results presented and critical analysed.

Finally, chapter 10 presents a summary of the main contributions of the thesis and outlines some of the possible areas for future studies.

In summary, chapter 1 introduces the domain of this research study by highlighting the research question, motivation, objectives and scope of study. A list of publications and an outline detailing work done is also given. The research begins by a critical review of relevant literature in chapters 2 and 3. Related works, WSN concepts and system under study are presented in chapter 2 while chapter
3 presents a detailed discussion comparing modelling approaches and possible solution techniques. In both chapters relevant terminologies are introduced.

Based on the concepts gathered, requirements of system under study and choices of modelling approaches and solution techniques, the models for the proposed system are progressively developed and solved. Chapter 4 introduces the proposed performability modelling approach and uses it to analyse performance of a simple infinite system model with CH breakdowns and repairs. This model is gradually developed in the following chapters to make it more realistic to the actual system. Chapter 5 introduces and analyses the effects of bounded queue capacity on system performance. The model is further advanced in chapter 6 by incorporating and analysing effects resulting from sleep operation dynamics on performance. In addition, chapter 7 extends the model by incorporating link failures in order to evaluate how environmental conditions affect the overall system performance. In chapter 8, an energy model is developed and used to evaluate energy consumption of the developed performability model of chapter 7.

As an extended discussion, chapter 9 presents an initial analysis of packet arrival distribution in WSNs by proposing an approach based on Kolmogorov-Smirnov Test Statistic on empirical data sets. Finally, chapter 10 concludes the work done.
Chapter 2

Literature Review

2.1 Introduction

Considering that WSNs are resource constrained, creating an infrastructure that connects the physical world and gather information for Internet of Things (IoT) adds more complexity to the performance WSN systems. This in turn has increased the demand for improved performance and dependability of WSNs in the wake of many application challenges Aboelaze and Aloul [2005], Akyildiz et al. [2007]. In many application environments high failure rate exhibited in WSNs limit their lifespan hence affecting their performance and availability thereby degrading overall network QoS. Combined performance and availability studies have successfully been used in modelling communication networks [Franken et al., 1994], [Trivedi et al., 2003], [Gemikonakli et al., 2006], [Do and Chakka, 2010], [Kirsal et al., 2011] over the years. It is possible to extend the same methodology for modelling WSNs in order to improve their QoS. Such studies are important in alleviating the impact on performance and availability resulting from frequent WSN node and channel failures. In this Chapter, a detailed literature review covering WSN architecture, existing performance and availability studies, and energy conservation schemes is carried out in order to develop accurate and effective analytical models for performance and availability evaluation of WSNs.
The remainder of the chapter is organised as follows: Section 2.2 presents a discussion on WSN architecture. Section 2.3 provides a detailed discussion of energy conservation in WSNs. In section 2.4, QoS and WSN performance are presented followed by a detailed discussion of the system under study in section 2.5. Finally, the chapter is summarised in section 2.6.

## 2.2 Wireless Sensor Network Architecture

### 2.2.1 Wireless Sensor Nodes

A Wireless Sensor Node is an end device used for monitoring event occurrence in the desired habitat. Several sensor nodes are usually deployed in the habitat of interest in an ad-hoc manner. In most occasions, sensor nodes are able to selforganize and configure themselves during deployment after which, they mostly stay stationary. The basic architecture of the sensor node given in Figure 2.1 is comprised of the following blocks [Islam et al., 2011], [Singhal et al., 2012]:

1. The central processing unit consisting of a microprocessor responsible for the coordination of the sensor node operations. It is responsible for all processing and decision-making
2. The power unit, which regulates and supplies the required energy to all the sensor node components to perform their necessary operations
3. The Radio transceiver (RF) responsible for data transmission between the nodes. Transceivers are responsible for relaying information through the wireless communicating media.
4. The sensing unit composed of sensors and Analogue to Digital Converters (ADCs). It is responsible for the detection of event occurrence within the habitat and the conversion of the physical phenomena into analogue electrical signals. The ADCs are then used to convert the signals from analogue to digital.
5. The memory unit that is used for data storage. In most cases, sensors
have programmable flash memory for storing programs and Random Access Memory (RAM) for temporary data storage.

A detailed study on the composition and operations of these units is presented in [Hill, 2003], [Villegas et al.], [Lymberopoulos and Savvides, 2008], [Kumari et al., 2013] and [Shahzad, 2014].

![Figure 2.1: Wireless Sensor Node Architecture](image)

### 2.2.2 Wireless Sensor Network

Depending on initial configurations during deployment, nodes may work as routers, gateways or end devices after deployment. Many topologies, including star, cluster/tree and mesh may be used to coordinate WSN operations after deployment [Martalò et al., 2009]. In star topology, each node maintains a single direct communication path with the gateway. However, this restricts the achievable network distance. In [Vlajic and Xia, 2006] cluster/tree topology was used. This allows a single path of communication to the gateway while at the same time allowing hoping through other intermediary nodes with routing capabilities. Nevertheless, the main drawback of this topology is loss of communication path among dependent nodes when the cluster coordinator fails. The remedy to this is use of mesh topology that enables multiple paths to the gateway. However, a part from
introducing complex routing, multi-path option also increases computational cost and network latency [Zhou and Krishnamachari, 2003], [Ebrahimi et al., 2011]. To this end, it is evident that the choice of topology is a trade-off that determines the expected performance outcome.

Figure 2.2 shows a basic clustered WSN deployment architecture. During deployment, nodes may be configured to function either as Full-Function Device (FFD) that effectively carries out all the operations, including data processing, routing & sensing or Reduced-Function Device (RFD) with limited operation capabilities [Salman et al., 2010]. RFD devices are only able to monitor their habitat and forward sensed information to the CH directly or through an FFD. In this topology, the end nodes send their information directly to the CH or through an FFD as illustrated in Figure 2.2. From the CHs, the information is forwarded directly or through an intermediary CH to the sink. The sink is a data collection point linking WSN and the fixed network for further processing. From the wired network, the processed information can then be made available and accessed by interested parties through the Internet.

Figure 2.2: Basic WSN Cluster Deployment Architecture

The main drawback for sensor operations after deployment is limited node power.
In some scenarios, sensors are deployed in hazardous environments where the batteries cannot be replaced once they are depleted. This calls for the need to optimise usage of available energy to promote longer operation time. Towards this, energy-efficient MAC and routing protocols implementing active/sleep operations were proposed in [Heinzelman et al., 2000], [Ye et al., 2002], [Lindsey and Raghavendra, 2002], [Van Dam and Langendoen, 2003], [Akkaya and Younis, 2003], [Li and Lazarou, 2004] [Al-Karaki and Kamal, 2005], [Ghosh et al., 2009], [Li et al., 2010], [Li et al., 2010], [Huang et al., 2012], [Tyagi and Kumar, 2013]. Other mechanisms like use of BCH have also been proposed [Hashmi et al., 2010].

Depending on the topology used, the nodes normally keep a list of their next routes to the sink. It is assumed that nodes always have knowledge of their neighbours and that least-cost route is usually employed depending on availability at the time of transmission [Islam et al., 2011]. Other sources of sensor failure may be through hardware and software failures. Software failures may be reconfigured, while hardware failures lead to complete destruction of the nodes and may only be remedied through replacement where possible [Kim et al., 2010]. There are schemes proposed for replacement through the provision of redundant nodes which are kept inactive until failure occurs [Munir and Gordon-Ross, 2011].

2.2.3 Resource Allocation for Wireless Sensor Network

Compared to fixed sensor networks in which only the individual sensing system is deployed at the place of interest, wireless sensor nodes in addition to the sensing system also integrate all other required resources necessary for complete operations. These include energy sources, radio communication transceivers, external memory and processor units. With all these components considered the wireless node resources are then constrained by the physical size and cost which are desired to be kept as minimal as possible [Shahzad, 2014]. This subsection presents a discussion on the main constraints of WSNs. The challenges include energy supply, available memory and processing capability of the sensor nodes.
2.2.3.1 Energy Sources in WSNs

The advancement of the development of Micro-Electromechanical Sensing (MEMS) devices supported by vast reduction in size and power consumption of CMOS circuitry has led to the production of ultra-low-cost sensing devices desired in a large number of applications. The low-cost and small-size requirements of the wireless sensors result into constraints of the type of energy sources and capacity that can be integrated into a sensor node. The three main supply categories include battery, solar sources and direct distribution.

Electromechanical energy storage in batteries is the predominant means of power supply to wireless devices today. The main forms of battery storage available for WSNs include Macro-Scale Batteries, which are considered very stable and versatile in all the small power sources hence preferred for most WSN applications. Next is a group of very small Micro-Scale Batteries. These give a small power output due to surface area limitations hence not recommended over the later.

While power sources are fundamentally energy reservoirs, power-scavenging sources are characterized by their power density instead of energy density. In comparison to the energy density of the power reservoirs with usable power that is dependent upon the time over which they can operate, the energy provided by power scavenging sources only depend on how long the source stays in operation.

Energy scavenging and harvesting differ slightly depending upon the sources used. In [Steingart, 2009], energy scavenging is used to reference environments where the ambient sources are unknown or highly irregular while in situations where the sources are well characterised and maintained, energy harvesting is used. In this category, a number of energy harvesting technologies have been used in WSNs successfully. These include the widely used solar cells both for recharging battery and capacitors used to power WSN. Successful studies on solar cell usage from simple to complex systems were presented in [Warneke et al., 2002], [Roundy et al., 2003], [Jiang et al., 2005b], [Dutta et al., 2006], [González et al., 2012], [Mukter et al., 2014]. Vibration methods that include piezoelectric materials, inductive and capacitive systems have also been used successfully [Roundy and Wright, 2004], [Lin et al., 2013], [Lee et al., 2014], [Lefeuvre et al., 2006] and
Thermoelectric energy generators (TEG) which produce electrical energy directly from heat have also been used successfully to power Wireless sensor nodes [Knight et al., 2008], [Knight and Collins, 2009], [Cassidy and Scruggs, 2013]. TEGs are deployed at locations with steep temperature differences in close proximity, such as interface between air and water, and, air and soil. They can also be employed on human or animal bodies to utilise the temperature difference between the bodies and the air [Dewan et al., 2014]. Detailed chronology of work done to improve these methods were presented in [Roundy et al., 2004], [Steingart, 2009] and [Dewan et al., 2014].

From the foregoing discussions, research work is ongoing in order to improve the efficiency of the existing energy sources used with wireless sensors in various application environments. These will further enhance availability and performance hence performability studies are recommended for best QoS in WSNs.

### 2.2.3.2 Memory in WSN

Advance developments towards low-cost and small-size sensor nodes have also affected the limits of the storage resources. Wireless nodes are equipped with memories having low capacities that limit their data storage ability. On the other hand, the choice of memory type may also be dictated by the level of power consumed when accessing data or just for maintaining data in memory. A detailed discussion on the choice of memory was presented in [Shahzad, 2014]. From the data sheets, a wide range of memory categories are used in various wireless sensor node platforms.

Though external flash and removable SD cards are used, memory available to individual sensor nodes is still restricted, limiting the amount of data that can be stored and/or processed at a given time. This calls for a good memory management scheme that can cope with the increasing traffic demands while at the same time offering desired QoS.
2.2.3.3 Micro-Controllers in WSN

In order to achieve energy efficiency required for operations, sensor nodes are developed with low-end processors (Micro-controllers) that enable low-cost and low-power consumption. The performance of the processor is controlled by an on-chip limited memory with operating frequencies between 1-to-4 MHz, thereby reducing computation capabilities of the processor [Gabriel and Popa], [Shahzad, 2014]. However, many controllers are static and able to provide frequencies between 0-to-8 MHz. Additionally, integrated on to the controller chip are Analogue to Digital Converters (ADCs) and digital Input/Output devices for providing connectivity to desired external devices like additional memories and sensors. The choice of a good micro-controller, therefore, considers a compromise on a number of parameters including:

1. **Voltage requirements and Power consumption**, which determine the amount of voltage supply needed to run the controller. This ranges between 2.7V and 3.3V in the majority of low-voltage micro-controllers. Low-power consuming controllers are most preferred for efficient energy saving. For example, from [Texas, 2003] it is noted that power consumed in sleep mode varies between controllers with a significant range of $1 \mu A - 50 \mu A$. This implies the need for making a better choice that may be a compromise with other factors.

2. **Wakeup time** is significant reducing delays that may result from frequent sleep given the controllers are expected to enter sleep mode most times of operation. A quick wake-up time will ensure the processor is not kept awake even during short periods of inactivity [Hill, 2003].

3. **Peripheral support** are used to provide interface between the controller and external devices through input/output port. These include digital sensors, transceivers, external memories and ADCs where analogue signals require conversion to digital signals. A variety of peripheral devices exists, but an excellent choice is necessary for good performance and better energy conservation.
2.2.4 Communication in Wireless Sensor Network

When energy harvesting is used, the technology and design considerations for the transceivers play a key role in attaining the stringent efficiency requirements for the low-peak power and ultra-low standby current a part from the general low power consumption known for battery-powered WSNs. Transceivers are generally known to be the highest power consumers hence extra care must be taken when choices are made as per application requirements. Considering that wireless sensors primarily collect and transmit raw data to a central station for further computation analysis, in some application scenarios, this may result to higher bandwidth requirements. Taking the case of a clustered WSN, the CH becomes a focal point for collecting all data and forwarding to the sink either directly or through intermediate nodes. Depending on the type of application monitored, the sampling frequency may vary from a few kHz up to hundreds of kHz with data resolutions from 12-to-16 bits [Bouzid et al., 2013], [Shahzad, 2014]. Based on the frequency and data resolution, the resulting bandwidth requirement will also depend on the number of sensors in the network that will directly determine the throughput requirements.

The choice of an appropriate transceiver that fulfils the required high data throughput, while at the same time ensuring low energy consumption remains a major challenge for WSN operations. Even though a number of transceivers have been proposed ranging from infra-red, mobile broadband and Worldwide Interoperability for Microwave Access (WiMAX), due to low-cost and ultra-low power consumption requirements, these do not satisfy all the prerequisites. However, Wi-Fi, Bluetooth Low Energy (BLE) and ZigBee appear promising in realising ultra-low power and low-cost requirements over a short range. Further quantitative evaluation suggests that ZigBee is the most energy efficient for communicating small amounts of data less than 500 bytes, which is a common phenomena in WSNs. A number of 802.15.4/ZigBee standard compliant transceivers are already in use with many sensors. These include transceivers CC2420, CC2520 and CC2538 among many others [Chipcon Product], [Texas, 2015].
2.3 Energy Conservation in Wireless Sensor Networks

In WSNs, energy is a critical resource requiring parsimonious use. In order to address this concern, energy conservation techniques have become vital in the design of WSN systems. Three main energy conservation techniques are identified in literature [Anastasi et al., 2009]:

1. **Duty cycling** involves alternating sensor operations between active and sleep periods depending on the network activity. A duty cycle may therefore be defined as a ratio of time period the sensor node takes in active operation.

2. **Data driven approaches** involve reducing the amount of data that is sampled by ensuring the sensing accuracy is kept within acceptable application levels. For example considering that sampled data usually exhibit strong spatial and/or temporal correlation [Vuran et al., 2004], the redundant information is eliminated in order to alleviate energy wastage.

3. **Mobility approaches** involve moving nodes used to collect data from static nodes thereby alleviating hopping scenarios where the nodes closer to the sink deplete their energy earlier [Li and Mohapatra, 2007], [Anastasi et al., 2009].

In this research, duty cycling conservation schemes are considered for modelling and energy evaluation of the CH operations. In Figure 2.3 [Anastasi et al., 2009], a taxonomy of the duty cycling schemes is presented. In this figure, duty cycling within the active node is referred to as power management while that of the entire network is referred to as topology control.

The novel idea of alternating node operations between Active and Sleep modes in WSN that began as a simple implementation of a timer in most protocols has improved over the years to be dynamically changed with traffic conditions and the nature of application area. Technological advancements have also seen the introduction of a second low power radio transceiver used to monitor the radio channel for incoming data packets and wake up the controller in time to

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receive the arriving packets. In order to maximise the gains of using sleep/active operations, this idea has been integrated in the development of both MAC layer and routing protocols [Heinzelman et al., 2000], [Ye et al., 2002], [Van Dam and Langendoen, 2003], [Li and Lazarou, 2004], [Lindsey and Raghavendra, 2002], [Al-Karaki and Kamal, 2005], [Tyagi and Kumar, 2013].

From Figure 2.3, two complementary approaches can be used to achieve duty cycling. Topology control exploits network redundancy in order to prolong the network lifetime. In this scenario, a minimum set of nodes is adaptively selected and maintained active to ensure network connectivity. The radio transceivers for the nodes not required for connectivity are switched to sleep mode in order to save energy. During operations, the transceivers for the selected nodes may also enter sleep mode in the absence of network activity and only wake up when required to receive incoming data packets thereby saving energy further. This increases network lifetime by a factor of 2-to-3 with respect to networks where all the nodes are always on [Anastasi et al., 2009], [Ganesan et al., 2004], [Mainwaring et al., 2002]. On the other side, node power management is wholly concerned with implementation of sleep/active schedules in the sensor node. Power management can be divided further into independent sleep/wake up protocols running on top of MAC layer protocol, or they may be fully integrated into MAC protocols. In the following subsections, detailed duty cycling sleep/wake up protocols are
presented.

2.3.1 Topology Control Protocols

In this section, a densely populated WSN with some level of redundancy is considered. Generally, a large number of sensor nodes are randomly deployed in the habitat by use of aeroplanes. In order to take care of failures occurring during deployment or operation times, a slightly larger number of nodes are initially deployed. The same idea is viable when topology control is employed in order to have sufficient nodes providing coverage alternately [Ganesan et al., 2004], [Anastasi et al., 2009]. Based on the application needs, topology control protocols dynamically adapt to the network topology allowing network operations using minimal sensor nodes. Mechanisms of choosing the time and nodes to be activated/deactivated are broadly classified into:

1. **Location driven protocols** identify the nodes to be activated based on theirs locations that are assumed known. A typical example of such is the Geographical Adaptive Fidelity (GAF) [Xu et al., 2001] and [Anastasi et al., 2009] which reduces energy consumption while keeping a constant level of routing fidelity. In GAF, the sensing area is divided into small virtual grids. Sensor nodes in adjacent grids, say $E$ and $F$, are able to communicate with one another. The nodes in a given grid, say $E$, are all assumed identical and only one of the sensor nodes stays activated at a time for routing purposes. The nodes choose among themselves, which one is to stay active.

In GAF, sensor nodes start by exchanging discovery messages after which they enter active phase and periodically re-broadcast the discovery message. When operating in either discovery or active states, a sensor node may enter the sleep state if it identifies another equivalent node is handling routing. From sleeping state, nodes wake up after some random time then goes back to discovery state. In order to conserve energy, load balancing is achieved by periodically choosing the lead node, which remains active for routing purposes. Node residual energy is used as the main metric for choosing the lead node for the grid. A detailed discussion on GAF is presented in [Xu
et al., 2001]. Some other location driven protocols like Geographic Random Forwarding (GeRaF) are also presented in the literature [Zorzi and Rao, 2003b], [Zorzi and Rao, 2003a], [Casari et al., 2005].

2. **Connectivity driven protocols** dynamically activate/deactivate sensor nodes in a manner that guarantee network connectivity and desired coverage [Kong and Yeh, 2007]. One example in this group is Adaptive Self-Configuring sEnsor Networks Topology (ASCENT) [Cerpa and Estrin, 2004]. In ASCENT, each node assesses its connectivity and adapts its participation in the multi-hop network topology based on the condition of the operating region. For example, when a node detects high packet loss, it sends out a request for more nodes to join the network to help relay massage. On the other side, if a node detects higher packet rate due to collision, then it reduces its duty cycle. Prior to joining a network the nodes first probe the local environment to establish if it is helpful to do so. The idea about ASCENT is that the nodes base their decisions on connectivity, and locally measured packet loss. ASCENT adaptively elects active nodes, which remain awake all the time while the rest of the nodes alternate between passive and sleep states following a random time. In the passive state, the nodes are only able to listen to the neighbouring communications in order to check if the adjacent nodes are active. In the event of a help message from the neighbouring nodes, the passive nodes will respond by joining active state until packet loss is reduced to acceptable levels and the condition is stabilized. More connectivity driven protocols have also been proposed. Some of these include An Energy -Efficient Coordination Algorithm for Topology Maintenance in Ad-Hoc Wireless Networks (SPAN) [Chen et al., 2002] and the Adaptive Fidelity Energy Conserving Algorithm (AFECA) that uses the observation about node density to increase radio sleep time [Xu et al., 2000].
2.3.2 Sleep/Wakeup Protocols

As highlighted above, a popular approach for increasing longevity of the WSNs is to alternate sleep/active operation periods of the sensor nodes. In this approach, sensor nodes enter the low-power mode (Sleep) many times in order to save power. However, the nodes periodically wake up in order to check for network activity [Keshavarzian et al., 2006]. In this section, main independent sleep/wake up schemes implemented on top of MAC protocol are presented. These are implemented at either the Network or Application layer. In [Anastasi et al., 2009], this category of protocols are further divided into three main groups presented in the following sections:

2.3.2.1 On-Demand Protocol

In this scheme, a node wakes up only when a neighbouring node requires communication with them, otherwise they remain in sleep mode. A second low power radio transceiver is used to monitor the channel continually for any packet arrivals and wakes up the main radio transceiver when the arrival occurs. In this scenario, the CH goes to sleep automatically following service end for the last job in the system and wakes up each time a new arrival occurs [Gu and Stankovic, 2005] and [Ameen et al., 2010]. Once in the active state, the CH remains active for as long as there are jobs in the system to be serviced. This implementation is found to save energy and is ideal for applications requiring very low duty cycle. The applications include all event-triggered scenarios such as surveillance of machine failures, fire detections, intrusion detection, etc.

In most cases, operations of On-demand scheme are achieved using two separate channels. The use of two radio transceivers ensure there is no transmission signal deferral on wake up channel if the other is in use for packet transmission thereby reducing the wake up latency. Existing On-demand protocols include Sparse Topology and Energy Management (STEM) [Schurgers et al., 2002b], which uses two different radio transceivers, one for wake up signals and the other for data packet transmission. However, for STEM, in order to eliminate communication
range limitations, the two radio transceivers do not use low power. In STEM, the source nodes initiate a talk by sending out stream beacon signals on the wake up channel of the intended recipient who in turn sends out a wake up acknowledgement and turns on its radio. As an advancement to STEM, authors in [Schurgers et al., 2002a] proposed STEM-T, which uses tone instead of beacon. In STEM-T, the tone is sent out to all the neighbouring nodes contrary to the intended receiver in the case of STEM. Another protocol in the same category is the Pipelined Tone Wakeup (PTW) developed to trade-off energy saving and latency [Yang and Vaidya, 2004]. PTW also uses two radios with a signalling tone on the wake up channel to wake up all the neighbouring nodes like in the case of STEM-T. However, in the case of PTW, the burden of tone detection is shifted from the receiver to the sender. The main hindrance to the radio trigger approach is the limited distance that the wake up signal can cover.

### 2.3.2.2 Scheduled rendezvous Protocol

In this scheme, all neighbouring nodes are required to wake up at the same time. In order to achieve this, the clocks for the sensor nodes are required to be synchronised. Typically, nodes periodically wake up to check for prospective communications after which they return to sleeping state until the next rendezvous time. The protocols in this category have different ways in which the sleep/wake up schedules are achieved. The commonly used method employs a fully synchronized approach [Ye et al., 2002] where all nodes wake up at the same time following a periodic pattern, say $\tau_{up}$, stays active for a predefined period, say $\tau_{on}$, then return to sleeping state until the next wake-up time. This scheme is further improved by allowing the nodes to switch off their radios when no activity is detected during operations [Van Dam and Langendoen, 2003]. A fully synchronised scheme has also been successfully implemented on MAC protocols like S-MAC [Van Dam and Langendoen, 2003] and T-MAC [Wei et al., 2004]. The main disadvantage of a fully synchronised scheme is that nodes wake up at the same time hence they all tend to want to transmit their data packets simultaneously thus resulting into frequent collisions. Moreover, due to fixed sleep and wake up periods, the scheme is not flexible to the varying network conditions.
Another approach uses staggered wake up pattern [Keshavarzian et al., 2006] where nodes are hierarchically arranged. While nodes at the same level and same region wake up at the same time as is the case with a fully synchronised approach, nodes at different levels wake up at different times. Staggered approach presents a number of advantages over a fully synchronised approach. These include reduced collision given only a subset of nodes are in the active state in the various levels at a given time instance. Holding onto the same reasoning, the active periods can also be made shorter, and thus saving energy. This method also allows lower hierarchy parent nodes to filter out unnecessary data before forwarding to the upper hierarchy parent node. However, like a fully synchronised approach, staggered wake up approach still faces drawbacks that include collisions since nodes at the same level still wake up at the same times. Because of fixed active and sleep periods, it is also not able to adapt to topology changes and traffic variations [Anastasi et al., 2009].

As advancement to the staggered wake up approach, authors in [Anastasi et al., 2006] have proposed an adaptive and low latency staggered scheme that sets the length of the active period to the minimum value consistently with the current network activity. In addition to minimising the energy consumption this scheme also lowers average packet latency with respect to the fixed staggered scheme. Packet collision experienced in the previous two cases is reduced by varying the length of active periods between nodes in the same level which are further associated with different parents [Anastasi et al., 2009].

More implementation schemes for scheduled rendezvous are studied in literature, including Flexible Power Scheduling (FPS) which takes time-slot approach where time is divided into slots, which are then arranged to form periodic cycles [Hohlt et al., 2004]. In this arrangement, nodes transmit/receive packets in their assigned slots hence maintain power only during that time-slot. An advanced FPS, namely Twinkle, which supports broadcast traffic and sink to sensor communication, was presented in [Hohlt and Brewer, 2006]. Other schemes include two staggered pattern and crossed staggered pattern [Keshavarzian et al., 2006]. In [Anastasi et al., 2009], the possibility of integrating these sleep/wake up schemes with other multi-parent schemes is highlighted. A proposal of such schemes was presented.
2.3.2.3 Asynchronous Protocol

Under this scheme, sensor nodes are allowed to schedule their own sleep/wake up times. In order to guarantee connectivity, sensor node neighbours have overlapped active periods within a specified number of cycles. This scheme employs a quorum based system widely used in the design of a distributed system. The two commonly used quorum systems are cyclic and grid quorum systems [Luk and Wong, 1997]. In quorum-based systems, a collection of sets is considered such that the intersection of any two sets is never empty [Colbourn et al., 2001].

A Quorum based asynchronous wake up was first proposed in [Tseng et al., 2003] where authors presented three different asynchronous sleep/wakeup schemes requiring modifications to the basic IEEE 802.11 power saving mode. In [Zheng et al., 2003], authors used a systematic approach to design an asynchronous wake up mechanism for ad-hoc networks, which is also applicable to WSNs. In this study, generation of wake up schedules is formulated as a block design problem and hypothetical bounds derived under different communication models. Based on the optimum results obtained from the theoretical framework, Asynchronous Wakeup Protocol (AWP) capable of detecting neighbouring nodes in a finite time without slot alignment requirement was developed. The protocol was found to be resilient to both packet collision and variations in network topology. The basic idea is to have each node associated with a Wakeup Schedule Function (WSF) used to generate a wake up schedule. The condition for two neighbouring nodes communicating is to ensure their wake up schedules overlap regardless of their clock difference. In order to conserve power nodes turn their radios on during active periods only. Other studies in this area include a cyclic quorum system using different sets [Wu et al., 2007].

Unlike the AWP which ensures wake up period overlap between neighbouring nodes, authors in [Wu et al., 2007] proposed an Asymmetric Cyclic Quorum (ACQ) system that guarantees neighbour discovery between each member node and the CH in a cluster and between the CHs in the network. This is because in a
clustered environment, there is no need to insist in all-pair neighbour discoveries between the cluster nodes. A detailed construction scheme is then presented, which resembles the ACQ system in $O(1)$ time. The operation is similar to that of Quorum based Power Saving (QPS) protocols [Tseng et al., 2003], [Wu et al., 2007] in which the time axis on each station is divided evenly into beacon intervals where the nodes may choose to stay active or sleep. As indicated before, in a quorum system, a cycle pattern defined is used to specify sleep/wakeup schedules within a given $(n)$ continuous beacon interval for each station. Since the pattern repeats every $(n)$ beacon intervals, $(n)$ is therefore the cycle length. The advantage of the QPS protocols is that a station is required to remain awake only $O(\sqrt{n})$ beacon intervals [Tseng et al., 2003], [Jiang et al., 2005a] every cycle and the guarantee of at least, one of the wake up intervals overlaps with that of another station.

Other approaches presented in literature include Transport-layer approach which apply quorum based wake up scheduling at the transport layer which can cooperate with any MAC-Layer protocol thereby allowing reuse of most MAC protocols [Wang et al., 2006] [Lai, 2010] Another approach is a mechanism called ”Disco” [Dutta and Culler, 2008] which is a simple adaptation of the Schedules based on Chinese Remainder Theorem [Niven and Mongomery, 1991] In this approach, it is shown that Disco can ensure asynchronous neighbour discovery in bounded time, even if nodes independently set their own duty cycles.

2.4 Quality of Service (QoS) and Wireless Sensor Network Performance

Guaranteeing QoS in WSNs has remained difficult and challenging due to sensor network resource constrains and diverse application requirements. The provision of WSN QoS can be broadly classified into application and network categories [Steine et al., 2015], [Bhuyan et al., 2010]. In the case of applications, QoS provision involves parameters specific to the applications. These may include sensor node measurements, deployment and coverage and the number and type of sensor
nodes required for particular applications. On the other hand, network QoS provision envisions how the supporting communication network will meet application needs while efficiently using network resources such as bandwidth and conserving the limited power available for network devices. While traditionally, QoS concentrated on the metrics of network level performance such as delay, throughput, jitter and others, in addition, QoS metrics for WSNs requires consideration of reliability and availability that aid in the provision of required QoS.

Recently, lots of studies have been proposed in the area of WSN performance ranging from MAC layer and routing protocols [Akyildiz et al., 2007], [Ghosh et al., 2009], [Almazydeh et al., 2010], [Huang et al., 2012] and network traffic engineering [Chiasserini and Garetto, 2004], [Bagula, 2010], [Kim et al., 2010] Furthermore, also reported are availability and reliability studies [Dini et al., 2007], [Almasaeid and Kamal, 2009], [Bruneo et al., 2010], [Hashmi et al., 2010], [Houaidia et al., 2011]. However, existing studies continue to address performance and availability concerns separately, implying that effects of performance measure on availability and vice versa are not considered. In this section, a detailed survey of existing studies on performance and availability/reliability is presented.

### 2.4.1 Performance Models for WSNs

Performance modelling and analysis continue to be of great practical and theoretical importance in supporting research as well as in the design, development and optimization of computer and communication systems and applications. The current trend towards the use of WSNs in various application areas also brings with it the need for more performance and availability modelling for an optimized deployment of WSNs. The study of modelling and performance evaluation of WSNs covers a diverse research area, including MAC protocols [Li et al., 2010], routing protocols [Almazydeh et al., 2010], Energy Efficiency [Ameen et al., 2010], Data gathering [Meghanathan, 2012], Topologies [Kamapantula et al., 2012] and radio transmission channels [Giuseppe et al., 2007], [Cheffena, 2012]. Lots of research in this area has covered WSN performance evaluation with a few trials on actual
modelling of WSNs varied subject areas.

Several energy-aware MAC layer and routing protocols for WSN exist in literature. Studies in [Chung and Hwang, 2010], [Ghosh et al., 2009], [Huang et al., 2012], [Simaiya et al., 2013] present a comparison and performance analysis of energy aware MAC layer protocols. In [Huang et al., 2012], a detailed evolution of MAC protocols is given. The authors further evaluate the designs of the protocols in terms of energy efficiency, data delivery performance and the overheads required to maintain a protocol’s mechanisms. In a recent study presented in [Lanjewar and Adane, 2014], authors have also presented a comparative study of MAC layer protocols in terms of energy efficiency, data delivery mechanisms and overheads incurred to maintain a protocol along with their advantages and disadvantage. In another study [Oller et al., 2013] a Wake-up radio system is developed and found to present an energy-efficient solution while providing a good trade-off between latency, packet delivery ratio and applicability. The performance of the wake-up radio system is then compared to two well-known WSN MAC protocols, namely B-MAC and non-beacon enabled IEEE 802.15.4. The results obtained indicate the wake-up radio system effectively outperforms the conventional WSN MAC approaches in terms of energy efficiency for realistic traffic loads expected in WSN.

In [Almazydeh et al., 2010] a simulation technique is used to evaluate the performance of known hierarchical routing protocols like LEACH, PEGASIS and VGA. It was noted that PEGASIS could greatly prolong the sensor network lifetime when the transmission range is limited. VGA, however, saves more energy than other protocols when the transmission range is further. In another study [Baghyalakshmi et al., 2010], it was indicated that TEEN, APTEEN, SPEED, RAP and RPAR minimize latency and conserve’s energy with their own design techniques. It can clearly be noted that a lot of trade-offs exist for consideration when choosing the right routing protocol that facilitates attaining required performance.

Topological studies have also been carried out to identify appropriate logical communication topologies between sensor nodes and the sink in a WSN [Zhou and Krishnamachari, 2003], [Vlajic and Xia, 2006], [Lee et al., 2009], [Ebrahimi et al.,
2011], [Wang et al., 2011a], [Mamun, 2012]. A number of topologies including Star, Cluster/Tree and Mesh have been used to coordinate WSN operations once deployed. In star topology, each node maintains a single communication path with the gateway either through direct connectivity or hopping through intermediary nodes [Vlajic and Xia, 2006], [Lee et al., 2009] However, this restricts the achievable network distance. The remedy to this is the use of Mesh topology, which enables multiple paths to the gateway. However, Mesh topology are known to increase network latency and routing challenges before reaching the gateway [Zhou and Krishnamachari, 2003], [Ebrahimi et al., 2011]

In [Mamun, 2012], authors presented a detailed qualitative comparison of different logical topologies for WSNs using different performance metrics including energy distribution and consumption, load distribution, redundancy, scalability, reliability, latency, Network connectedness, Lifetime and topology management overhead. From the results, though the chain oriented topology is more promising; it requires special attention in some areas to make it more efficient. Cluster based topologies performed very well in most of the areas, and it forms a bridge between the under performing topologies and challenges of the high-performing topologies. In order to conserve power further in a cluster based topology, there has been a growing interest in unequal clustering techniques to improve the overall network lifetime and combat the hotspot problem prevalent in multi hop WSNs. Towards this, authors in [Ever et al., 2012] proposed an unequal clustering algorithm (UHEED), which is an advancement of a Hybrid Energy-Efficient Distributed (HEED) clustering approach for ad hoc sensor networks [Younis and Fahmy, 2004].

In [Chiasserini and Garetto, 2004], a Markov model for WSNs whose nodes may enter sleep mode was presented, and used to investigate the system performance in terms of energy consumption, network capacity, and data delivery delay. It was also used to investigate the trade-offs between performance metrics and sensor dynamics in sleep/active modes of WSNs. In [Qiu et al., 2011], it was noted that due to limited hardware consumption, optimizing node packet buffer and maximizing performance are necessary to improve transmission QoS in WSNs. In the same studies, a packet buffer evaluation method using Queuing Network Models was
proposed where, blocking probabilities and system performance indicators of each node were then calculated using an approximate iterative algorithm for blocking probabilities. In this study, the buffer capacity for a single node was evaluated, and it was concluded that sink nodes require higher-performance capacities.

In order to address convergence-related issues, a new structure was proposed, which converges WSNs and Passive Optical Networks (PON) in [Wang et al., 2011b]. The performance of the structure is modelled and analysed using two M/M/1 queues in tandem. The results indicated how WSN and PON dimensions affect the average queue length, hence may be used as a guideline for resource allocation. In the preceding discussions, none of the authors considered the possibility of node failure during operations. Instead, they all assumed that network nodes do not fail during operation.

In an earlier study in [Ali and Gu, 2009], authors presented a performance modelling approach for WSNs as a queue network with on and off servers representing a sensor nodes active and sleep operation states. The traffic flow and operation state of a given node $j$ is modelled and analysed as a single server following Jackson’s network with node breakdowns and repairs. In this model, Time Division Multiple Access (TDMA) media access protocol with slot reuse is considered for the network. Using the model, a joint distribution of the sensor node queue length for the network is determined. From the results, the probability distribution of the number of active nodes and blocking probability of node activation are determined. However, in this study the actual node failures and repairs are not considered.

The use of the WSN-free radio band within industrial environments faces a lot of challenges, including significant noise from varying temperatures, strong vibrations and strong electromagnetic noise caused by large motors [Sexton et al., 2005], [Tang et al., 2007]. In a bid to address these concerns, discussions on challenges, design principles, and technical approaches of WSNs for industrial applications were presented in [Gungor and Hancke, 2009]. In a recent study, authors in [Cheffena, 2012] presented a novel complete dynamic wideband channel model, which takes into account the noise, interferences, and multipath propagation effects present in industrial environments. The model developed is then used to
evaluate the performance of the industrial WSNs based on IEEE802.15.4 physical layer standards in terms of bit error rate. The advantage of using link diversity to improve the link quality in harsh industrial environment is also demonstrated.

2.4.2 Availability and Reliability Models for WSNs

The main drawbacks to the provision of high availability demanded by WSN applications include limited lifetime, service attacks by intruders, software and hardware failures just to mention a few. There has been recent research in this area [Masoum et al., 2008], [Kim et al., 2010], [Munir and Gordon-Ross, 2011]. In [Thein et al., 2008], [Hashmi et al., 2010] use of BCHs is presented as a form of redundancy when a cluster head fails. However, in both cases’ performance degradation due to replacement and transfer delays between failing CH and BCH is not accounted for. In [Munir and Gordon-Ross, 2011], a Markov model characterizing fault-tolerant sensor node for applications with high reliability requirements is proposed based on the novel concept of determining the coverage factor.

In yet another study [Bruneo et al., 2010], reliability and reproducibility of WSN were investigated, and it was concluded that star topology showed better reliability and reducibility but at the cost of limited network size. This limitation was solved using cluster topology for multi-hop communication, which is also limited by a central point of failure at the CH automatically disconnecting child nodes from the sink node. Further studies on the effect of unreliable WSN links on dependability parameters and the adoption of nonlinear battery discharges were also proposed. Research by [Almasaeid and Kamal, 2009] proposed to minimize the number of additional nodes needed to repair the connectivity by achieving a certain level of fault tolerance using on the minimum K-connectivity algorithm.

In order to improve coverage and connectivity when nodes begin to fail, the use of mobile robots to replace the failed sensor nodes with new ones was proposed [Dini et al., 2007]. Here the robot strategically places the new sensor nodes in central locations that would enable maximum habitat coverage. In another study [Song et al., 2010], the design and implementation of a reconfigurable robot is presented. The robot can serve as a mobile node for wireless sensor networks.
making it adaptable to changing terrain conditions in real-world applications. A jumping WSN robot node for use in repair of broken network connections was also proposed by [Jun et al., 2012]. This robot provides a powerful maintenance support for applications in unfriendly environments.

It is evident that failing nodes can be repaired through software reconfiguration and replacement using mobile nodes in case of complete failure. Manual replacement of nodes is also possible in some non-hazardous application environments. In addition, deployment of extra sensor nodes kept inactive until need arises during failure has also been used [Anastasi et al., 2009].

### 2.4.3 Performability Models for Wireless Sensor Networks

Previous research indicates limited work in the area of WSN performance and availability modelling. Apart from modelling, other studies have attempted to tackle known issues of energy, routing, topology, reliability and dependability in an attempt to optimize performance of WSNs.

In [Houaidia et al., 2011] and [Jun et al., 2012], mechanisms for repairing and replacing failing nodes to sustain longer network life time were developed. Together with recent work in the areas of performance and availability, this is a positive indication of a possibility to integrate performance and availability for purposes of modelling the system’s behaviour and analyzing performance in the presence of failures and repairs. The modelling of WSNs for performability evaluation has not been considered before. Such models if successfully done can provide efficient and reliable configurations to optimize various aspects of WSNs.

Owing to the nature of WSN operations that require frequent topological reorganizations and reconfiguration normally caused by active/sleep operation modes and sensor node failures, it is appropriate to consider the effects resulting from repairs, reconfigurations and replacement of failed sensor nodes where applicable. For such scenarios, pure performance models that ignore failures, repairs and recovery are known to overestimate the system’s ability to perform [Trivedi et al., 2003]. On the other hand, pure availability analysis tends to be conservative
since performance considerations are not taken into account [Trivedi et al., 2003]. In order to obtain realistic composite performance and availability measures associated with failure and recovery behaviour, performability modelling has been proposed and used successfully to model and analyse related communication and computing systems over the years [Mitrany and Avi-Itzhak, 1968], [Chakka and Mitrani, 1996], [Kirsal and Gemikonakli, 2009], [Ever et al., 2009], [Trivedi et al., 2003], [Ever et al., 2013]. However, there is no record of previous research on performability evaluation of WSNs. This is mostly attributed to several deployment challenges, top among them battery power depletion, which normally reduces the lifespan of WSN networks. With successful research being done to improve the life span of the nodes, WSNs tend to inhibit similar characteristics of communication networks hence the available modelling and solution techniques may be successfully used to model these networks.

To obtain realistic solutions, state space models have been employed successfully to solve complicated systems exhibiting transitions between various independent states [Thein et al., 2008], [Sheng-li et al., 2009]. These models may be broadly categorized as; Markovian, Non-Markovian and Non-Homogeneous Markov models. In the literature, Quasi Birth and Death processes (QBDs) have been used extensively to model performance and reliability of various systems [Chakka, 1995], [Ever, 2007], [Gemikonakli, 2014].

### 2.5 System under study

In this study, a WSN of $Y$ stationery, identical nodes is considered. The nodes are organised into a group of $K$ Clusters, each with one CH coordinating cluster operations. A cluster is formed up of the CH itself, FFD and RFD nodes working as end devices. In order to enhance reliability and availability of the network, the CH operations are rotated among strategically deployed FF nodes. The choice of the CH is based on node energy levels, and other metrics deemed appropriate [Hashmi et al., 2010], [Chiasserini and Garetto, 2004], and [Li, 2011]. To conserve energy, CHs rotationally enter sleep mode after transferring operations to the
next CH. For this purpose, use of the best energy-saving protocols like UHEED [Ever et al., 2012] is assumed. The system is also assumed to have redundant sensor nodes deployed at inception but kept inactive until the need to replace a failing node arises [Munir and Gordon-Ross, 2011]. It is further assumed that all nodes are equipped with omnidirectional antennas with same radius \( d \) and can communicate directly with the CH based on the IEEE802.15.4/ZigBee standards. In order to reduce the energy consumption further, nodes are capable of choosing an arbitrary transmission power level as long as the radius \( d \) is not exceeded.

Information sensed at the nodes is forwarded to the CH, which finalises cluster data aggregation. The CHs may also generate data packets based on their observations. The total information is then transmitted by the CH to the sink directly or through another intermediary CH. It is assumed that at least one path exists towards the sink [Chiasserini and Garetto, 2004]. Like other communication networks, this system is subject to failures, which may result from hardware, software and channel link errors. Figure 2.4 shows the system scenario in consideration.

![Network topology of the reference scenario](image)

**Figure 2.4:** Network topology of the reference scenario

A closer look at the functions of the CH indicates several possible operative states, including active and sleep modes implemented in a couple of protocols in order to conserve the limited power available for node operations.
2.5.1 WSN Clustering

In most cases, the formation of WSNs is by deployment of large magnitudes of small sensor nodes in the habitats. Once deployed, the sensor nodes that are pre-loaded with necessary software then discover their neighbours and self-configure themselves into a desired network. Due to inability to cope with energy and resource constraints of WSNs, hierarchical networks are preferred over flat networks. Lots of clustering algorithms have been developed in order to address the constraints of WSNs [Anker et al., 2008], [Mamun, 2012], [Liu, 2012], [Jadidoleslamlam, 2013]. Clustering is preferred in WSN deployment because of its ability to achieve network scalability, energy efficiency, prolonged life time, reduced communication overheads and ease of management in large scale WSNs [Jadidoleslam, 2013].

This arrangement allows all the cluster nodes to have an opportunity to operate as a CH in a rotational arrangement [Ameer Ahmed Abbasi, 2007]. The rotation has also been enhanced by the introduction of BCHs used for creating redundancy in the event of failure before rotation time is reached [Hashmi et al., 2010], [Ameer Ahmed Abbasi, 2007], [Gupta and Younis, 2003]. In every cluster, the nodes communicate directly with the CH. They operate as end devices collecting data and transmitting to the CH, making the CH become the central point for data aggregation. The CH is also responsible for transmission of all cluster traffic to the base station directly or hopping through some other intermediary CHs.

Some of the benefits of using clustering include route localization, which reduces routing table stored at individual nodes [Akkaya and Younis, 2005], conservation of communication bandwidth by limiting inter-cluster interactions to CHs and avoiding redundant exchange of messages among sensor nodes [Ameer Ahmed Abbasi, 2007]. Clustering also benefits from the ability to stabilize topology at sensors’ level thereby reducing topology maintenance overhead. Under this, sensors would only concern themselves with connection to the CH since changes at the inter-CH never affect them. [Ameer Ahmed Abbasi, 2007], [Hou et al., 2005].

Noting that the major concern is to guarantee reduced energy consumption and longer life time, research by [Vlajic and Xia, 2006] indicated that a good choice of a clustering scheme is necessary. In this paper, authors also recommend a maximum of five hops a CH can be deployed away from the sink. However in
every cluster, two hops is recommended for deployment of nodes away from the CH.

In [Ameer Ahmed Abbasi, 2007], a clear outline of clustering objectives is given. These include increasing connectivity and reducing delay, which ensures a good path for connecting all CHs to the sink at reduced delay, minimum cluster count, which enables limiting number of hop counts to the sink and maximizing network longevity, which determines the lifetime of the network. Together with taxonomies of clustering attributes, Figure 2.4, this work depicts WSN clusters as a key medium to the provision of WSN network QoS [Bhuyan et al., 2010] and concentrates on modelling integrated performance and availability of these networks when subjected to a number of metrics.

Clustering hierarchy continues to be used enormously because of the advantages it offers in the resource management and scalability of the network among others [Jadidoleslamy, 2013]. However, QoS handling, mobility effects and redundancy management for ensuring network reliability remains an important trade-off for improving performance in WSNs [Liu, 2012].

### 2.5.2 Cluster Head Selection

The process of choosing the CH involves consideration of a number of metrics, which vary between protocols. Commonly used metrics include the initial and residual energies. During first deployment, same power level is assumed for all the nodes. Using the algorithms that vary from one protocol to another, a CH is selected among strategically placed contending nodes that are preferably 1-hop apart in the neighbourhood. Examples of the protocols used for clustering include LEACH [Heinzelman et al., 2000], where CHs are rotationally selected from the cluster nodes in order to distribute communication energy within the cluster to all the nodes. LEACH has since been improved a lot in order to enhance its performance. The LEACH family include TL-LEACH [Loscri et al., 2005], E-LEACH & M-LEACH [Xiangning and Yulin, 2007], V-LEACH [Yassein et al., 2009] and others.
Another clustering protocol is the HEED [Younis and Fahmy, 2004] which chooses the CH based on the nodes residual energy and the intra-cluster communication cost. In addition, HEED ensures even distribution of the CH throughout the network. Like in the case of LEACH, HEED also does CH election periodically. As an improvement to HEED, Distributed Weight-based Hierarchical Clustering Protocol (DWEHC) was developed. DWEHC improves on HEED by building balanced cluster sizes through creation of a multi-level structure for intra-cluster communication while at the same time limiting the number of sensor nodes that can be attached to a parent node. Other clustering protocols include Position-based Aggregation Node Election protocol (PANEL), Unequal Clustering Size (UCS), Energy Efficient Clustering Scheme (EECS) and others have been developed. A detailed summary of the clustering protocols is presented in [Liu, 2012]. Further details on clustering is presented in [Anker et al., 2008], [Mamun, 2012] and [Jadidoleslamy, 2013].

2.6 Chapter Summary

Significant studies addressing various challenges facing WSNs have been done. However, the ever expanding use of WSN technology in diverse application environments resulting from the availability of technologically advanced low-cost sensor devices has presented further resource constrains on WSNs hence opening more research opportunities. Serious challenges facing WSN include limited power available to the sensor nodes, constrained memory capacity, and processing power, and radio channel interferences.

In order to address these concerns, existing studies have continued to look into the various areas, including topology-related studies that have carried on presenting energy aware routing protocols to help conserve the little available energy and at the same time provide required redundancy. MAC protocols have also been developed to control sensor sleep/wake up schedules. In addition, advanced energy aware transceivers for controlling sleep/wake up periods are proposed. Furthermore, development of micro-battery power sources has promoted the development
and use of sensor networks in all aspects of life.

In order to optimize the use of WSNs, more research studies have continued to address performance and availability/reliability. However, QoS has remained a trade-off of performance metrics depending on the application area. In this section, a detailed study on WSN resource constraints and other limitations directly affecting performance and availability of the network is carried out. A lot of previous and current research work tend to concentrate on independent availability, and performance studies oblivious of the results of such studies. A composite study of availability and performance is hence necessary if optimum QoS is to be achieved.
Chapter 3

Modelling Approaches and Solution Techniques

3.1 Introduction

The rapidly growing demand for use and deployment of WSNs continues to pose a great challenge to service delivery considering the diversity of application environments that demand varying service level requirements. In addition, resource constrained sensor nodes require performance and availability optimization in order to achieve meaningful results within the desired operation periods. Therefore, it is essential to develop a new traffic model that integrates performance and availability to address known issues emanating from independent availability and performance studies. For this purpose, this study considers analytical models for integrated performance and availability of clustered WSNs based on single server systems representing the behaviour of the CH when subjected to both internally and externally originating traffic.

The following sections provide the underpinning theoretical knowledge and discussion on the various solution approaches and their limitations for the proposed system models with bounded and unbounded queues. In this study, the use of Queueing theory and Markov Chain analysis is undertaken to evaluate the offered
joint performance and availability models. The rest of the chapter is organised as follows: Section 3.2 present evaluation methods for performance of computer and communication systems. In Section 3.3, solution approaches for two dimensional state space are discussed followed by the chapter summary in section 3.4

3.2 Evaluation Methods for Performance of Computer and Communication Systems

3.2.1 Pure Performance Evaluation Models

Performance evaluation has been used extensively to study computer and communication systems [Chakka, 1995], [Ever et al., 2012]. In this study, the same approach has been extended to study and evaluate performance of WSNs. Under independent performance studies, it is assumed that systems never fail during operation hence system unavailability is of no consequence. The study of such systems has been analysed using queuing models with one or multi server systems [Chakka et al., 2000], [Ever et al., 2009], [Wang et al., 2011b], and [Kirsal, 2013]. In WSN systems, mainly single server queuing systems have been used [Chiasserini and Garetto, 2004], [Qiu et al., 2011], and [Wang et al., 2011b]. Cutting-edge mathematical techniques that are computationally efficient have made analytical modelling approach the most preferable. Using this approach, the exact or approximate solutions of the system models can be obtained. Fast computations and formulae obtained are the main advantages of analytical modelling. Queuing theory and Markov Birth and Death Processes successfully used in performance evaluation Chakka and Mitrani [1996], Ever et al. [2009], Kirsal et al. [2011] are used in this study. They are fundamental to modelling representations of systems with stochastic processes. Terminologies used in queuing theory for performance modelling are detailed in the model of WSN cluster communication systems given in Figure 3.1.

In queuing theory, a single queuing station system consists of one finite or infinite queues and one or more identical service stations. The term queue is used to mean
a waiting line, hence queuing theory can be defined as a theory of waiting lines. In Figure 3.1, when a new job arrives, it is either taken in for service directly or placed in the queue if the CH is busy. For CHs with unbounded queues, all jobs are stored making the queue size grow as necessary to accommodate all arriving jobs. On the other hand, if the CH is bounded, the arriving jobs are only admitted if the queue has an empty slot. Any job arriving when the queue is full is lost. In such systems, the CH (server) can only serve one job at a time hence its state at a given time is either busy or idle. Kendall’s notation, $A/B/c/K/m/Z$ has been used to describe queuing systems [Chakka, 1995], [Ever, 2007], [Vrije, 2003]. The notations represent the following:

1. $A$ - represent the distribution of job arrivals
2. $B$ - represent the distribution of service time
3. $c$ - represent the number of servers
4. $K$ - represent queue capacity
5. $m$ - represent the population size
6. $Z$ - represent the queue discipline.

The commonly used disciplines in computer and communication systems are...
First-In First-Out (FIFO), and First-Come First-Served (FCFS). In order to determine the distribution of inter arrival and service times, Markov processes are usually used [Vrije, 2003].

A Markov process is a stochastic process whose future state is purely dependent on the current state. In other words, the prospective probabilistic behaviour of the process depends only on the present state of the process and is not influenced by its past history [Vrije, 2003], [Gunter et al., 2006]. The main classes of stochastic processes are Markov chains and Markov processes, where a Markov chain is defined as a discrete-time process for which the future behaviour, given the past and the present, only depends on the present but not on the past. However, a Markov process is defined as a continuous-time version of a Markov chain. The discrete state Markov processes where the transitions are restricted to neighbouring states, are called birth-death processes [Ever, 2007]. The discrete states of these processes are usually represented by integers, and from state \( n \) it can only change to state \( n + 1 \) or state \( n - 1 \) representing arrival and departure of a job to or from the system respectively. A typical birth and death process [Ever, 2007] for an M/M/1 queuing system is illustrated in Figure 3.2.

![Figure 3.2: Markov Birth and Death Process](image)

From Figure 3.2, \( b_i \) and \( d_i \) illustrate birth and death transitions originating from state \( i \) respectively. Using the relations given in [Trivedi, 2002b] it is possible to compute the steady state probabilities \( (P_i) \) using equations 3.1 and 3.2.

\[
P_{i+1} = \frac{b_i}{d_{i+1}} P_i
\] (3.1)
and

\[ \sum_i P_i = 1 \]  

(3.2)

Employing the steady state probabilities, well established queuing theory formulæ can be used to obtain various performance evaluation measures.

The proposed clustered WSN, can be modelled using a single queueing system with the CH centrally collecting and processing packets arriving from cluster nodes. Additionally, the CH also forwards to the sink, packets originating from other CHs located outside the sink’s communication radius. Let \( X = \{X(t), t \geq 0\} \) be a continuous time Markov chain describing transition states of the CH with respect to packet arrivals. The process \( X(t) \) defined on a probability space \((\Omega, \mathcal{F}, \text{Pr})\), with countable values in the state space \( E \) such that for a finite set \( 0 \leq t_1 < t_2 < \cdots < t_n < t_{n+1} \) of “times” and a corresponding set \( i_1, i_2, \ldots, i_{n-1}, i, j \) of CH states in \( E \), the \( \text{Pr}\{X(t) = i, X(t_{n-1}) = i_{n-1}, \ldots X(t_1) = i_1\} > 0 \) [Anderson, 2012]. The states of the process \( X \) can therefore be given by:

\[
\text{Pr}\{X(t_{n+1}) = j | X(t_n) = i, X(t_{n-1}) = i_{n-1}, \ldots, X(1) = i_1\} = \text{Pr}\{X(t_{n+1}) = j | X(t_n) = i\}
\]

(3.3)

Letting \( s \) be an instance of time and considering all \( s \) and \( t \) such that \( 0 \leq s \leq t \) and all \( i, j \in E \) the conditional probability \( \text{Pr}\{X(t) = j | X(s) = i\} \) of equation 3.3 depends only on \( t - s \), and not on \( s \) and \( t \) individually. This property captures the homogeneity of packet distribution at the CH. The conditional probability can now be expressed by equation 3.4 and the transition function of the process \( X(t) \) given by 3.5.

\[
\text{Pr}\{X(t) = j | X(s) = i\} = \text{Pr}\{X(t - s) = j | X(0) = i\}
\]

(3.4)

\[
p_{ij}(t) = \text{Pr}\{X(t) = j | X(0) = i\}, \quad i, j, \in E, t \geq 0
\]

(3.5)

From [Vrije, 2003], the transition between states of the CH are guided according
to the following rules:

1. The system may jump into state $i$ following the arrival or departure of a data packet. It then stays in state $i$ for an exponentially distributed time with a mean of $\frac{1}{v_i}$ independently of how the system reached state $i$ and how long it took to get there. Here the mean distribution time, $\frac{1}{v_i}$, may be caused by arrival of the next packet, $\frac{1}{\lambda_i}$, or the departure from the system after service $\frac{1}{\mu_i}$.

2. If the system leaves state $i$, it jumps to state $j$ ($j \neq i$) with probability $p_{ij}$ independently of the duration of the stay in state $i$, where $\sum_{j \neq i} p_{ij} = 1$ for all $i \in E$.

The convention $p_{ii} = 0$ for all states of $i$ is convenient and natural. This ensures that the sojourn time in a state is unambiguously defined.

From pure performance point of view, it is usually assumed that systems never fail. Such system models are found to be optimistic considering that systems are always expected to fail, and the failures may have a great impact on the overall system performance. In WSNs, systems usually fail either due to hardware, software or when their energy source is completely depleted. Any kind of failure in this case may result into node, cluster, or total network failures. Moreover, network portion failures greatly impact on overall network coverage and may degrade system performance further.

### 3.2.2 Pure Availability Evaluation Models

The availability of WSN systems is becoming increasingly important due to increased dependency of various WSN applications requiring continuous monitoring and also the need for fault-tolerant design. Availability is closely related to reliability, and is defined in ITU-T Recommendation E.800 [ITU-T, 2008] as “the ability of a system to be in a state to perform a function or an operation at a given instant of time, or at any instant of time within a given time interval, assuming that the external resources, if required, are provided.” The main difference between the reliability and availability is that the reliability refers to failure-free
operation during an interval, while availability refers to failure-free operation at a
given instant of time [Trivedi, 2002b], usually the time when a device or system is
first accessed to provide a required function or service. Availability may further
be categorised as:

1. **Instantaneous Availability or Point availability** $A(t)$ of a component
(or a system) is defined as the probability that the component/or system
is properly functioning at time $t$ [Trivedi, 2002b], [Ever, 2007], and may be
described mathematically as:

$$A(t) = R(t) \int_0^t R(t - x)m(x)dx$$

(3.6)

where $R(t)$ is the probability of having no failure in interval $(0, t)$ and $m(x)$
is the repair density. The equation shows that the system is available either
if no failures occurs in interval $(0, t)$, or failure occurs but repair of the
system is completed before time $t$ [Trivedi, 2002a].

In the absence of a repair or a replacement, availability $A(t)$ is simply equal
to the reliability $R(t)$ of the component.

2. **Limiting Availability** defined as the steady-state availability ($A$) is the
limiting value of $A(t)$ as $t \to \infty$. From the literature [Ever, 2007], it may
be expressed mathematically as:

$$\lim_{t \to \infty} A(t) = A = \frac{1}{\xi + \frac{1}{\eta}} = \frac{MTTF}{MTTF + MTTR}$$

(3.7)

where $\xi$ and $\eta$ are failure and repair rates respectively, and $\frac{1}{\xi}$ and $\frac{1}{\eta}$ are Mean
Time To Failure ($MTTF$) and Mean Time To Repair ($MTTR$) respectively.

3. **Interval (Average) Availability** defined as the expected fraction of time
the system is up in a given interval $(0 \to t)$ may be given by:

$$A_I(t) = \frac{1}{t} \int_0^t A(x)dx$$

(3.8)
In [Trivedi, 2001], it is explained that the three availabilities relate as given in equation 3.9

\[
\lim_{t \to \infty} A_I(t) = \lim_{t \to \infty} A(t) = \frac{\eta}{\eta + \xi}
\]  

(3.9)

In order to study system reliability and availability, three model types are identified as Combinatorial, State space and Hierarchical models [Trivedi, 2001], [Ever, 2007]. In combinatorial models, three model types: reliability block diagrams, reliability graphs and fault trees are commonly used. These model types are similar since they capture conditions that make a system fail in terms of the structural relationships between the system components. Reliability block diagrams (RBD) implemented either in series, parallel or in k-out-of-n configurations represent the logical structure of a system with regard to how the reliability of its components affects the system reliability. An RBD can be used to model availability if the repair and failure times are all independent. The assumption of independence and series-parallel structure allows very fast computation of reliability and availability measures. However, many system models in practice do not follow the series-parallel structure. Symbolic Hierarchical Automated Reliability/Performance Evaluator (SHARPE) software package developed by Sahner and Trivedi in 1986 allows easy specification and solution of such models [Trivedi and Malhotra, 1993], [Trivedi, 2001].

Reliability graph models are considered to consist of a set of nodes and edges (and directed arcs), where the edges represent components that can fail or structural relationships between the components [Trivedi, 2001]. The graph contains one node, the source (meaning no arcs enters it), with no incoming edges and one node, the sink (also called destination or terminal nodes) with no outgoing edges. The arcs are assigned failure distributions. A system represented by a reliability graph fails when there is no path from the source to the sink. The edges can be assigned failure probabilities, failure rates or unavailability values or functions, the same as reliability block diagrams. A reliability graph is equivalent to a non-series-parallel reliability block diagram. In the reliability graph, the components are the arcs, while in the block diagram, the components are the boxes. The non-
series-parallel block diagram cannot be directly analysed by (or even specified for) SHARPE, but the reliability graph can. The price for more generality is the increased complexity of solution.

A fault tree is a pictorial representation of the sequence of events/conditions to be met for a failure to occur [Sahner et al., 1996], [Sathaye et al., 2000]. It uses AND, OR, and $k$ of $n$ logic gates to represent the combination of events in a tree-like structure. In order to represent situations where one failure event propagates failures along multiple paths in the fault tree, fault trees can have repeated nodes. There exists several efficient algorithms for solving fault tree [Sathaye et al., 2000]. Examples include; algorithms for serial - parallel systems (for fault tree without repeated components), a multiple inversion (MVI) algorithm called the LT algorithm for obtaining the sum of disjoint products (SDP) from mincut set [Muppala and Trivedi, 1992] and the factoring /conditioning algorithm that works by factoring a fault tree with repeated nodes into a set of fault trees without repeated nodes [Sathaye et al., 2000], Satyanarayana and Prabhakar [1978]. In [Doyle and Dugan, 1995], [Doyle et al., 1995], it is shown that binary decision diagrams (BDD)-based algorithms can be used to solve very large fault trees.

In previous studies [Sathaye et al., 2000], Trivedi [2002b], it is noted that reliability block diagram, reliability graph and fault trees cannot easily handle more complex situations such as failure/repair dependencies and shared repair facilities. State space representations have successfully been used to model such complex systems. A state space model is a description of a configuration of states used as a simple model of the system under study. State space models consist of states and transitions between the states. Gracefully degrading systems may be able to survive the failure of one or more active components and continue to provide service at a reduced level. Some commonly used techniques for modelling of gracefully degradable systems include Markov reward model (MRM), Markov chains, Stochastic reward nets and Petri nets [Trivedi, 2001] and [Sathaye et al., 2000].

The advantage of using non-state-space models seen above is that they are efficient to specify and solve. However, the solution of these models assumes the components are independent. For instance, in a block diagram, fault-tree or reli-
ability graph, the components must be completely independent of one another in their failure and repair behaviour. A failure in one component cannot affect the operation of another component, and components cannot share a repair facility. Markov models provide the ability to model systems that violate the assumptions made by the non-state-space models as seen but at the cost of a state space explosion. A system having \( n \) components may require up to \( 2^n \) states in a Markov chain representation [Trivedi, 2001].

Trivedi mentions two ways of dealing with state space explosion problem as tolerance or avoidance [Trivedi, 2001]. Complex system tolerance must apply to specification, storage and solution of the model. If the storage and solution problems can be solved, the specification problem can be solved by using more concise (and simpler) model specifications that can be automatically transformed into Markov models. Complex models can be avoided by using hierarchical model composition [Trivedi, 2002b]. The ability of SHARPE to combine results from different kinds of models also makes it possible to use state-space methods for those parts of a system that require them, and use non-state-space methods for the more well-behaved parts of the system.

In practical system design, a pure availability model may not be enough for gracefully degrading communication and computer systems considering that they tend to be very conservative given they do not explicitly consider different levels of performance of system states. A composite model for both availability and performance is therefore necessary as the system degrades over time. A more realistic analysis method was introduced in [Beaudry, 1978] and a conceptual framework of performability introduced by Meyer [Meyer, 1980]. This modelling approach is very useful for systems as they degrade and experience moments of breakdowns and failures.

### 3.2.3 Performability Evaluation Models

Unlike in the preceding two sections, performability technique is used in this section to yield a single monolithic model [Beaudry, 1978], [Neuts and Lucantoni, 1979], [Beaudry, 1978]. Such an overall model of system behaviour can potentially
yield more accurate results than the disjoint models. Where the main concern is performability over a specified time period, reliability issues are considered. On the other hand, if performability at an instant of time is required, then availability issues are considered. In both cases, the model specifies the amount of work to be done within a given interval when the system is affected by failures and repairs.

Significant amount of work has been reported on the development of techniques for evaluating performance and availability of computer and communication systems [Trivedi and Malhotra, 1993], [Sathaye et al., 2000], [Haverkort et al., 2001], [Trivedi, 2002b]. Other useful approaches for performability modelling and evaluation have also been presented in [Ever et al., 2013], [Ever et al., 2012], [Sahner et al., 1996], [Heimann et al., 1990], [Chakka, 1995], [Ever, 2007]. These include Markov Reward Models (MRM), Integrated performability model/or queues with server breakdown, Iterative Modelling approach, Completion Time Approach and Multi State Combinatorial Approach. In [Trivedi and Malhotra, 1993], queues with server breakdowns are considered same as integrated performability models though treated differently because of the extensive research realised in this area. Here priority queuing system for different customer types and queuing systems with vacations are considered for modelling servers with failures and repairs. In the same studies [Trivedi and Malhotra, 1993], authors discussed some tools, which may be used for performability and reliability analysis.

Despite numerous studies reported in the area of WSN reliability, availability and performance, there are no attempts for combined WSN performance and availability studies. The rising concerns due to pure performance and availability/reliability have therefore remained as major challenges to WSN applications despite extensive studies in the area of performability. The existing performability modelling tools can be used efficiently to obtain fairly accurate results for WSN systems with failures and breakdowns. In [Chakka, 1995], [Chakka and Mitran, 1996] Chakka et al., 2007] and [Kirsal, 2013] QBDs have extensively been used to model Multi-Server systems with finite and infinite queuing capacities and has been proposed in this study to model WSN systems.

QBDs is a special class of finite and infinite state Continuous Time Markov Chains (CTMC) characterized by a probability matrix, and combines a large degree
of modelling expressiveness with efficient solution methods [Chakka, 1995] and [Ever, 2007]. In this approach, semi finite and infinite lattice strips of Markov states with certain regularities are used to model the system. The transitions into and out of particular states are then used to obtain the balance equations for the various states. For these systems, the process becomes ergodic only if it is irreducible, and the corresponding balance equations of state probabilities have a normalisable unique solution. This approach has extensively been used in various research studies [Chakka, 1995], [Chakka, 1998], [Ever, 2007], [Kirsal, 2013].

Using two-dimensional state space representation, the system state at time $t$ can be described using a pair of integer valued random variables, $I(t)$ and $J(t)$, specifying sever configuration (Multi-Server operative states) and the number of jobs in the system respectively. For example, if there are $N+1$ server configurations, represented by the values $I(t) = 0, 1, 2, \ldots, N$, these $N+1$ can be used to represent the possible operative states of the model. The model assumptions are considered to ensure that $I(t), t \geq 0,$ is an irreducible Markov process. $J(t) \leq L$ is the total number of jobs in the system at time $t$, including all jobs in service. Then $Z = [I(t), J(t)]; t \geq 0$ is an irreducible Markov process on a lattice strip (a QBD process), that models the system. Its state space is $(0, 1, \ldots, N) \times (0, 1, \ldots, L)$ (where $L$ can be finite or infinite). Once the steady-state probabilities of such systems are computed, various performability measures such as mean queue length, blocking probabilities, throughput, mean response time and others may be derived using queuing theory knowledge. In [Chakka, 1995], [Ever, 2007] and [Kirsal, 2013], similar models have been analysed for exact performability modelling of homogeneous and heterogeneous multi-server systems and some repair strategies.

In order to address the concerns of pure performance and availability arising in WSN systems, two-dimensional representations of steady states are used in this research. Random variables $I(t)$ and $J(t)$ are specified according to the characteristics of the model and the behaviour of the system under study.
3.3 Solution Approaches for Two-Dimensional State Space

Existing mathematical solution approaches [Chakka, 1995], [Ever, 2007] can be used effectively to solve obtained steady-state probabilities of the Quasi-Birth-Death models. However, the choice of appropriate solution technique is vital considering that various approaches may exhibit certain weaknesses and strengths that may hinder their usage with some models. Some popular mathematical solution approaches that are likely candidates for solving WSN models include Product Form, Seelens Method, Matrix-Geometric Method, Gauss-Seidel iterative method, MRM, Spectral Expansion and System of Linear equations. In order to ascertain validity of the models used in this study, Spectral Expansion and System of Simultaneous equations are comparatively used where possible and validated with a dedicated simulation program.

When two-dimensional representation is considered, the steady-state probabilities may be defined using row vectors $v_j$ as $j = 0, 1, ..., L$ where $L$ can be finite or infinite. This notation shows that elements of vector $v_j$ are the steady-state probabilities when there are $j$ jobs in the system.

3.3.1 Discussion on Existing Solution Methods

This section briefly discusses some well known solution methods for finding the stationary distribution of Quasi-Birth-Death processes, some of which are comparatively presented in the previous works [Chakka, 1995], [Ever, 2007], [Kirsal, 2013] and [Gemikonakli, 2014]. Product form method when applied under certain assumptions provide simple exact solutions, making queuing networks a very important, widely used tool for modelling parallel and distributed systems [Henderson, 1990].
3.3.1.1 Seleens Solution Approach

Seleens method gives an approximate solution for QBD and Quasi Simultaneous-Multiple-Births and Simultaneous-Multiple-Deaths (QBD-M) processes Chakka [1995]. The Markov Chain is first truncated to a finite state, which is an approximation of the original process. Then, it is used together with a dynamically adjusted relaxation factor in an efficient iterative solution algorithm [Seelen, 1986], [Ever, 2007]. In each iteration, the algorithm computes one of the state probabilities. That means the computation time required may be proportional to the queue capacity. However, it is stated that, the number of iterations needed for accurate results, does not depend on the queue capacity [Seelen, 1986], [Ever, 2007]. The dependency of computation time on the number of servers is also not stated [Ever, 2007]. The use of an appropriate value for the relaxation parameter is important to obtain the most accurate results. In contrast, this method does not define any solutions to determine the value of the relaxation parameter [Seelen, 1986], [Chakka, 1995], [Ever, 2007] and [Kirsal, 2013].

3.3.1.2 Gauss-Seidel iterative Approach

Block Gauss-Seidel iterative was developed purposefully for solving linear system of equations iteratively [Poblet-Puig and RODRÍGUEZ-FERRAN, 2010]. In this approach the equations are examined at a time in a sequence making the computation appear serial since each component of the new iterate depends upon all previously computed components and the order in which the equations are examined [Barrett et al., 1994]. It was successfully used by [Meier-Hellstern, 1989] for the analysis of a queue arising in overflow models. In this method, the infinite-state problem is first reduced to a linear equation involving vector generating functions and some unknown probabilities. The computations appear to be serial. Since each component of the new iteration depend upon all previously computed components, the updates cannot be done simultaneously [Ever, 2007]. There is no mention of the dependence of the number of iterations to queue capacity or the number of servers [Horton and Leutenegger, 1994] [Dayar, 1998], [Ever, 2007]. In addition, even with the use of successive over-relaxation, conver-
gence of this approach is slow hence not as popular as the Matrix-geometric and Spectral Expansion solution methods.

### 3.3.1.3 Matrix-Geometric Solution Method

In matrix-geometric solution method, first a non-linear matrix equation is formed from the system parameters. Then the minimal non-negative solution for the rate matrix (R) is computed by using an iterative algorithm. Denoting the stationary probability of the process being in state \((i, j)\) by \(\pi_{(i,j)}\), and applying vector notation \(\pi_n = (\pi_{(n,0)}, \ldots, \pi_{(n,0)})\), the probability vector can be expressed as:

\[
\pi_{n+1} = \pi_n R, \ldots, \quad n \geq 1
\]  

(3.10)

The iterative computation of the rate matrix \(R\) is therefore, a key element in matrix-geometric solution [Latouche and Ramaswami, 1999], [Gemikonakli, 2014]. This method has probabilistic interpretation for each step of the computations [Neuts, 1981], [Ever, 2007], [Kirsal, 2013]. The main drawback of this method is that the number of iterations for computing \(R\) cannot be predetermined and there is a great computational requirement to obtain \(R\). Another observation is that in matrix-geometric method, for some values of certain parameters, the computational requirements are uncertain and relatively large [Ever, 2007].

Matrix-geometric solution method and Spectral Expansion methods are the most commonly used solution techniques for QBD Markov models [Chakka, 1995], [Dini et al., 2007]. Matrix-geometric method is developed by Neuts [Neuts, 1981] and the Spectral Expansion method has been applied to the solution of steady-state solution of QBD Markov models by Mitrani and Chakka [Mitrani and Chakka, 1995], [Chakka, 1995], [Chakka and Mitrani, 1996] These two methods have been critically analysed and their performances compared in [Mitrani and Chakka, 1995], [Haverkort and Ost, 1997], [Tran and Do, 2000], [Ever, 2007]. From the previous studies, it is stated that the spectral expansion method is a better solution method, especially when more heavily loaded systems are studied and when batch arrivals (or departures) are included in the model.
3.3.2 Systems of Simultaneous Equations

In modelling the proposed systems, convenient use is made of state transition diagrams to depict CTMC. The labelled directed graphs represent the states of the CTMC identified by $i$ and $j$ transition rates. The CTMC can formerly be described using the infinitesimal generator matrix $G = (g_{i,j})$ and the initial state probability vector $p(0)$; denoting the system state at time $t \in \tau$ [Haverkort, 2001]. The probability vector $p$ can be defined by $p = (\lim_{t \to \infty} p_{i,j}(t))$. As $t \to \infty$, the Kolmogorov forward equation becomes $pG = 0$. The steady-state probability can be defined as $p = (p_0, p_1, p_2, \ldots)$. In order to obtain the steady-state probabilities, only the computation of system of linear equations is required:

$$pG = 0, \quad \sum_{i \in E} p_i = 1 \quad (3.11)$$

To derive the balance equations for the proposed system, transitions into and out of the various CH states are considered. Figure 3.3 illustrate possible transitions into and out of state $j$ [Mitrani, 1998].

![Figure 3.3: Transitions into and out of state $j$](image)

At steady-state the balance equation argument given in equation 3.12 is used to derive equations of the system. The equations can be interpreted to say that the average fraction of steps on which the chain makes a transition out of state $j$ is equal to the average fraction steps on which the chain makes as transitions into state $j$ Mitrani [1998]. Considering that $1 - g_{j,j}$ is equal to the sum of all one-step
transition probabilities out of state \( j \), it is possible to present the flow equation in the form

\[
p_j \sum_{i=0, i \neq j}^{\infty} g_{j,i} = \sum_{i=0, i \neq j}^{\infty} p_i g_{i,j} : i, j \in E
\]  

(3.12)

The resulting set of linear equations, considered together with the normalisation equation given in 3.11 are computed to obtain the limiting probabilities. Once obtained, it is possible to use the steady-state probabilities to compute various performance measures like MQL, throughput and delay. Since this technique can be quite costly in terms of computation time, a sparse matrix is used in place of a full matrix to enable computation using non-zero variable only [Gemikonakli, 2014].

### 3.3.3 The Spectral Expansion method

Spectral Expansion is an emerging solution technique which is useful in performance and dependability modelling of discrete event systems. It solves some Markov models that arise in several practical system models. It is an exact solution technique for the steady-state analysis of certain two-dimensional Markov processes in semi-infinite or finite lattice strips. These processes are mostly arising in performance and dependability problems of computing and communication systems. In this method, first, the necessary matrices are computed by following the given algorithm. Then eigenvectors and eigenvalues are computed to obtain a system of linear equations. In other studies [Chakka, 1995], [Chakka and Mittrani, 1996], [Chakka, 1998], spectral expansion method was employed to solve several non-trivial and complicated modelling problems occurring in computer and communication systems. Performance measures are evaluated, and optimization issues are addressed. Furthermore, a comparative study is performed to show that the spectral expansion algorithm has an edge over most methods, including the matrix-geometric method in computational efficiency, accuracy and ease of use. Spectral Expansion method has also been used solve several complicated modelling problems in other areas [Ameer Ahmed Abbasi, 2007], [Ever
et al., 2009], [Ever et al., 2012], [Kirsal, 2013].

For good results, computations of accurate eigenvalues and eigenvectors are necessary when using spectral expansion method since the performance measures can be quite sensitive to these [Ever, 2007]. There are various libraries (F02, F03, F04,F07, F12, . . . ) that provide routines to compute very sensitive eigenvalues and eigenvectors for given matrices. Such libraries are provided by the NAG (Numerical Algorithms Group) library [Group, 2006].

In this approach, three transition rate matrices; A, B and C are first determined from the performability models developed. Matrix A is defined as the matrix of instantaneous transition rates from state \((i, j)\) to state \((k, j)\) with zeros on the main diagonal. These are the purely lateral transitions of the model \(Z\) explained in subsection 3.2.3. Matrices B and C are transition matrices for one-step upward and one-step downward transitions respectively. The two-dimensional process \(Z\) evolves with the following instantaneous transitions:

i.) \(A_j(i, k)\): Purely lateral transition rate, from state \((i, j)\) to state \((k, j)\), \((i = 0, 1, \ldots, N; k = 0, 1, \ldots, N; i \neq k; \text{ and } j = 0, 1, \ldots, L)\), usually caused by a change in the operative state (i.e. a change in random variable \(I(t)\)).

ii.) \(B_j(i, k)\): One-step upward transition rate, from state \((i, j)\) to state \((k, j + 1)\), \((i = 0, 1, \ldots, N; k = 0, 1, \ldots, N; \text{ and } j = 0, 1, \ldots, L)\), usually caused by a job arrival into the queue.

iii.) \(C_j(i, k)\): One-step downward transition rate, from state \((i, j)\) to state \((k, j - 1)\), \((i = 0, 1, \ldots, N; k = 0, 1, \ldots, N; \text{ and } j = 0, 1, \ldots, L)\), usually caused by the departure of a serviced job.

The transition rate matrices do not depend on \(j\) for \(j \geq M\), where \(M(M \geq 1)\) is a threshold having an integer value [Ever, 2007], [Mitrani and Chakka, 1995], [Chakka, 1995], [Mitrani, 1998]. This implies the matrices remain similar as indicated in equations 3.13, 3.14 and 3.15.

\[
A_j = A : \quad L \geq j \geq M \quad (3.13)
\]
\[ B_j = B : \quad L - 1 \geq j \geq M - 1 \]  
(3.14)

\[ C_j = C : \quad L \geq j \geq M \]  
(3.15)

The spectral expansion method is applicable for systems with unbounded queuing capacities (i.e. \( K \leq L < \infty \)) as well as systems with bounded queuing capacities (i.e. finite \( L \geq K \)). The solution presented is also valid for steady states of multi-server systems. Following the spectral expansion solution, the steady-state probabilities of the system considered can be expressed as:

\[ P_{(i,j)} = \lim_{t \to \infty} P(I(t) = i, J(t) = j); \quad 0 \leq i \leq N \text{ and } 0 \leq j \leq L \]

where \( N \) and \( L \) represent the number of operative states and finite or infinite queue capacities respectively. Let us define certain diagonal matrices \( D^A_j, D^B_j, D^C_j, D^A, D^B \) and \( D^C \) of size \((N + 1) \times (N + 1)\) as follows:

\[ D^A_j(i, i) = \sum_{k=0}^{N} A_j(i, k); \quad D^A(i, i) = \sum_{k=0}^{N} A(i, k); \]

\[ D^B_j(i, i) = \sum_{k=0}^{N} B_j(i, k); \quad D^B(i, i) = \sum_{k=0}^{N} B(i, k); \]

\[ D^C_j(i, i) = \sum_{k=0}^{N} C_j(i, k); \quad D^C(i, i) = \sum_{k=0}^{N} C(i, k); \]

and \( Q_0 = B, \quad Q_1 = A - D^A - D^B - D^C, \quad Q_2 = C \). For both bounded and unbounded queuing systems, all state probabilities in a row can be defined as:

\[ \mathbf{v}_j = (P_{0,j}, P_{1,j}, \ldots, P_{N,j}); \quad j = 0, 1, 2... \]  
(3.16)

Here, for a bounded system, \( j \) is limited by finite \( L \). In this case, when the queue is full, the arriving jobs are lost. The matrices given above are used in the spectral expansion solution for both bounded and unbounded queuing systems. The steady-state balance equations for unbounded queuing systems with \( 0 \leq j \leq L - 1 \) and \( L \geq j \geq M - 1 \) are given by:

\[ B_j = B : \quad L - 1 \geq j \geq M - 1 \]  
(3.14)

\[ C_j = C : \quad L \geq j \geq M \]  
(3.15)
$L$ can now be written as:

$$v_0[D_0^A + D_0^B] = v_0A_0 + v_1C_1$$

(3.17)

where $D_0^A$ and $D_0^B$ are diagonal matrices formed when the system is empty.

$$v_j[D_j^A + D_j^B + D_j^C] = v_{j-1}B_{j-1} + v_jA_j + v_{j+1}C_{j+1}; \quad 1 \leq j \leq M - 1$$

(3.18)

$$v_j[D^A + D^B + D^C] = v_{j-1}B + v_jA + v_{j+1}C; \quad j \geq M$$

(3.19)

and the normalizing equation is given as follows:

$$\sum_{j=0}^{\infty} v_je = \sum_{j=0}^{\infty} \sum_{i=0}^{N} P_{i,j} = 1.0$$

(3.20)

where $e$ is the column vector of infinite elements each of which sums up to 1. From Equation 3.19 one can deduce that

$$v_jQ_0 + v_{j+1}Q_1 + v_{j+2}Q_2 = 0; \quad j \geq M - 1$$

(3.21)

Furthermore, the characteristic matrix polynomial $Q(\lambda)$ can be defined as:

$$Q(\lambda) = Q_0 + Q_1\lambda + Q_2\lambda^2$$

(3.22)

$\lambda$ and $\psi$ are eigenvalues and left-eigenvectors of $Q(\lambda)$ respectively. Note that, $\psi$ is a row-vector defined as:

$$\psi = \psi_0, \psi_1, \ldots, \psi_N, \quad \lambda = \lambda_0, \lambda_1, \ldots, \lambda_N \text{ and } \psi Q(\lambda) = 0; \quad |Q(\lambda)| = 0.$$ 

Finally, for an unbounded system, when the stability condition is satisfied [Chakka,
1995], [Ever, 2007] one can obtain the general solution.

\[ V_j = \sum_{k=0}^{N} (a_k \psi_k \lambda_k^{j-M+1}); \quad j \geq M - 1 \] (3.23)

and in the state probability form as:

\[ P_{i,j} = \sum_{k=0}^{N} (a_k \psi_k(i) \lambda_k^{j-M+1}); \quad j \geq M - 1 \] (3.24)

where, \( \lambda_k(k = 0, 1, ..., N) \) are \( N + 1 \) eigenvalues that are strictly inside the unit circle [Chakka, 1995], [Ever, 2007] and \( a_k(k = 0, 1, ..., N) \) are arbitrary constants which can be scalar or complex-conjugates. All the \( a_k \) values and the \( v_j \) vectors can be obtained using the process in [Chakka, 1995].

For the case of bounded queue with \( 0 \leq j \leq L \), the balance equations are:

\[ v_0[D_0^A + D_0^B] = v_0A_0 + v_1C_1 \] (3.25)

\[ v_j[D_j^A + D_j^B + D_j^C] = v_{j-1}B_{j-1} + v_jA_j + v_{j+1}C_{j+1}; \quad 1 \leq j \leq M - 1 \] (3.26)

\[ v_j[D_j^A + D_j^B + D_j^C] = v_{j-1}B + v_jA + v_{j+1}C; \quad M \leq j < M \] (3.27)

\[ v_L[D_L^A + D_L^C] = v_{L-1}B + v_LA \] (3.28)

The normalisation equation is given as:

\[ \sum_{j=0}^{L} v_j e = \sum_{j=0}^{L} \sum_{i=0}^{N} P_{i,j} = 1.0 \] (3.29)
From Equation 3.27 it is possible to deduce that

\[ v_j Q_0 + v_{j+1} Q_1 + v_{j+2} Q_2 = 0; \quad (M - 1) \leq j \leq (L - 2) \] (3.30)

and the characteristic matrix polynomials can be expressed as:

\[ Q(\lambda) = Q_0 + Q_1 \lambda + Q_2 \lambda^2; \quad \bar{Q}(\beta) = Q_2 + Q_1 \beta + Q_0 \beta^2 \] (3.31)

where:
\[ \psi Q(\lambda) = 0; \quad |Q(\lambda)| = 0; \quad \phi \bar{Q}(\beta) = 0; \quad |\bar{Q}(\beta)| = 0 \]

\(\beta\) and \(\phi\) are eigenvalues and left-eigenvectors of \(\bar{Q}(\beta)\) respectively. Note that, \(\phi\) is a vector defined as \(\phi = \phi_0, \phi_1, \ldots, \phi_N\), \(\beta = \beta_0, \beta_1, \ldots, \beta_N\).

Furthermore,

\[ v_j = \sum_{k=0}^{N} (a_k \psi_k \lambda_k^{j-M+1} + b_k \phi_k \beta_k^{L-j}); \quad M - 1 \leq j \leq L \] (3.32)

This can be represented in the state probability form as given in equation 3.33.

\[ P_{i,j} = \sum_{k=0}^{N} (a_k \psi_k \lambda_k^{j-M+1} + b_k \phi_k \beta_k^{L-j}); \quad M - 1 \leq j \leq L \] (3.33)

where \(\lambda_k(k = 0, 1, \ldots, N)\) and \(\beta_k(k = 0, 1, \ldots, N)\) are \(N + 1\) eigenvalues each, that are strictly inside the unit circle [Chakka, 1995], [Ever, 2007], and, \(b_k(k = 0, 1, \ldots, N)\) are arbitrary constants which can be scalar or complex-conjugate just like \(a_k\). The \(v_j\) vectors can be obtained as explained in the previous case. From the \(P_{i,j}\), a number of steady-state availability, reliability and performability measures can be computed quite easily. For example, mean queue length (MQL) can be
obtained as:

\[ MQL = \sum_{j=0}^{L} \sum_{i=0}^{N} P_{i,j} \]  

(3.34)

where \( L \) can be finite or infinite depending on whether the case concerned is bounded or unbounded. For cases where \( L \) is finite, the percentage of jobs lost \((PJL)\) can be obtained by using the following equation:

\[ PJL = 100 \times \sum_{i=0}^{N} P_{i,L} \]  

(3.35)

Once the steady-state probabilities are computed, it is possible to calculate some other system performance measures such as mean response time and throughput in addition to \( MQL \) and \( PJL \) shown above.

### 3.4 Chapter Summary

This chapter provides a description and analysis of various modelling approaches and solution techniques used for single and multi-server systems. Pure performance, pure availability/reliability and performability modelling techniques are critically analysed and compared. From the analysis, composite measures of performance and availability prove to be more realistic and accurate modelling techniques for both single and multi-server systems since they are usually fault-tolerant.

A critical comparison of the existing solution approaches for solving multi dimensional state space models is presented. The main limitations of the various techniques and the advantages Spectral Expansion method has over the other methods are outlined. A brief explanation of the system of simultaneous equations and spectral expansion techniques is also presented.

In the study of WSNs performability, in addition to the frequent node and chan-
nel failures, the transition states are further complicated by the introduction of sleep scheduling states. These require additional computations which sometimes prevent expression of state probabilities in terms of each other. In such cases use of simultaneous linear equations are employed as discussed in section 3.3.2. The proposed models for the single CH can then be solved using simultaneous linear equations. Alternatively, spectral expansion exact solution technique can be modified and used to solve the proposed models for CH performability. In some system scenarios, the two solution approaches are used concurrently thereby validating each other.

In order to ascertain the correctness of the models, an event based scheduling approach has been used for developing a dedicated simulation program for the system under study and used to validate the analytical solutions obtained. The developed program is also applicable when it is not possible to obtain accurate results due to mathematical intractability as the system becomes more complex.
Chapter 4

Performability Modelling and Evaluation of Unbounded Clustered WSN

4.1 Introduction

Wireless sensor nodes are known to have constrained resources due to their size and nature of application areas. The storage capability of the sensor nodes is restricted by the available limited memory. In practice, most WSN deployments assume infinite queue capacity hence loss of information is not anticipated. Such networks are also impacted by the frequent node failures and consequently, performance degradation occurs.

This chapter presents a model of a clustered WSN with an infinite queue capacity CH. The proposed model that considers system failures and repairs /replacement is further used to evaluate system performance and availability in terms of MQL and Response Time ($R_T$). To start with, data delivery models are considered in details for purposes of determining data arrival distribution patterns and choosing preferred system model. Based on the system model, data packet arrival distribution is then modelled following a single server queuing system. The resulting
arrival pattern is then used to further develop the performability model.

The novelty of this study include:

1. Justifying the significance of performability studies by comparing results obtained from independent and composite studies.

2. Modelling packet arrival distribution at the CH

3. Proposing a performability modelling methodology for WSNs using existing solution approaches.

4. Using the proposed models to evaluate performance measures in terms of $MQL$ and $R_T$.

In order to illustrate the significance of this study, performance results obtained using a pure model are compared with results from the performability model. The rest of the chapter is organised as follows: Section 4.2 provides a detailed discussion of the proposed model. Section 4.3, presents a two dimensional Markov representation of the proposed model. Experimental results and discussions are presented in section 4.4. Finally section 4.5 presents a detailed summary of this chapter.

4.2 Model Description

In section 2.5, the system under study presented is formed using several clusters. The network topology given in figure 4.1 depicts the actual implementation of the scenario. System operations and traffic distribution remain as discussed in these sections. In this chapter, a single cluster is considered for performability studies. The model proposed, incorporate an infinite queue at the CH to depict a typical WSN deployment scenario. To make the model more realistic, it also integrates failures and repairs. The model used is developed considering highlighted concerns in earlier studies in the same area [Chiasserini and Garetto, 2006], [Masoum et al., 2008], [Kim et al., 2010], [Houaidia et al., 2011], [Jun et al., 2012] as outline in chapter 2.
In this chapter, use of Open Queuing Networks (OQN) is discussed to model the behaviour of a WSN cluster based on previous works of [Chiasserini and Garetto, 2006], [Qiu et al., 2011], [Wang et al., 2011b], [Chakka, 1995]. In [Chiasserini and Garetto, 2006], open queuing networks are successfully used to model a Markov sensor network, in which nodes may enter sleep mode. The system performance is analysed in terms of energy consumption, network capacity and data delivery delay. In [Chakka, 1995], the analysis of overall packet arrival rate approximations is considered at the servers using various methods and Poisson approximation is specified as an accurate approach in large-scale networks in which the nodes receive arrival streams from a number of other nodes. This is based on the fact that the superposition of many independent and relatively sparse processes converge to Poisson distribution as the number of component processes tend to infinity. In [Qiu et al., 2011], authors successfully modelled WSN cluster node behaviour using M/M/1/N with holding nodes, and the model was found to be consistent with real data. In another study [Wang et al., 2011b], authors successfully modelled the convergence of WSNs with PON using two M/M/1 queues in tandem. The model is then used to derive required performance metrics.

In order to improve reliability of WSNs in harsh environments, studies in [Munir and Gordon-Ross, 2011] propose a fault-tolerant sensor node model for appli-
cations with high reliability requirements. In [Munir and Gordon-Ross, 2011], sensor failure probabilities are assumed to follow exponential distribution. Note that in this study, times between failures and repair/replacement times are assumed to be exponentially distributed. To allow a Markovian chain analysis, it is possible to assume that the time to failure of all components has an exponential distribution. This signifies that the distribution of the next failure time of a component does not depend on how long the component has been operating. The next break-down is the result of some suddenly appearing failure (software, signal, configuration-related failures), not of gradual deterioration. In the next section, a model of data packet arrival at the CH is presented.

4.2.1 Data Delivery Models for Wireless Sensor Networks

In WSNs, the phenomenon characterizes application interests in a manner that allows the applications to be oblivious to the underlying sensor network infrastructure and protocols. Considering the models that govern the generation of the application traffic, it is possible to classify sensor networks in terms of the data delivery required by the application interest. These include continuous, event-driven, query-driven and hybrid data delivery models [Tilak et al., 2002].

In the continuous data delivery model, sensors communicate their data continuously at pre-specified rates. In [Heinzelman et al., 2000], it is shown that clustering is most efficient for static networks where data transmission is continuous. However, in dynamic networks, clustering may purely be suitable at some degree of mobility. For the event-driven data models, the sensors report information only when an event of interest occurs. In this case, the application is interested only in the occurrence of a specific phenomenon or set of phenomena. To illustrate this, let us consider temperature monitoring in a green house. Temperature levels are set such that when the target reference is reached, an alarm is raised to trigger specific conditioning activities. On the other side, query-driven model sensors only report their results in response to an explicit request from the application either directly or indirectly through other sensor nodes. Finally, some application networks allow the co-existence of the three data delivery models. This is
presently becoming more common in WSNs where varieties of occurrences are of interest. An example may be in an agricultural setup where events of interest include, event-driven temperature monitoring, query-driven pest control, and continuous monitoring of soil moisture content and humidity [Park and Park, 2011], [Li and Xu, 2015]. For purposes of traffic routing, unicast communication strategy is applied considering sensor nodes have a direct communication link with the CH as illustrated in figure 4.2.

4.2.2 Determination of Arrival Distribution

Mixed opinions are presented in the literature on the appropriate packet inter-arrival distribution times for WSNs. Some scholars consent to use of other inter-arrival distributions [Wang and Zhang, 2008] [Wang, 2010] while others have shown that it is possible to use Poisson distribution for tractability reasons [Tilak et al., 2002] [Chiasserini and Garetto, 2006] [Chung and Hwang, 2010] [Zhou et al., 2011] [Qiu et al., 2011] [Zhen et al., 2014]. Other probabilistic distributions like Bernoulli, Log-normal and Gamma have also been mentioned in some areas. From [Wang et al., 2012], a comparison of measured inter-arrival times and theoretical exponential distribution graphs confirm that exponential distribution closely model inter-arrival times except in low periodic traffic conditions. However, depending on the choice of the model (discrete-time or continuous), geometric distributions could also be used for discrete systems.

In addition to data delivery models mainly characterized by application demands, other factors that may influence WSN arrival processes including; data collection models like mobile sinks and configurations of the MAC layer protocols. Depending on the type of MAC protocols used, sleep scheduling dynamics for asynchronous and synchronous schemes also determine possible arrival distributions. As an example, use of contention based MAC protocols like T-MAC, CSMA/CA employ exponential back-off algorithms to avoid collision during channel contention making delay times non-deterministic [Van Dam and Langendoen, 2003]. Nevertheless, Bernoulli and Poisson arrival processes are widely used in WSNs.

In this study, event-based applications where nodes send data only if certain
physical events of interest occur are considered. In this case, the generated data are often sporadic. Considering that the occurrence of such physical events are never frequent, the probability that the event occurs at a given time is governed by a Poisson process, and the inter arrival time is exponentially distributed [Wang et al., 2012]. In this thesis, Poisson distribution has been considered in all the models for tractability reasons.

4.2.3 Choosing the Preferred Model

In a cluster based WSN topology, the CH is the centre of communication between the cluster nodes and the sink. All cluster nodes are assumed to be directly connected to the CH. The CH connects directly or through other CHs to the sink forming an overall cluster tree network. The nodes independently monitor their habitat and contend with others for channel availability to relay their observed data to the CH. It is assumed that the CH is not aware of the next arrival source until the arrival actually occurs. The arrival of packets at the CH is assumed to follow Poisson distribution with mean rate $\lambda$ and service time assumed exponentially distributed with rate $\mu$ [Zhang and Li, 2012]. Service priority is based on First Come First Served (FCFS).

In this model, the total data at the CH arrive from within the cluster (internal sources) and externally from other CHs (external sources) forwarding data to the sink. From IEEE 802.15.4/Zigbee standards, a maximum of 36 nodes is recommended per cluster for better performance [Ergen, 2004]. This study employs more than 30 nodes inclusive of the CH. Since a large number of independent Poisson streams are received from the nodes, the resulting superposition of all the arriving jobs at the CH from internal and external sources follows Poisson distribution [Chakka et al., 2007] with rate $\lambda_k$ where $k$ is the index of the CH (node $k$).

From the preceding discussions, the CHs operation is similar to that of an OQN with input and output entries. When operating at steady state, average flow entering the CH queue is same as the flow leaving the queue. The behaviour and operation at each CH is similar and may be independently modelled using an
M/M/1 queuing system following Jacksons theorem that treats each node in an OQN as a single server.

4.2.4 Modelling Data Packet Arrival at the Cluster Head

This section presents the queue model of the WSN topology presented in Section 2.5. The resulting job arrival at the CH is a collection of jobs from the cluster nodes, the sensed information by the CH itself and forwarded data from other CHs. The jobs are assumed to be i.i.d random variables with same rate $\lambda$. The operation is assumed to be similar at all other CHs. For this study, it is assumed that there are $N$ CHs, $k = 1, 2, \ldots, N$, through which the sink may be reached. The behaviour of a single CH, i.e. node $k$, is modelled as an OQN using M/M/1 queueing system. The cluster is assumed to have same number of cluster nodes ($N$) in each cluster denoted by $n = 1, 2, \ldots, N$. Considering that the number of cluster nodes plus neighbouring CHs are taken to be more than 30, it is possible to assume that the resulting superposition of all the job arrivals at node $k$ from internal and external sources follow Poisson distribution with mean arrival rate $\lambda_k$ [Ever et al., 2009].

Figure 4.2 shows the proposed queuing model for analyzing the single CH behaviour. $\lambda_n$ and $\lambda_r$ represent the internal and external arrival rates at node $k$ (CH) respectively. Similarly, $q_{n,k}$ and $q_{r,k}$ are routing probabilities from internal cluster nodes and external CHs respectively. Once the jobs are processed at node $k$, they are transmitted directly or forwarded upward to the sink through node $r$. Here node $r$ represents the next CH towards the sink. The operation at the forwarding node $r$ is similar to that at node $k$. Since the nodes are prone to failures, it is assumed that when a node fails, it is taken into the repair process immediately [Hashmi et al., 2010], [Liu et al., 2011]. This could be through software reconfiguration or replacement of failing nodes. Service times, failure times, and repair times are all assumed to be exponentially distributed with rates $\mu_k$, $\xi_k$, and $\eta_k$ respectively. The interruption policy is such that service is resumed from the point of interruption or repeat with re-sample.

Jobs leaving node $k$ are rerouted to node $r$ with the probability $q_{k,r}$ for service at
node $r$. If jobs are not routed to node $r$ then $q_{k,r} = 0$. It is assumed without loss of generality that as far as the queue length distributions are concerned $q_{k,k} = 0$, $(k = 1, 2, \ldots, N)$. In addition, $q_{k,N+1} = 1 - \sum_{r=1}^{N} q_{k,r}$ is the exit probability from the system after a job is serviced at node $k$. The exit probability $q_{k,K+1}$, is assumed to be non zero for at least one value of $k$. This implies that at any given time, at least one of the CHs has direct link with the sink. Let $Q$ be the routing probability matrix of size $N \times N$, such that, $Q_{k,r} = q_{k,r}; (1 \leq k, r \leq N)$. To analyse the performability of this system, steady-state conditions are considered.

In order to model the total packet arrival at the CH, node $k$, Poisson approximation approach is employed. Suppose that the network is in steady state. Let $\lambda_k$ be the average number of jobs arriving into, and departing from, node $k$ per unit time. The arriving jobs are coming from internal $\sigma_k$ and external ($\lambda_r$) sources. On the average, $\lambda_r$ leave node $r$ per unit time; of these, a fraction $q_{rk}$ go to node $k$. Therefore the rate of traffic from node $r$ to node $k$ is $\lambda_r q_{rk} \ (r = 1, 2, \ldots, N)$. Similarly, the rate of traffic from node $k$ to node $r$ is $\lambda_k q_{kr}$. A related traffic model was presented in [Mitrani, 1998]. The total arrival rate ($\lambda_k$) at the CH node $k$ as the sum of external and internal traffic rates can be expressed using
equation 4.1.

\[ \lambda_k = \sigma_k + \sum_{r=1}^{N} \lambda_r q_{rk}; \quad k = 1, 2, \ldots, N \]  

(4.1)

Here \( \sigma_k \) represent the sum of all internal arrivals and may be expressed using equation 4.2.

\[ \sigma_k = \sum_{n=1}^{N} \lambda_n q_{nk} \quad n = 1, 2, \ldots, N \]  

(4.2)

The term \( \sum_{r=1}^{N} \lambda_r q_{rk} \) in equation 4.1 represents the externally arriving jobs from other CHs as mentioned earlier.

In order to define the total arrival rates for each node, the row vectors \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N) \) and \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_N) \) can be employed. Let also \( E_k \) be the unit matrix of size \( N \times N \) then;

\[ \lambda (E_k - Q) = \sigma \]  

(4.3)

Letting the effective average service rate at the CH be \( \hat{\mu}_k \), and taking into account the losses resulting from failures and repairs it can be shown that \( \hat{\mu}_k \) is given by equation 4.4, [Chakka and Mitrani, 1996], [Thomas and Mitrani, 1995], [Sheng-li et al., 2009].

\[ \hat{\mu}_k = \mu_k \cdot \eta_k / (\eta_k + \xi_k) \]  

(4.4)

where \( \xi_k \) and \( \eta_k \) are failure and repair rates for node \( k \) respectively.

In order for the system to reach steady state operations, the effective service rate must be greater than the effective arrival rate at the CH. Thus \( \hat{\mu}_k > \lambda_k; k = 1, 2, \ldots N \) is the condition for steady-state analysis. In earlier studies, [Chakka and Mitrani, 1996], [Mitrany and Avi-Itzhak, 1968], [Thomas and Mitrani, 1995]
the MQL for such systems are given by equation 4.5.

\[ MQL = \frac{\lambda_k[(\xi_k + \eta_k)^2 + \xi_k \hat{\mu}_k]}{\xi_k + \eta_k)(\eta_k \hat{\mu}_k - \lambda_k(\xi_k + \eta_k))} \]  \hspace{1cm} (4.5)

With the values of MQL and \( \lambda_k \) known, and considering that no jobs are lost, the \( R_T \) for the CH can be computed using equation 4.6.

\[ R_T = \frac{MQL}{\lambda_k} \]  \hspace{1cm} (4.6)

### 4.3 Two Dimensional Markov Representation of the Proposed Model

In this section, presented is the performability model for the system under study. Considering that all the sensor nodes forward information to the CH, in this model, the routing probability matrix \( Q \) has a special form and the total amount of arrivals to the CH can be calculated as \( \lambda_k = C\lambda \), where \( C \) is the number of source nodes in a WSN sending packets to the CH, and \( \lambda \) is the average packet generation rate of the source nodes [Wang et al., 2011b]. There are similar studies on M/M/1 with breakdown and repairs though not in WSN area [Chakka and Mitrani, 1996], [Thomas and Mitrani, 1995], [Sheng-li et al., 2009]. The state transition diagram for the CH is given in figure 4.3.

In this model, it is assumed that data packets will to continue to arrive during failures. However, service is only possible when the server is operational. The system state at time \( t \) may be described using a pair of integer valued random variable \( I(t) \) and \( J(t) \) specifying the CH failure and repair configurations, and the number of jobs in the system respectively. The operative states \( I(t) \) in this case represents the assumed failed and working periods of the CH. \( Z = [I(t), J(t)]; t \geq 0 \) is an irreducible Markov process on a lattice strip (a QBD process), that models the system. Its state space is \((0,1) \times (0,1,\ldots)\). Similar models [Ever et al., 2009], [Kirsal and Gemikonakli, 2009], [Chakka and Mitrani,
Figure 4.3: State transition diagram for CH performability model

1996], [Mitrany and Avi-Itzhak, 1968], [Chakka, 1995] are analysed for exact performability evaluation of various multi-server systems with single repairman and for both finite and infinite L for some repair strategies. It is possible to extend the exact solution methodology for performability evaluation of WSNs.

Since the possible operative states of the CH and the number of data arrivals are represented in the horizontal and vertical directions of the lattice respectively, the transition matrices can be derived as:

i  \( A \) is the matrix of instantaneous transition rates from \((i, j)\) to state \((l, j),(i = 0, 1; l = 0, 1; i \neq l; j = 0, 1, \ldots)\), with zeros in the leading diagonal, caused by a change in the state [Kirsal and Gemikonakli, 2009], [Ever et al., 2009]. These are the purely lateral transitions of the process \( Z \). Matrix \( A \) clearly depends on parameters \( \xi \) and \( \eta \). The state transition matrices \( A \) and \( A_j \) are of size \((2) \times (2)\) and can be given as:

\[
A = A_j = \begin{bmatrix}
0 & \eta \\
\xi & 0
\end{bmatrix}
\]

ii Matrices \( B \) and \( C \) are transition matrices for one step upward and one step
downward transitions respectively [Kirsal and Gemikonakli, 2009], [Ever et al., 2009]. The transition rate matrices do not depend on $j$ for $j \geq M$, where $M$ is a threshold having an integer value [Ever et al., 2009]. The respective transition matrices are:

\[
B = B_j = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \quad \text{and} \quad C = C_j = \begin{bmatrix} 0 & 0 \\ 0 & \mu \end{bmatrix}
\]

Elements of matrix $B$ are dependent on the data arrival rate ($\lambda$) at the CH while elements of matrix $C$ depend on the CH service rate ($\mu$).

Spectral Expansion solution can be employed and the details of the method used can be found in [Chakka and Mitrani, 1996] [Thomas and Mitrani, 1995]. From the state probabilities, a number of steady-state availability, reliability, performability measures can be computed. For illustration, $MQL$ and $R$ are employed. These can be obtained using equations 4.5 and 4.6 where $MQL$ is the expected value of $J(t)$.

4.4 Numerical results and discussions

In this section numerical results are presented for the model considered. A comparative analysis is performed for two different solution approaches, namely Poisson approximation and Spectral Expansion. The results are very close and further verification with simulation results are also in good agreement with a maximum discrepancy of less than 2%.

A dedicated software written in C++ language was used to simulate the actual system. Simulation results were then compared with the analytical results obtained by applying spectral expansion and Poisson approximation solution approaches to the Markov model of the system. All results obtained reveal good agreement with both spectral and Poisson approximation techniques. VC++ 10.0, and the NAG library (for spectral expansion only) were used to achieve all the results presented in this study.
The following parameters were used to obtain all results presented in this section unless otherwise stated. The values of failure and repair/replacement rates are chosen to ensure that repair/replacement rate $\eta_k = 0.5/\text{hours}$ equivalent to mean repair time of 2 hours, which is much higher than the failure rate $\xi_k = 0.001/\text{hours}$ translating to mean run time of 1000 hours before failures. Service rate is taken as $\mu_k = 300/\text{hour}$ and arrival rates are chosen carefully to ensure the system remains stable. The results are shown for Poisson approximation, spectral expansion method and simulation in all the diagrams.

Figure 4.4 shows $MQL$ as a function of $\lambda$. In order to analyse the effects of arrival rate on the cluster size, the experiment is achieved by varying the number of nodes in each run. The results indicate that fewer nodes are able to accommodate higher arrival rates in contrast to the system getting saturated at low arrival rates when many nodes are used to cover the habitat. In figure 4.5, similar results obtained using a pure performance model is presented. MQL results obtained are kept much lower compared to those obtained from the integrated studies considering failures. The distinct difference in the levels confirm the effects of failures and repair/replacement in a typical WSN network.

In both cases, in order to determine the best performance with optimum coverage, a trade-off exists between MQL and appropriate number of nodes to be used for deployment while at the same time observing the IEEE/ZigBEE802.15.4 recommended limit of 36 nodes per cluster. In other words, the model can be used to specify the size of a cluster when a specific flow is expected from the sensing nodes.

Figure 4.6 shows the response time as a function of the number of nodes for various $\lambda$ values. The three solution techniques show that the results are in good agreement with best response times realised with fewer nodes in the cluster. The model may therefore be used to select an appropriate response time operation region for WSNs. Noting that the unit of response time is dependent upon the units of arrival and service rates. Since these values can be dependent on the type of application, in this study, a generic approach is adopted.

In most cases, a WSN cluster is initially populated with the maximum required
Figure 4.4: MQL Vs Arrival rate-Integrated System

Figure 4.5: MQL Vs Arrival rate-Pure Performance
nodes for best coverage. In figure 4.7, the arrival rate is maintained at specific values of $\lambda = 5, 6, 7, 8$. It is observed that there is a maximum limit for required sensor nodes per cluster in order to maintain appropriate traffic that the CH can handle. Similar results obtained using a pure performance model is shown in figure 4.8. In this scenario, the results are also kept minimal compared to those obtained using the model with failures. The margin difference between the two results show the effects caused by system failures and how much the pure model overestimates systems ability to perform as expected.

In figure 4.9, MQL is presented as a function of the failure rate, when arrival rate is varied from $\lambda = 5, 6, 7, 8$ and sources maintained at $K = 30$. Results show the effects of failures clearly. The purpose of this experiment is to establish failure levels that may not adversely compromise system performance. The importance of repair facility and high reliability demonstrate acceptable levels of system availability that ensure system performance is not compromised significantly. From this figure, low failure rates below $\xi = 0.001$ are desired. Above this, the system availability and performance is greatly compromised as illustrated with the
Figure 4.7: Effects of variable sources on MQL - Integrated Model

Figure 4.8: Effects of variable sources on MQL - Pure Model

mismatch and large variations in $MQL$. 
4.5 Chapter Summary

In this chapter, an analytical model for a clustered WSN with unbounded queue capacity CH is presented. First, the process of arrival distribution at the CH is determined and used to develop a queue model for packet arrivals at the CH. A performability model whose inputs are arriving data packets is then developed to capture the system behaviour in different operative states. In order to resolve the proposed system model, Markov chain was used to analyse and evaluate performance and reliability/availability.

Although Markov chain analysis is used for performance and reliability/availability evaluation of various WSN applications in the literature, to the best of our knowledge, this is the first attempt to combine performance and availability metrics. Using a generic system model, it is proved that existing solution techniques can be used to model WSN networks. Results indicate that performability modelling is significant in the establishment of WSNs. In this study, two analytical modelling approaches are employed in addition to the simulation program used for
validation. The results obtained from the three approaches are in good agreement with a maximum discrepancy less than 2% observed in figures 4.4, 4.6 and 4.7. The model presented is useful for optimization of WSN clusters.

In order to make the model more realistic, a finite queue capacity CH is considered in the next chapters. It is also possible to extend this model for intra and inter cluster traffic studies.
Chapter 5

Performability Modelling and Evaluation of a Clustered WSN with a Bounded Queue

5.1 Introduction

Unlike other wireless and wired communication networks, WSNs have limited storage memory both for operating systems and temporary data storage. Under normal operations, this implies restricted storage capacity resulting into arriving packets being dropped and lost whenever the queue capacity is full. This situation may become worse in clustered networks where FFD rotate CH operations. The self-similarity characteristic of WSNs also makes the situation more challenging during heavy traffic bursts [Liu and Ju, 2010]. Depending on application data intensity, different application categories may require varying queue capacities in order to achieve desired performance. It is therefore important to analyse and identify appropriate storage memory necessary for various WSNs application.

In this chapter, a model for a clustered WSN considering CHs with bounded queues is presented. Like the model presented in chapter 4, this model also takes into account failures, repairs/replacement and restoration of the CH during oper-
ations. Similarly, the use of BCH is implied for providing redundancy during CH failures. Once the model is developed, it is used to compute performability measures such as MQL, throughput, response time and system blocking probability. Results obtained using a pure performance model is also presented to illustrate effects of such systems.

The rest of the chapter is organised as follows: Section 5.2 presents a detailed description of the extended model. The queue model is presented in section 5.3 followed by a two dimensional Markov representation of the model in section 5.4. A detailed discussion of the obtained results is presented in section 5.5 and finally, the chapter is summarised in section 5.6.

5.2 Model Description

In this chapter, the system description and packet arrival models remain the same as presented in sections 2.5 and 4.2.4 respectively. However, the performability model of section 4.3 is improved to incorporate a finite queue capacity in order to make it more realistic to an actual system. The improved model is presented in figure 5.1.

Accordingly, WSNs were meant for low to medium rate applications hence memory was not a major concern. Conversely, the introduction of video and image sensors in addition to bursty high data rate applications have caused additional challenges [Akkaya and Younis, 2003]. Data intensive applications that send information to a central server are particularly constrained due to the large queue capacities required for temporary storage of sensed data [Chang et al., 2007]. This becomes more challenging in clustered networks where the CH has to queue lots of data from internal and external sources for onward transmission to the sink.

In addition to the limited queue capacity, WSN traffic does exhibit self-similarity. Self-similar traffic is well studied in the literature [Sikdar et al., 2002], [Shuo et al., 2008], [Liu and Ju, 2010] and is known to affect queue size greatly where traffic bursts are experienced. In [Liu and Ju, 2010], authors proposed an Adaptive Weighted Fair Queuing (A-WFQ) by adapting the weight of a queue according to
the queuing delay that relates to the Hurst parameter of the Self-Similar traffic. Here the Hurst parameter ($H$) is used to measure the degree of long range dependence. For example, let $(X_i, i = 1, 2, \ldots, n)$ be equally spaced samples of some Hurst parameter of a self-similar process, if $\text{Var}(X_1 + \cdots + X_n)$ of long range dependent sequence grows at speed $n^{2H}$, where $H \in (1/2, 1)$, then the number $H$ is the Hurst parameter of the sequence. In this study, observations made include delay and rapid growth of queue size when traffic has a big Hurst parameter thereby leading to higher delay and packet loss when the queue is full. However, other than delay, the study did not address other performance measures. Furthermore, the study assumes a failure-free system environment that is not always the case in WSNs.

In [Qiu et al., 2011], authors proposed a new evaluation method for optimizing packet queue capacity of nodes using a queuing network model to improve the transmission QoS. The CH behaviour was modelled using M/M/1/N queuing system with break-down and repair/replacement. In order to evaluate the congestion situation in the network, and also get real effective arrival rates and transmission rates for the model, holding nodes were introduced in the queuing network model. However, in this study, authors did not assess system throughput, blocking probability and possibility of packet loss when the queue is full. Moreover, they also assumed a perfect working network without failures.

In another study [Tang, 2013], authors presented an analytical traffic model for an unreliable WSN that models the dynamics of traffic flow from source node through a set of intermediate nodes to the sink using single server queues connected in tandem. For performance analysis purposes, they decompose the servers into individual nodes. In their model, a finite queue capacity is considered at the CHs. If the queue capacity for the upstream node is full, the node intending to transmit is forced to hold back its packet until an opportunity arises. Arriving packets are lost if they find the queue is full. This is then used for the analysis of blocking probability of the system. In this study, only failures resulting from power failures are considered and assumed following exponential distribution. Other forms of failures are not considered. Moreover, the recovery considered is for the selection of a new CH. The paper does not discuss the mechanisms used.
Other approaches for memory management in WSN are presented in [Manjiri, 2013]. The authors first identify the concerns & research challenges related to memory management that require consideration when designing operating systems for WSNs. The authors also reckon the significance for additional memory for applications running real-time traffic for purposes of improving overall network QoS. In addition, they also highlight possible future approaches for memory management. However, in this paper, no consideration is made to performance concerns in relation to system failure.

From the preceding discussions, it is evident that there is a need for a planning and deployment tool that takes into account the storage limitation of sensor nodes used for CHs operations. In addition, the tool should incorporate an integrated performance and availability modelling and evaluation in order to reduce any effects that may result from independent studies. For clustered WSNs, such a tool would be significant for performance tuning and upgrades once the network is operational.

5.3 Queuing Model for the System

In this section, a model incorporating the effects caused by introducing a finite queue length on the CH is presented. All the other parameters remain as discussed in section 4.2.4. Similarly, the total packet arrival rate at the CH remains as given in equation 4.1. Since a finite queue is considered, jobs arriving when the queue is full are lost. The blocking probability($P_B$) when the queue is full, the effective arrival rate ($\lambda_{k,e}$) at the CH (node $k$) and the rate at which the jobs are lost ($\lambda_{k,l}$) due to blocking can be computed by

$$P_B = \sum_{i=0}^{N} P_{i,L}$$

(5.1)

$$\lambda_{k,e} = \lambda_k (1 - P_B)$$

(5.2)
\[ \lambda_{k,l} = \lambda_k P_B \]  \hspace{1cm} (5.3)

where \( P_{i,L} \) is the probability of being in the operative state \( i \) when the queue is full, \( N \) is the number of operative states and \( L \) is the maximum queue capacity.

The total arrival rates at each CH remain as defined in equation 4.3. Letting the effective average service rate at the CH be \( \hat{\mu}_k \), and taking into account the losses resulting from failures and repairs, the effective service rate can then be computed using equation 4.4 [Thomas and Mitrani, 1995], [Sheng-li et al., 2009].

For steady state, the effective service rate must be greater than the effective arrival rate at the CH. Thus \( \hat{\mu}_k > \lambda_{k,e} \); \( k = 1, 2, \ldots K \) is the condition for steady state analysis.

### 5.4 Two Dimensional Markov Representation of the Proposed Model

In this model, since all the sensor nodes forward their information to the CH, the matrix \( Q \) has a special form and the total amount of arrivals to CH can be calculated as \( \lambda_k = C \lambda \), where \( C \) is the number of source nodes introduced in section 4.3 and \( \lambda \) is the average packet generation rate of the sensor nodes [Wang et al., 2011b]. The CH state transition diagram is given in figure 5.1. The operative states, \( F \) and \( R \) represent failed and fully active states respectively.

The model treats sleep and breakdown states as short and long breakdown periods respectively since data will continue to arrive in both states. However, service is only possible when the server is fully operational. The system state at time \( t \) may be described using a pair of integer valued random variable \( I(t) \) and \( J(t) \) specifying operative states of the CH and the number of jobs within the system respectively. The operative states \( I(t) \) in this case represents the assumed failed and working periods of the CH. \( Z = [I(t), J(t)]; t \geq 0 \) is an irreducible Markov process on a lattice strip (QBD process), that models the system. Its state space
is \((0,1) \times (0,1,\ldots,L)\). In order to solve this model, Spectral Expansion exact solution methodology presented in section 3.3.3 is employed for performability evaluation of the WSN system model.

![State transition diagram for CH performability](image)

Figure 5.1: State transition diagram for CH performability

Since the possible operative states of the CH and the number of data arrivals are represented in the horizontal and vertical directions of the lattice respectively, the transition matrices can be derived as:

i.) \(A\) is the matrix of instantaneous transition rates from \((i,j)\) to state \((l,j)\), \((i = 0,1; l = 0,1; i \neq l; j = 0,1,\ldots,L)\), with zeros in the leading diagonal, caused by a change in the state [Ever et al., 2009]. These are the purely lateral transitions of the model \(Z\). Matrix \(A\) clearly depends on parameters \(\xi\) and \(\eta\). The state transition matrices \(A\) and \(A_j\) are of size \((2) \times (2)\) and can be given as shown below.

ii.) Matrices \(B\) and \(C\) are transition matrices for one step upward and one step downward transitions respectively [Ever et al., 2009]. When there is no job within the system, the elements of matrix \(C\) are zero. The transition rate matrices do not depend on \(j\) for \(j \geq M\), where \(M\) is a threshold having
an integer value [Ever et al., 2009]. The respective transition matrices are shown below:

\[
A_j = \begin{bmatrix} 0 & \eta \\ \xi & 0 \end{bmatrix} \text{ and } B_j = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \text{ and } C_j = \begin{bmatrix} 0 & 0 \\ 0 & \mu \end{bmatrix}
\]

Elements of matrix \(B\) are dependent upon the data arrival rate (\(\lambda\)) at the CH while elements of matrix \(C\) depend upon the CH service rate (\(\mu\)).

Once the state transition matrices are established, spectral expansion solution technique is then employed to derive steady-state probabilities for the model. From the state probabilities, a number of steady-state availability, reliability, and performability measures can be computed. For illustration, Attention is given to the blocking probability described by equation 5.1, the \(MQL\), throughput (\(\gamma\)), utilization (\(u\)), and \(R_T\) which may be computed using equations 5.4 through 5.7 respectively. From the model, service is only possible when there are jobs within the system.

\[
MQL = \sum_{j=0}^{L} j \sum_{i=0}^{N} P_{i,j} \quad (5.4)
\]

\[
\gamma = \sum_{j=1}^{L} \sum_{i=1}^{N} \mu P_{i,j} \quad (5.5)
\]

\[
u = 1 - \sum_{i=0}^{N} P_{i,0} \quad (5.6)
\]

\[
R_T = \frac{MQL}{\gamma} \quad (5.7)
\]

where \(P_{i,j}\) is the probability of the system being in state \(i\) with \(j\) jobs at a given time during operation.
5.5 Numerical Results and Discussions

In this section, numerical results for the model obtained using Spectral Expansion solution approach are presented. The results are verified using a dedicated event-driven simulation software for the actual system developed in C++ language and validated using well known mathematical solutions. The simulation software has also been verified to match the M/M/1/L system performance as presented in [Cassandras et al., 2008], [Chakka, 1998]. Finally, the steady-state results are compared with results obtained in chapter 4 in order to understand the effects of limited memory capacity on CHs. Table 5.1 lists a summary of steady-state performance metrics used for this study.

Table 5.1: Performance Metrics Explained

<table>
<thead>
<tr>
<th>Performance Metrics</th>
<th>Brief Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Arrival rate ($\lambda_{k,e}$)</td>
<td>Total arrival rate excluding blocked packets</td>
</tr>
<tr>
<td>Rate of jobs lost ($\lambda_{k,l}$)</td>
<td>Rate of data packet loss due to blocking</td>
</tr>
<tr>
<td>Mean Queue Length ($MQL$)</td>
<td>Average packets in the queue at steady state</td>
</tr>
<tr>
<td>Throughput ($\gamma$)</td>
<td>Packets departing from CH after service</td>
</tr>
<tr>
<td>Utilization ($u$)</td>
<td>Fraction of time CH is busy</td>
</tr>
<tr>
<td>Response Time ($R_T$)</td>
<td>Mean time packets take in the system</td>
</tr>
<tr>
<td>Blocking Probability ($P_B$)</td>
<td>Probability arriving packets find queue full</td>
</tr>
</tbody>
</table>

5.5.1 Parameter Choice

In this section, parameter choices are discussed and a summary of simulation parameters used is presented in Table 5.2. The parameters shown are considered throughout the evaluation of the system unless otherwise stated. In choosing the input parameters, a generic system was considered. Though in most research work, arrival rates of 1 packet/second is used, variation of arrival rates between 1 to 10 packets/second has also been recorded [Zhou et al., 2011]. In other areas, mean arrival rates have been varied between $\lambda = 1 - 15$ packets/hr [Li et al., 2011].

Assuming a 36 sources full capacity cluster operation for monitoring moisture...
content in an agricultural farmhouse, configured with the same mean arrival rate of \( \lambda = 8 \) packets/hr from each cluster node (\( \lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_C \)), the effective arrival at the CH becomes \( \lambda_k = C\lambda = 288 \) packets/hr, where \( C \) is the number of of source nodes introduced in section 4.3. For stability, the CH requires a slightly higher mean service rate per hour. Considering that the CH has in addition internal data and control processes, a service rate of \( \mu_k = 300 \) packets/hr was arbitrarily chosen in order to ensure steady-state condition is reached when the CH is operating near full capacity. Arrival rates following Poisson distribution are varied between \( 1 - 8 \) packets/hr from each node to ensure the system remains stable. An arbitrary queue length of \( L = 50 \) packets was also chosen for this study throughout the experiments.

Sensors are usually attached with a \( 2 \times AA \) battery pack of \( 2.7 - 3.3 \) volts capable of continuous operation for 3.25 days as given in the CC2420 transceiver data sheet. In this study, it is assumed that good mechanisms for availability are put in place, and battery depletion is not the cause of failures. Use of backup for CHs and solar charging systems [Munir and Gordon-Ross, 2011], [Li et al., 2011] are just a few examples of such mechanisms. In order to model these systems, mean failure (\( \xi \)) and repair (\( \eta \)) rates were assumed to be \( \xi = 0.001/hr \) and \( \eta = 0.5/hr \) translating to mean failure and repair occurring after every 1000hrs and 2hrs respectively. The small repair rate ensures the system does not stay longer in failed state. These values are maintained during the experiment except where specified.

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter Type</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Arrival rate ( \lambda )</td>
<td>0 - 14</td>
</tr>
<tr>
<td>2.</td>
<td>Service rate ( \mu )</td>
<td>300</td>
</tr>
<tr>
<td>3.</td>
<td>Failure rate ( \xi )</td>
<td>0.001 - 0.01</td>
</tr>
<tr>
<td>4.</td>
<td>Repair rate ( \eta )</td>
<td>0.5</td>
</tr>
<tr>
<td>5.</td>
<td>Queue capacity ( L )</td>
<td>10, 30, 50, 100, 500, 1000</td>
</tr>
</tbody>
</table>
5.5.2 Results and discussions

In this section, results of a pure performance model for a bounded CH is presented in figure 5.2 for comparison purposes with the proposed performability model. In all the four scenarios presented, MQL, response time, and Blocking probability are all kept lower compared to results obtained using performability model figures 5.3(a), 5.4(a), and 5.5. The difference in results show the effects of failures and repairs/replacement and can be used for system design, deployment planning, and optimization studies.

From figure 5.3(a), the MQL is presented as a function of arrival rate \( \lambda \). For every run, a fixed number of nodes is chosen and the arrival rate \( \lambda \) is varied between 0 to 14 packets/hr. It is observed that for steady-state operations, the MQL is kept below 5 jobs after which the system becomes unstable, and the MQL increases sharply. This is contrary to the infinite system studied in chapter 4 which indicate a slightly higher MQL value (10 jobs) before the system becomes unstable. From the results, systems with low traffic intensity (up to \( \lambda = 5 \) packets/hr) are able to accommodate more source thereby providing a wider coverage compared to higher traffic intensity system that are restricted to fewer sources hence limited coverage area.

Assuming an infinite queue capacity for WSN CH may therefore impact negatively on system performance hence the need to optimize operations using the available resources. This becomes a trade off when coverage and optimum performance are of concern as highlighted in chapter 4.

In figure 5.3(b), the effect of varying queue capacity is compared. The number of nodes is maintained at 30 throughout the experiment. The following queue capacities were used; \( L = 10, 30, 50, 100, 500, 1000 \). During each run, the arrival rate is varied from 1 to 9 packets/hr, and the MQL is recorded appropriately. It is observed that when queue capacity is low, then MQL remains very small, since more jobs are lost. However, as the queue capacity (\( L \)) is increased, a limit is approached beyond which further increments do not cause any meaningful change to MQL. Appropriate queue capacity is therefore desired for optimum system performance.
In figure 5.4(a), the average response time for a finite system is noted to be much less compared with the infinite systems of figure 4.6 since only a few packets may wait in the queue at any given time. Though these results are generic, the response times may easily be customised for particular WSN application requirements for purposes of deployment planning and operation management.
In figure 5.4(b), the response time is given as a function of the number of nodes for various failure rates. For this experiment, queue capacity of $L = 10$ is used. Arrival rate ($\lambda$) is a constant but overall arrival at the CH increases due to increasing number of nodes. It can be observed that response time is higher when the system exhibits high failure rates. As the number of nodes are increased, the arrival rate also increases followed by a gradual increase in the response time. As the number of nodes are further increased, a level of arrival rate is reached after which any additional node results into a rapid increase in response time. For optimum operation at higher failure rates it is preferable to maintain more
than 20 active nodes per cluster but not more than the desired maximum of 36 nodes. From figures 5.4(a) and 5.4(b) it can be deduced that favoured response time for better performance falls below 0.025 hrs. Systems configured for much lower response times are mostly preferable for WSNs.

From figure 5.5, it is observed that at low arrival rates blocking probability remains low with nearly all queue capacities. However, during heavily traffic load, systems with low queue capacities exhibit higher blocking probabilities observed for $L = 10$. For queue capacities of $L = 500$, the blocking probability approaches zero, thus confirming that nearly all packets are served as the queue capacity tends to infinity. The results in figure 5.5 show the performance of realistic WSN systems, which do not have infinite queue capacity. From figures 5.3(b) and 5.5, it is possible to recommend different queue capacities for various applications, which are dependent on the volume of data generated and whether they are mission-critical. In table 5.3, generalised queue capacity recommendations for different applications are given. In order to optimise system performance, utilization was maintained between 0.1 to a maximum of 0.9.
Table 5.3: Proposed Buffer sizes for various application categories

<table>
<thead>
<tr>
<th>Application Categories</th>
<th>Buffer Sizes</th>
<th>Application Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Data Intensive</td>
<td>10 - 30</td>
<td>Smart Agriculture</td>
</tr>
<tr>
<td>Medium Data Intensive</td>
<td>30 - 50</td>
<td>Body Area Networks (BANs)</td>
</tr>
<tr>
<td>Data Intensive</td>
<td>50 - 100</td>
<td>Volcanic Erruption, Wild Fire</td>
</tr>
<tr>
<td>High Data Intensive</td>
<td>100 - 500</td>
<td>Video &amp; Data (Intelligent transport Systems)</td>
</tr>
<tr>
<td>Very High Data Intensive</td>
<td>Above 500</td>
<td>Real time Multi-Media Applications</td>
</tr>
</tbody>
</table>

5.6 Chapter Summary

In this chapter, a solution technique for modelling and performability analysis of clustered WSNs with bounded queues is presented. The study is focused upon the behaviour of the CH as it receives and processes internal and external job arrivals while at the same time, prone to possible breakdowns, repairs/replacement, and restoration during operations. The CH is successfully modelled using an M/M/1/L open queuing network, and its steady-state probabilities derived using spectral expansion solution technique.

Numerical results are presented comparatively with results obtained from simulation runs for various performability measures. The results which are in good agreement with discrepancies under 2% clearly show the effects of bounded CH queues and confirm the importance of performability modelling for WSNs. From results, it is deduced that finite queues limit acceptable data packets at the CH at any given time and results to loss of packets arriving when the queue is full. Variations in required queue capacities are application dependent with data intensive applications demanding more storage as highlighted in table 5.3.

This study can be further extended to model intra and inter cluster traffic with consideration to priority queues for mission-critical applications in a mixed application environment. An example of this may be monitoring the spread of pests in the agricultural farm where other applications like temperature, humidity, fire and intrusion detection are also of interest. It is possible to identify and include additional operative states of the CH in the model in order to capture more realistic system behaviour. These may include sleep mode, channel failure,
reduces operation state etc. Furthermore, the effects of performance and availability measures on energy consumption can also be incorporated in optimisation studies.
Chapter 6

Performability Modelling and Evaluation of Clustered Wireless Sensor Networks Implementing Sleep Operation Dynamics

6.1 Introduction

Following rapid technological development of Micro Electro-Mechanical (MEM) sensing devices WSNs have become more desirable for information gathering and building reliable and efficient infrastructures for data and communication systems thus serving as a base infrastructure for the Internet of Things (IoT) technologies. However, in the wake of high application demands in diverse environments, performance and availability/reliability of WSNs have continued to suffer because of sensor energy limitations [Chiasserini and Garetto, 2006]. As a result, quality of service offered by WSNs is continuously degraded.

In order to conserve energy, alternating sensor node operations between sleep and active modes is widely used as introduced in section 2.3. Conversely, the mechanisms used for implementing sleep schedules have introduced more challenges to
the performance and dependability of WSN systems.

This chapter analyses the effects introduced by sleep/active duty cycling operations on WSN performance and availability/reliability. First, the models are developed considering the existing sleep scheduling mechanisms [Van Dam and Langendoen, 2003][Anastasi et al., 2009][Tyagi and Kumar, 2013]. Quasy Birth and Death (QBD) processes are used in turn to model the system for performability studies similar to approaches used in Chakka [1995]. The remainder of this chapter is organised as follows: section 6.2 present operation dynamics of sleep and active modes of WSNs, section 6.3 describes the system model. The two-dimensional representations of the proposed sleep scheduling models for WSNs are presented in section 6.4 followed by Numerical results and discussions in section 6.5. Finally, the chapter summary is presented in section 6.6.

6.2 Operation Dynamics of Sleep and Active Modes of Wireless Sensor Networks

In this section, detailed properties for sleep/active periods of WSN operations are investigated and justified. Alternate sleep and active periods are being used to conserve limited node energy. Various approaches are used in the literature to model these operation states. In the active state, the node can operate normally while the sleep state corresponds to the lowest value of node power consumption [Chiasserini and Garetto, 2006], [Zhang and Li, 2012], and [Zhen et al., 2014].

Generally, the implementation of energy-saving schemes has been achieved using MAC protocols. These include CSMA/CA, S-MAC, Time out MAC (T-MAC) and others. Even though CSMA/CA has been modified for use in a number of network platforms, the contention procedure introduces a significant overhead in energy consumption that limits its use for WSNs. The latter two were specifically developed to reduce the idle listening of the sensor nodes in order to reduce energy consumption. The main goal is to turn off the radio transceiver whenever there are no packet arrivals. In S-MAC, sleep and active periods are fixed while in T-MAC these periods are dynamically dependent on the traffic load [Zhen et al.,
From previous studies, it has been confirmed that T-MAC outperforms S-MAC and CSMA in terms of energy-saving [Zhen et al., 2014], [Van Dam and Langendoen, 2003]. Interesting to note is the fact that T-MAC reduces idle listening by transmitting all messages in bursts of variable lengths while allowing sleeping in between the bursts. On the contrary, S-MAC has a fixed duty cycle. S-MAC is a single frequency contention based protocol that divides time into fairly large frames. In this protocol, every frame has two parts as Active and Sleeping parts. During the active part, the sensor can communicate with its neighbours and send any queued messages. During the sleeping part, a node turns off its radio to preserve energy.

In other studies such as [Chiasserini and Garetto, 2006], [Zhang and Li, 2012], the temporal evolution of the sensor states has been modelled in terms of operating cycles. Each cycle comprises a sleep phase (S) and an active phase (A). During phase (S), the sensor is in sleep mode, when the sensor switches to active mode phase (A), it schedules a time in the future when it goes back to sleep. The scheduled periods of active and sleep are expressed in time slots and may be modelled as random variables geometrically distributed with parameters $q$ and $p$ respectively [Chiasserini and Garetto, 2006]. This assumption was made for tractability reasons and justified through results showing close approximation to the sensor behaviour going into periodic sleep as is the case with real systems.

At the time of entering sleep mode, a sensor prolongs its active phase (A) if its queue is not empty [Chiasserini and Garetto, 2006], [Van Dam and Langendoen, 2003]. Under this circumstance, phase (A) is extended until all the data packets in the queue are processed. The extended phase is known as reduced active phase (N). It is observed from [Chiasserini and Garetto, 2006] that introducing phase N allows sensors to adapt to traffic conditions thereby preventing possible network instability due to overloading. A similar model was used in [Zhang and Li, 2012]. However, in this work, the alternate durations the sensor may take in full active phase (A) and semi active phase (N) are assumed to be exponentially distributed with mean values of $1/\alpha$ and $1/\beta$ respectively.
6.2.1 Justification for Exponential Distribution for Active/Sleep Operation Periods

Two main approaches used for implementation of Sleep/Active power saving in WSN include software and hardware schemes [Van Dam and Langendoen, 2003],[Anastasi et al., 2009]. In the software-based scheme, the low duty cycle is represented as a periodic wake-up scheme where a node routinely switches between active communication epoch and power efficient sleep state. In hardware-based approaches, pure asynchronous rendezvous schemes are employed. These approaches allow sensors to remain within the sleep state most of the time, but only waking up when probed by the neighbours. In this arrangement, nodes no longer use duty cycling but are instead equipped with a low-power wake-up receiver module which continuously monitors the channel. In order to communicate with a neighbour, a node first sends a wake-up call. After successful reception and decoding of the wake-up call the wake-up receiver sends an interrupt signal to the node itself, which then fires up its primary radio to engage in efficient high speed communication with the sender. After the transmission, both nodes activate their wake-up receivers, resuming their usual activities and going back to sleep mode. Though this approach promises good energy saving in the long run, existing sensor nodes only have one radio system hence they may require hardware alterations, which may be expensive. The extra hardware also consumes additional energy to run the supplementary circuitry [Gu and Stankovic, 2005].

In another study [Anastasi et al., 2009], an Adaptive Staggered sLEEp Protocol (ASLEEP) for efficient power management in wireless sensor networks was proposed for periodic data-acquisition applications. This protocol dynamically adjusts the sleep schedules of the nodes to match the network demands even in time-varying operating conditions. Moreover, it does not require a-priori knowledge of the network topology or traffic pattern. The novelty of this approach is the fact that it overrides the use of fixed duty cycling and implements an adaptive duty cycling scheme that automatically adjusts the Sleep/Wake up periods depending on the observed operating conditions.

Considering that under normal operations, the active period of the node is fully
dependent upon the traffic intensity, an appropriate choice is necessary to regulate idling times. In addition, using ASLEEP protocol and hardware wake-up receivers is preferred for conserving node energy consumption. In the two approaches, timings for sleep/active periods are not predetermined but left to depend upon network traffic and operation dynamics.

While implementing the two schemes; hardware wake-up receivers and ASLEEP protocol, it can be observed that the distribution of sleep/active times is a continuous process that follows i.i.d random variables. They do not depend upon the knowledge of the present or past sleep/active times, but their arrivals are purely dependent on present network operation dynamics. For tractability, it is possible to assume without loss of generality that both active and sleep times are exponentially distributed. In these study, hardware wake-up receivers and software, ASLEEP schemes are considered for modelling the distribution of sleep/active schedules. In the case of hardware, the nodes enter sleep mode after serving the last packet and wake up on arrival of a new data packet. However, for ASLEEP, sleep and active times are assumed exponentially distributed with random variables having a mean values $1/\alpha$ and $1/\beta$ respectively as explained above. The models developed for this study are based upon earlier studies in [Chiasserini and Garetto, 2006], [Zhang and Li, 2012]. The novelty of our model is the inclusion of the failed states and consideration of the bounded queues.

Considering the use of the two possible sleeping mechanisms; software ASLEEP and hardware wake-up receivers, two models were developed.

1. Transition into sleep mode at the completion of last service and wake up at packet arrival. In this case, beta and alpha are not required as the arrival and service end times of the packets are used.

2. Duty cycling using ASLEEP protocol. In this scenario, beta and alpha play a major role in determining the required operation periods while in the various states.

In the first case above, nothing else is required except the distribution of inter-arrival and service times. However, in the second case, the events leading to sleep and active operation states are considered. These include:
1. **Active State**

While operating normally, the CH schedules a time in the future when it will go into sleep mode. Conversely, when this period ends, the following are considered for determining the next action to take, which may either reduce or increase the value of alpha:

a. If there are no more packets to be served, then the CH transits into sleep mode.

b. If there are packets left to be served, the CH enters into the reduced active state \((N)\). In this state, the CH stops receiving incoming data packets. Nevertheless, it continues to serve the remaining data packets. As soon as the service for the last packet is completed, the CH enters sleep mode \((S)\). During sleep mode, the CH does not involve in any activity.

c. During active operation, if the CH has no more requests in the system to serve, it will get into the idle state for a period after which it will automatically enter sleep mode if no more data packets arrive.

2. **Sleep State**

Each time the CH goes into sleep mode, it re-schedules a time in the future when it will transit back to active mode. The sleep time is assumed exponentially distributed with rate \(\beta\). At the end of the sleep period, the CH checks the availability of data packets, and changes state to active mode otherwise it prolongs sleep state in order to save energy. The dynamic change of Sleep/Active periods based on traffic conditions can therefore be used to determine the parameter \(\beta\).

### 6.3 Model Description

In order to build the models, the network scenario presented in Figure 4.1 is considered. The queue model also remains the same as presented in section 5.3. Based on the discussions in sections 6.2 on sleep scheduling mechanisms, two
models are proposed as an advancement to the earlier model presented in Figure 5.1. The proposed models are presented below in line with the sleep scheduling scheme employed.

### 6.3.1 On-Demand Wake-Up Scheme

In Figure 6.1, On-demand wake-up schedule is considered. In this scheme, a second radio transceiver that consumes very low power is used for continuous monitoring of the channel for any packet arrivals while the main radio transceiver is put to sleep. The scheme enables the CH to enter sleep mode automatically following service end for the last job within the system and wakes up only when a new arrival occurs [Miller and Vaidya, 2005] [Gu and Stankovic, 2005] and [Ameen et al., 2010]. Service and arrival distribution times remain as discussed in section 4.2.3. When wake-up radio transceivers are used, the CH remains in the active state, as long as there are jobs within the system to be serviced.

![Figure 6.1: CH Phase transition with On-Demand Protocol](image)

### 6.3.2 Adaptive Duty-Cycling Scheme

Adaptive duty cycling schemes are known for adjusting sleep wake up periods depending on the observed operating conditions. Several implementation approaches have been proposed in the literature [Van Dam and Langendoen, 2003] [Anastasi et al., 2009] [Yang and Heinzelman, 2013].

In Figure 6.2, a block diagram of the possible CH operative states is presented. The operative states are broadly represented in three categories as; Active mode, Sleep mode and Node Failure mode. The active mode is further divided into Full...
operation (Denoted by phase R) and Reduced operation (Denoted by phase N) phases. While in normal operations, the CH switches back and forth between Active and Sleep modes. The two modes operate for a time period modelled as random variables exponentially distributed with parameters $\alpha$ and $\beta$ respectively. While in the active mode, the CH may either be in phase (R) or in phase (N). In phase (R), the CH may receive and transmit data packets or idle if there are no data packets to be processed. Depending on the prevailing conditions the CH may transition to any of the following states:

a.) The CH goes to sleep mode (phase S) if the active mode period expires and there are no jobs remaining within the system to be processed.

b.) If there are jobs remaining within the system at the expiry of the active period, the CH transitions to reduced operation’s phase. In this phase, the CH may only process and transmit the remaining data packets and immediately enter sleep mode at the transmission of the last data packet.

c.) While operating in any of the Active phases, the CH may fail as a result of corrupt software configuration, battery power depletion, and hardware malfunctions. Where repair is possible, the CH is restored back to full active mode once repair is complete. Failure and repair times are assumed exponentially distributed with rates $\xi$ and $\eta$ respectively. During failures, it is assumed that a backup CH installed at inception takes up all the responsibilities of the CH until a new choice of the CH is made from the remaining cluster nodes.

During Sleep mode, the CH is completely cut off from network activities. It does not receive or transmit data packets. It is assumed to be in its lowest power consumption state. The CH only switches back to full active phase at the expiry of the sleep period. If the CH fails while in sleep mode, it is taken into repair and then re-instated into the system as a full-function device.

In order to explicitly explain the models proposed, the following assumptions and notations were introduced for the sensor node under investigation.

a.) The duration a CH takes operating in full-active phase (R) is a random
time distributed exponentially with a mean of $1/\alpha$. During this period, the CH may:

i.) Generates packets following Poisson distribution with rate $\lambda$.

ii.) Receive data packets originating internally from cluster nodes and externally from other CHs. Packet arrival distribution at the CH is assumed to follow Poisson process with rate $\lambda_k$.

iii.) Process and transmit or relay data packets with random exponential time with a mean of $1/\mu$.

iv.) Idle while listening to the wireless channel in readiness to receive arriving data packets. In this state, all the internal circuitry is kept ready to operate.

b.) At the expiry of full-active phase R, the CH transitions either to sleep or Reduced-active phase. In order to enter reduced active phase, there must be at least one data packet in the system waiting to be processed. In reduced active phase ($N$) the CH may only process and transmit the remaining data packets in the system following a random exponential time with a mean of $1/\mu$. Internal data generations and external data arrivals are stopped to enable the CH prepare to enter sleep mode after the transfer of all data packets in reduced-active phase. Phase N, therefore, allows the
sensor to adapt to traffic conditions and prevent network instability due to overloading [Chiasserini and Garetto, 2006].

e.) The duration the CH takes in sleep mode is distributed exponentially with a mean of $1/\beta$. While in sleep mode, the sensor is completely cut off from the rest of the network. At the expiry of the sleep mode, the sensor automatically reverts to full-active phase (R).

d.) The CH can only be engaged with one activity at a time, and the order of service is based on First-Come First-Served basis ($FCFS$).

e.) Considering the memory limitation of the CH, a finite queue length of $L$ is assumed.

6.4 Two Dimensional Representation of the Sleep Scheduling Models for WSNs

This section presents sleep scheduling models used for WSN performance and availability evaluation. In developing the models, this study considers On-demand and adaptive duty cyclic sleep scheduling schemes introduced in sections 6.3.1 and 6.3.2 above.

6.4.1 Model for On-demand sleep Scheduling

In the case of On-demand sleep scheduling, the important implementation feature is the replacement of the idle state with sleep state. The model is therefore, similar to that presented in Figure 5.1. The only difference is that the period spent in the idle state is replaced with sleep state. All other performance parameters remain as presented in section 5.4. The new model with sleep state is presented in figure 6.3.

The solution approach in this model is similar to that of Figure 5.1. The state probabilities also remain the same. However, the probability of being in the sleep
Figure 6.3: CH Performability Model based on On-Demand Protocols

state can now be given using equation 6.1.

\[
P_{SLP} = 1 - \sum_{j=1}^{L} \sum_{i=0}^{N} P_{i,j} + P_{FM0} 
\]

(6.1)

Where \( P_{SLP} \) and \( P_{FM0} \) are probability of being in the sleep state, and node failed state respectively. In both states, there are no jobs within the system. Figure 6.4 shows the graph of mean times the CH spend in active and sleep operation states with respect to variable nodes. The results were obtained using parameters justified in section 5.5.1. A random choice for parameter values was made for this experiment; arrival rate of \( \lambda = 5 \) pckts/hr, and queue capacity of \( L = 50 \) packets. From the figure, sleep time is higher when fewer nodes are used but reduce linearly as the nodes are increased over time. On the contrary, the CH remains active for a longer period when more nodes are used. The lowest times are recorded when least nodes are deployed. We also note that the mean active time remains low under 35 sources after which it rises sharply as sources are further increased. This is in line with 36 nodes per cluster recommended by IEEE802.15.4/Zigbee standards.
6.4.2 Model for Adaptive Sleep Scheduling Schemes

In the case of adaptive duty cycling schemes, the model in Figure 5.1 is improved to consider reduced operation phase $N$. The improved performability model is presented in Figure 6.5. The model has three main operation phases namely: Node failure phase $F_M$, Active phase $R$, and Reduce operation phase $N$. $L = 0, 1, \ldots, L$ represent the number of jobs in the system at any given time during operations. In phase $N$, $S_0$ is the sleep state reached after serving all data packets in phase $N$. The sleep state may also be reached at the expiry of active phase $R$ operations as discussed in section 6.3.2. $N_i$ through $N_L$ represent the number of packets waiting in the queue for service at the expiry of the full active period of phase $R$.

From the model, packet arrival is possible during phase $R$ of the full active mode and in the node failure phase $F_M$. In phase $N$ of the active phase, the CH can only serve the remaining data packets. In order to solve this model for steady state probabilities, spectral expansion exact solution technique and system of simultaneous linear equations are employed and validated using a simulation program.
The system state at time \( t \) may be described using a pair of integer valued random variable \( I(t) \) and \( J(t) \) specifying the CH operative states and the number of jobs within the system respectively. The operative states \( I(t) \) in this case represents the Failure phase, Full operation phase, and Reduced operation phase of the CH. \( Z = [I(t), J(t)]; \quad t \geq 0 \) is an irreducible Markov process on a lattice strip (QBD process), that models the system. Its state space is given by \((0, 1, 2) \times (0, 1, \ldots, L)\). Similar models in [Chakka, 1998] [Ever et al., 2009] are analysed for exact performability evaluation of various multi server systems with single repairman and for both finite and infinite \( L \) for some repair strategies. It is possible to extend the exact solution methodology for performability evaluation of WSNs. In addition, a system of linear simultaneous equations was also employed to solve the model.

### 6.4.2.1 Spectral Expansion Solution Approach

In order to use spectral expansion exact solution approach, transition matrices are first derived from Figure 6.5 as detailed below;

- Matrix \( A \) is the matrix of instantaneous transition rates from \((i, j)\) to state
\((l, j), (i = 0, 1, \ldots, N; l = 0, 1, \ldots, N; i \neq l; j = 0, 1, \ldots, L)\), with zeros in the leading diagonal, caused by a change in the operative state [Ever et al., 2009]. These are the purely lateral transitions of the model \(Z\). \(A\) depends on parameters \(\xi, \eta, \beta\) and \(\alpha\). The state transition matrices \(A\) and \(A_j\) are of size \((3) \times (3)\) as shown.

\[
A = \begin{bmatrix} 0 & \eta & 0 \\ \xi & 0 & \alpha \\ \xi & \beta & 0 \end{bmatrix} \quad \text{and} \quad A_j = \begin{bmatrix} 0 & \eta & 0 \\ \xi & 0 & \alpha \\ \xi & 0 & 0 \end{bmatrix}
\]

ii Matrices \(B\) and \(C\) are transition matrices for one step upward and one step downward transitions respectively [Chakka, 1995] [Ever et al., 2009]. When there is no job in the system, the elements of matrix \(C\) are all zero. The transition rate matrices do not depend on \(j\) for \(j \geq M\), where \(M\) is a threshold having an integer value. Below are the respective transition matrices:

\[
B = B_j = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad C_j = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix}
\]

Elements of matrix \(B\) are dependent on the data arrival rate \((\lambda)\) at the CH while elements of matrix \(C\) depend on the CH service rate \((\mu)\). Once the state transition matrices are established, use was made of spectral expansion solution technique explained in section 3.3.3 to derive steady-state probabilities for the model. From the state probabilities, computation of a number of steady-state availability, reliability and performance measures are easily achievable.

**6.4.2.2 Using system of Simultaneous Linear Equations**

In this approach, use is made a balance equation argument based on equivalence of inflow and outflow transitions into a state as given in equation 3.12 to develop a set of simultaneous linear equations. Using the resulting Kolmogorov forward
equations, the generator matrix \( \mathbf{G} \) and the state probability vector \( \mathbf{P} \) are derived as explained in section 3.3.2. The resulting linear equations are then considered together with normalization equation given in part two of equation 3.11 for computation of the limiting probabilities. Using the steady-state probabilities, it is possible to compute desired performance parameters like mean queue length, throughput, delay, and others.

In this study, taking a visual balance equation from the performability model of Figure 6.5, a set of steady-state linear equations 6.2 - 6.13 are obtained. The obtained set of equations are further used to determine the state transition matrix \( \mathbf{G} \) given in table 6.1. The following equations illustrate transitions into the various operative states within the system. From the performability diagram, transitions into states with jobs between 1 < \( j \) ≤ \( L - 1 \) are similar. Equations 6.2 - 6.5 present transition states in Node failure mode Phase (\( F_M \)).

**For number of jobs \( j = 0 \)**

\[
F_0(\eta + \lambda) - \xi R_0 - \xi S_0 = 0 \tag{6.2}
\]

**For number of jobs \( j = 1 \)**

\[
F_1(\eta + \lambda) - \xi R_1 - \xi S_1 - \lambda F_0 = 0 \tag{6.3}
\]

**For number of jobs 1 < \( j \) ≤ \( L - 1 \)**

\[
F_j(\eta + \lambda) - \xi R_j - \xi S_j - \lambda F_{j-1} = 0 \tag{6.4}
\]

**When the queue is full \( j = L \)**

\[
\eta F_L - \xi R_L - \xi S_L - \lambda F_{L-1} = 0 \tag{6.5}
\]

The next set of equations 6.6 - 6.9 present the equations for transition states in full active operation phase (\( R \)).
For number of jobs \( j = 0 \)

\[
R_0(\xi + \lambda + \alpha) - \mu R_1 - \eta F_0 - \beta S_0 = 0 \quad (6.6)
\]

For number of jobs \( j = 1 \)

\[
R_1(\xi + \lambda + \alpha + \mu) - \mu R_2 - \lambda R_0 - \eta F_1 = 0 \quad (6.7)
\]

For number of jobs \( 1 < j \leq L - 1 \)

\[
R_j(\xi + \lambda + \alpha + \mu) - \mu R_{j+1} - \lambda R_{j-1} - \eta F_j = 0 \quad (6.8)
\]

When the queue is full \( j = L \)

\[
R_L(\xi + \mu + \alpha) - \lambda R_{L-1} - \eta F_L = 0 \quad (6.9)
\]

The last set of equations 6.10 - 6.13 present the balance equation arguments for transition states of the reduced active operation phase \( N \).

For number of jobs \( j = 0 \)

\[
S_0(\xi + \beta) - \alpha R_0 - \mu S_1 = 0 \quad (6.10)
\]

For number of jobs \( j = 1 \)

\[
S_1(\xi + \mu) - \mu S_2 - \alpha R_1 = 0 \quad (6.11)
\]

For number of jobs \( 1 < j \leq L - 1 \)

\[
S_j(\xi + \mu) - \mu S_{j+1} - \alpha R_j = 0 \quad (6.12)
\]
When the queue is full $j = L$

$$\mu S_L - \alpha R_L - \xi F_L = 0 \quad (6.13)$$

Using steady-state probabilities from both solution approaches, we obtain performance measures of interest using equations 6.14 - 6.19. Other performance measure including Blocking probability, Mean queue length, throughput, utilization and response time are obtained as discussed in section 5.4 using equations; 5.1, 5.4, 5.5, 5.6 and 5.7 respectively.
Table 6.1: Transition Matrix G

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<td>F_0</td>
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<td>F_{L-2}</td>
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<td>(\eta + \lambda)</td>
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<td>-\eta</td>
<td>(\xi + \alpha + \lambda)</td>
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P is an ((L+1) x 1) vector given by: \( P = (0, 0, 0, 0, 0, \ldots, 0, 0, 0, 0, 0, 1) \) and

Vector "X" for the unknowns is a (3L x 1) and is given by:

\[ X = (F_0, R_0, S_0, F_1, R_1, S_1, F_2, R_2, S_2, \ldots, F_{(L-2)}, R_{(L-2)}, S_{(L-2)}, F_{(L-2)}, R_{(L-1)}, S_{(L-1)}, F_L, R_L, S_L) \]
1. Percentage of jobs lost (PJL):

This account for the number of jobs lost due to blocking. The percentage may be computed using equation 6.14 given below.

\[ PJL = 100\% \times \sum_{i=0}^{N} P_{i,L} \]  

(6.14)

2. Probability the system is in idle state (PR0):

During full active phase, equation 6.15 provides the probability that the system is idling waiting for any job arrivals.

\[ PR_0 = 1 - (\sum_{j=1}^{L} \sum_{i=0}^{N} P_{i,j} + P_{F,0} + P_{S,0}); \quad i = 0, 1, \ldots, N; \quad j = 1, 2, \ldots, L \]  

(6.15)

where \( \sum_{j=1}^{L} \sum_{i=0}^{N} P_{i,j} \) represent the busy state of the system, \( P_{F,0} \) represent the probability of node failed state and \( P_{S,0} \) the probability of the system being in sleep state. In \( P_{F,0} \) and \( P_{S,0} \) states, the system does not have any job being processed or waiting for service in the queue.

3. Probability of being in Full Active Phase (R):

The probability the system is operating in full active phase (R) may be computed by;

\[ PR = \sum_{j=0}^{L} P_{i,j}; \quad i = 1; \quad j = 0, 1, 2 \ldots, L \]  

(6.16)

4. Probability system fails with jobs in the system (PF,j)

\[ P_{(F,j)} = \sum_{j=1}^{L} P_{i,j}; \quad i = 0; \quad j = 1, 2, \ldots, L \]  

(6.17)
5. Probability system in sleep state \( (P_{S0}) \):

\[
P_{S0} = 1 - \left( \sum_{j=1}^{L} \sum_{i=0}^{N} P_{i,j} + P_{FM0} + P_{R0} \right); \quad i = 0, 1, \ldots, N; \quad j = 1, 2, \ldots, L
\]  

6. Probability the system is in reduced operation state \( (P_{Nj}) \):

\[
P_{Nj} = \sum_{j=1}^{L} P_{i,j}; \quad i = 2; \quad j = 1, 2, \ldots, L
\]  

6.5 Numerical Results and Discussions

In order to show the effectiveness of the proposed model and analyze the effects introduced by sleep state scheduling schemes on the performability of homogeneous WSNs, numerical results are presented throughout this section. The first set of solution comparatively present results obtained using the proposed analytical techniques; Spectral Expansion and the System of Simultaneous Equations. Both results are further validated using results obtained from a dedicated discrete event driven simulation program developed in C++. In this work, attention is dedicated to the influences caused by sleep scheduling on performance parameters that may include; mean queue length, response time, throughput, blocking and transition into the various operative states. Detailed discussion on the obtained results are given in the following sections.

6.5.1 Parameter Choices

In order to choose active and sleep time, a ratio of the duty cycle enabling more sleep time is considered. From literature sleep and active time rates of 0.01, 0.05 and 0.1 per hour have been used in [Chiasserini and Garetto, 2006], [Li, 2011] respectively. In this study, a choice of sleep/active period is determined by first
running two separate experiments in which one of the parameters is held constant as the other is varied and appropriate values determined. The rates of sleep and active times considered for best performance are $\beta = 0.6/\text{hr}$ and $\alpha = 0.2/\text{hr}$ respectively.

In this study, it is assumed that batteries do not run out of power during operation. This is so because use of good mechanisms (e.g. Solar system) for harvesting power are employed for recharging limited battery power [Li, 2011]. In addition, BCHs are used for redundancy purposes. The failures considered for this case are therefore, purely software related or caused by acts of adversaries. In order to model such systems, mean failure ($\xi$) and repair ($\eta$) rates of $\xi = 0.001/\text{hr}$ and $\eta = 0.5/\text{hr}$ were employed respectively. In all the experiments, the parameter values used remain the same as given in table 6.2 unless specified otherwise.

Table 6.2: Simulation parameters and values

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\xi$</th>
<th>$\eta$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>Queue capacity (L)-packets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>300</td>
<td>0.001</td>
<td>0.1-0.9</td>
<td>0.2</td>
<td>10, 30, 50, 100,</td>
<td></td>
</tr>
</tbody>
</table>

6.5.2 Validation of results

In the following figures, results obtained from the three solution approaches; System of Simultaneous Equations, Spectral Expansion and simulation are compared. For this purpose, a comparison of MQL, Response Time and Throughput is presented against varying arrival rate $\lambda$ shown in Figures 6.6(a), 6.6(b), and 6.7 respectively. From the figures, it is notable that the system is more stable when the low arrival rates less than $\lambda = 6/\text{hr}$ are employed in contrast to higher arrival rates. This is significant for power saving in application areas with low to medium traffic intensities. As observed, the results from the three solution approaches closely follow one another with discrepancies of less than 5% within the desired traffic region.

From Figure 6.6(a), MQL is maintained low below $\lambda = 6/\text{hr}$. Further increases in arrival rate result to the system becoming unstable, and the solution approaches giving varying results. A similar trend is observed with Response time and
Figure 6.6: Validation of Performance Results

Figure 6.7: Throughput against Arrival Rate

Throughput, Figures 6.6(b) and 6.7 respectively. Using spectral technique, results become more irregular with arrival rates beyond service rate when compared...
to those obtained using system of simultaneous linear equations and simulation approaches.

In the subsequent sections, system operation in different states is illustrated with respect to changing arrival rates. Furthermore, presented are analyses of how sleep scheduling affects system performance.

Figure 6.8 presents operative states within the system in the active mode. In Figure 6.8(a), a comparison of the various active operative states is presented. At low arrival rates, the system stays idle most of the time while the active period with jobs is low. As arrival rate is gradually increases, active state probability advances steadily up to a highest point beyond which, no further changes are observed as arrival rates are increased. The converse is the same for idle probability that reduces to the lowest possible point. On the other hand, minimal variations are observed in full active, sleep, and reduced states as arrival rates are increased. Figure 6.8(b) show the increasing trend of the probability of being in the reduced operation state as arrival rate is increased during operation.

In figure 6.9, the effects of sleep variation on system performance is analysed. In
this experiment, the number of nodes, the failure rate ξ, and queue capacity L are fixed at 25 sources, 0.001/hr and 10 packets respectively. Sleep time is then varied between β = 0.1 – 0.8 during each run as performance parameters of interest are observed. From Figure 6.9(a), MQL is maintained below the queue capacity of L = 10 sources at arrival rates below λ = 12/hr. Above this rate, MQL increases
rapidly with higher sleep rates experiencing higher MQL values as shown. From Figure 6.9(b), throughput increases linearly below arrival rate of $\lambda = 12/hr$ after which it remains constant as arrival rates are increased further. Again at sleep rates of $\beta = 0.8/hr$, throughput is highest and lowest at $\beta = 0.1/hr$. In Figure 6.9(c), system response time is kept below 0.05 hours when arrival rate is less than $\lambda = 10/hr$ but sharply rises as arrival is increased then exhibit minimal changes, remaining nearly constant with further increments. Figure 6.9(d) shows blocking analyses. The system blocking probability is kept very small when operating below arrival rate of $\lambda = 10/hr$. However, further increments result into nearly all jobs being blocked. Like in the previous cases at $\beta = 0.8/hr$, the probability of arriving jobs being blocked is much higher when compared to lower sleep rate values.

Figure 6.10: Effects of Sleep Time on Operation Times

From Figure 6.10, Full active operation state variation with changing sleep rate is shown. At low arrival rates, each experimental run shows the CH spending most of its time in the full active state. Beyond arrival rates of $\lambda = 10/hr$, the full active time is reduced a bit as shown in Figure 6.10(a). Finally in this category, Figure 6.10(b) shows how sleep rate influences actual CH sleeping time. From this figure, it is deduced that the CH sleeps mostly when sleep rate $\beta = 0.1/hr$
and reduces to a minimum as the rates are increased to $\beta = 0.8/hr$.

In the next set of experiments, system failure is analysed as sleep rate ($\beta$) and arrival rate ($\lambda$) are varied. The remainder of the parameters are maintained as given in table 6.2. The results are shown in figure 6.11.

![Figure 6.11: Node Failure Probabilities](image)

(a) Node Fails during Operations  (b) Node Fails with Jobs in the System

Figure 6.12: Node Fails with Jobs in the System

Figure 6.11(a) shows the overall probability of node failure during operation. Figure 6.11(b), provides the details of node failure with jobs within the system. It is notable that probability of Failure with jobs within the system is higher at
high arrival rates in all the cases. In comparison, the failure rate is also slightly more at $\beta = 0.8/hr$ than at lower values. Conversely, probability of the CH failing with no jobs within the system is higher at low arrival rates and lower at higher arrival rates as illustrated in Figure 6.12.

### 6.6 Chapter Summary

Alternating Sleep/Active operation period in WSNs has widely been used to conserve the limited energy resource. While this may save energy significantly, it may also interfere negatively with desired system performance and in some cases even facilitate more energy consumption. Several approaches have been used as discussed in chapter 2.3.2. In this chapter, the effects of sleep scheduling on system performance are investigated. First operation dynamics of implementing sleep in WSNs is analysed and results used to model the system behaviour. The developed models are then solved using two analytical approaches; Spectral Expansion and System of simultaneous linear equation which are further validated using a dedicated simulation program developed in C++. Results obtained from the three approaches closely match with discrepancies of below 5%.

In the case of On-Demand sleep scheduling, energy is significantly saved when low arrival rates are prevalent. This, however, changes when high-traffic rates are predominant due to significant use of wake-up energy. A mechanism is therefore, necessary to categorize traffic intensity for purposes of regulating appropriate sleep periods. Adaptive sleep scheduling has been used but this as well presents various performance degradations. By specifying operation duty cycle, the system operation is restricted within a given space. This situation may adversely affect system performance due to restricted memory resources and even worse if frequent link failures are not avoidable. The results obtained can be used by system designers to develop a more resilient system that regulates sleep schedules in line with traffic intensity.

Using the obtained state probabilities it is possible to extend this study to model CH energy consumption. The need for priority queuing in a multi-application
environment may also be incorporated. In the next chapter, in addition to sleeping effects, link failure is introduced to enable a more realistic study of WSN performability in the presence of unreliable links.
Chapter 7

Performability Modelling and Evaluation of Unreliable Links and Sleeping Effects on Clustered WSN

7.1 Introduction

In most application areas, WSNs cooperatively monitor the habitat and through the wireless medium, transmit the information to the sink for onward processing. The sensing nodes that contend for the communication channel are further facing link failures hindering information flow between the sensors and the sink nodes. This condition can become worse if power levels for the sensor nodes are low. These two situations may eventually degrade WSN performance and availability/reliability. Therefore, it is important to analyse performance and availability effects resulting from channel failures in addition to the challenges introduced by bounded queues and node failures introduced in chapters 5 and 6. In order to obtain realistic results, this study extends the models presented in the Figures 5.1 and 6.1 by incorporating channel failures and employ Quasy Birth and Death (QBD) processes for performability studies of WSN systems using Spectral
Expansion and System of linear equations solution approaches.

The propagation of radio signals may be affected by several factors that eventually degrade signal quality or in some cases complete outage. These are more prevalent when propagating wireless signals with low power radios, typically used in WSNs. This makes WSN radio links very unpredictable. Since WSNs are applicable in diverse areas, the underlying factors also differ considerably. However, the factors leading to unreliable links may be broadly classified into three areas as:

1. **Interference effects**, which results from concurrent transmissions from other nodes within the WSN

2. **Environmental effects** that lead to multi-path propagation resulting from background noise

3. **Hardware transceivers** that may distort sent signals due to their internal noise [Nnebe, 2014].

Studies in [Martínez-Sala et al., 2005], [Hrovat and Javornik, 2013], and [Nnebe, 2014] present analysis of path loss in different application areas.

A review of channel modelling for wireless body area network in medical communication was presented in [Taparugssanagorn et al., 2008]. In the study, path loss is noted to be very high when the receiver antenna is placed on a different side from the transmit antenna. In addition, the propagating wave is noted diffracting around the human body rather than passing through it. In another study [Cheffena, 2012], authors presented channel modelling and evaluation of industrial wireless sensor networks. The study presents a model that takes into account the noise, interferences, and heavy multi-path propagation effects present in harsh industrial environments. The model is finally used for performance evaluation of IEEE 802.15.4 in terms of bit error rate. A similar study is presented in [Gungor and Hancke, 2009] with the main aim of discussing implementation challenges and design criteria for industrial WSN. However, in these studies, node and channel failure effects on system performance are not considered.

In an earlier study [Chiasserini and Garetto, 2006], authors proposed a Markov model for radio interference in WSNs and used it to evaluate channel contention.
between transmitting nodes. However, in this study, CSMA/CA was employed and collision-free data transmission was assumed. Furthermore, path loss-related concerns are not taken into account. In all the cases above, performance degradation resulting from complete channel failures and repairs are not considered. The main focus of this chapter is therefore, to model channel failures and repairs in addition to the CH failures. The rest of the chapter is organised as follows; Section 7.2 presents the system model, followed by section 7.3, which presents a two-dimensional Markov representation of the proposed model as well as analytical solution approaches used to solve the model. The numerical results are presented in section 7.4 and finally the chapter summary is given in section 7.5.

7.2 Model Description

Similar to the previous chapters, we consider the network scenario presented in Figure 4.1. The queue model also remains the same as presented in section 5.3. However, in this study channel failure is introduced to the performability model presented in Figure 5.1. In order to mimic the real system scenario closely, this study also considers systems implementing on-demand sleep schedule mechanisms discussed in section 2.3.2.1. In these schemes the sensor nodes do not idle, instead, they enter sleep mode as soon as the last job in the system is serviced [Gu and Stankovic, 2005] [Miller and Vaidya, 2005] [Ameen et al., 2010] [Umbdenstock et al., 2013]. The nodes are equipped with a second low power consuming transceiver used for monitoring the channel for incoming packets and in turn waking up the node in good time to receive the packets. Figure 7.1 is a phase transition diagram used to mimic the behaviour of the CH. The CH is assumed to operate in two main modes, “Active mode (phase R) and Sleep mode (phase S).” While operating in any of the two phases, the CH or the channel may fail and enter node failed state (phase FM) or channel failed state (phase FC) respectively as illustrated in Figure 7.1. In the event the system goes into either FM or FC phases, it is assumed the system enters repair facility and then it is restored back to normal operation immediately after repair is completed.
Based on the transition diagram and other traffic operation dynamics, the previous performability model presented in Figure 5.1 is developed further to incorporate channel failures. More details and proposed solution approaches for the new model are discussed in the following sections.

7.3 Two Dimensional Markov Representation of the Proposed Model

From Figure 7.1, a performability model incorporating channel failure and sleep state is developed. Figures 7.2(a) and 7.2(b) are shown comparatively to highlight the sleep and idle states. In the latter, idle times are considered as sleep periods for the CH. Like in the previous cases, the horizontal axis (i), represent system operative states; channel failure \( FC \), node failure \( FM \), and active state \( R \). The active state is further divided into full active \( R_1, R_2, \ldots, R_L \) and Sleep state \( S_{(LP)} \). The vertical axis represents the number of jobs (j) in the system at a given time. \( L \) represents the CH queue capacity. In this model, full service is available only in the active state \( R \). While in node failed state \( FM \), the CH is only able to receive data packets. However, in both states, arriving packets are admitted into the CH, as long as there are empty spaces in the queue. Any packet arriving
when the queue is full is dropped and assumed lost. When the channel fails, the CH will not be able to receive or transmit data packets. The CH and the nodes are forced to hold their packets until the channel is repaired and restored before they can continue with normal operations. The total packet arrival rate at the CH remains as given in section 4.3, i.e., $\lambda_k = C\lambda$. Service rate $\mu$, node failure rate $\xi$ and repair rate $\eta$ remain as introduced in section 4.2.4. In addition, channel failure and restoration is assumed to follow exponential distribution with rates $\zeta$ and $\theta$ respectively. In order to solve this model, two analytical solution approaches; Spectral Expansion and System of linear equations are used and validated using a dedicated simulation program. The solution approaches are explained in the following subsections.

![Figure 7.2: CH Performability Models](image)

Note: The difference in Figures 7.2(a) and 7.2(b) is in the operative states $R_O$ and $S_{LP}$. The values of channel repair rates, $\theta$ and $\theta_1$ are the same in both diagrams.
7.3.1 Spectral Expansion Solution Approach

From the performability model of Figure 7.2(b), the three transition matrices $A$, $B$, and $C$ are determined. The system state at time $t$ may be described using a pair of integer valued random variable $I(t)$ and $J(t)$ specifying the CH operative states, and the number of jobs in the system respectively. The operative states $I(t)$ in this case represents the channel and node failed states and active period of the CH. $Z = [I(t), J(t)]; \ t \geq 0$ is an irreducible Markov process on a lattice strip (QBD process) that models the system. Its state space is $(0, 1, 2) \times (0, 1, \ldots, L)$.

In order to solve the proposed system model, Spectral Expansion exact solution methodology used for performability evaluation of related models and system of linear simultaneous equations are employed.

Matrix $A$ is defined as the matrix of instantaneous transition rates from state $(i, j)$ to state $(l, j)$ with zeros on the main diagonal. These are the purely lateral transitions of the model $Z$. Matrices $B$ and $C$ are transition matrices for one-step upward and one-step downward transitions respectively. However, transition rate matrices do not depend on $j$ for $j \geq M$, where $M$ is a threshold having an integer value [Chakka, 1995]. The matrices $A, B$ and $C$ are of size $3 \times 3$. $A_j, B_j$ and $C_j$ matrices represent system state when $j < M$.

\[
A = A_j = \begin{bmatrix} 0 & \theta_1 & \theta \\ \zeta & 0 & \eta \\ \zeta & \xi & 0 \end{bmatrix}
\]

\[
B = B_j = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \quad \text{and} \quad C = C_j = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mu \end{bmatrix}
\]

Elements of matrix $B$ are dependent on the data arrival rate ($\lambda$) at the CH while elements of matrix $C$ depend on the CH service rate ($\mu$). With state transition matrices established, spectral expansion solution technique is then used to derive steady-state probabilities for the system, which may be expressed as $P_{i,j} = \lim_{t \to \infty} P(I(t) = i, J(t) = j); 0 \leq i \leq N, 0 \leq j \leq L$, where $L$ is a finite...
queue length and \( N \) represent the number of operative states.

### 7.3.2 Using Simultaneous Linear Equations

The system of simultaneous linear equations is based on the balance equation argument that “Outflow transition rate from a state \((i, j)\) = Inflow transition rate into state \((i, j)\)”. Using this principle, a set of Kolmogorov linear equations [Gemikonakli, 2014] are derived at steady state using equation 7.1 below.

\[
P_i \sum_{j \neq i} g_{i,j} = \sum_{j \neq i} P_i g_{j,i}; \quad i, j \in S
\]  

(7.1)

The steady-state probability can be defined in terms of the state transition rate matrix \( G \) and the state probability vector \( P = \lim_{t \to \infty} P_{i,j}(t) \). As \( t \to \infty \), the Kolmogorov forward equation becomes \( PG = 0 \). The steady-state probability can then be defined as; \( P = (P_0, P_1, \ldots, P_N) \). Together with the normalization equation \( \sum_{i \in S} P_i = 1 \), these set of equations can be solved to obtain the limiting state probabilities.

For this study, taking visual balance equations from the performability model given in Figure 7.2(b), a set of steady-state linear equations 7.2 to 7.13 for the model are derived using the flow balance equation 7.1 above. Once derived, the steady-state linear equations are further used to determine the state transition matrix “\( G \)” given in table 7.1.

To derive the linear equations, we consider transition into and out of the various states independently. From Figure 7.2(b), transitions into operative states with jobs between \( 1 < j \leq L \) are similar. Therefore, these transitions are not shown here. The balance equation for operating in channel failure state \((FC)\) are given by the following:

**For number of jobs \( j = 0 \)**

\[
(\theta_1 + \theta)F_{CO} = \zeta S_{LP} + \zeta F_{MO} \Rightarrow (\theta_1 + \theta)F_{CO} - \zeta S_{LP} - \zeta F_{MO} = 0
\]  

(7.2)
For number of jobs $j = 1$

$$(\theta_1 + \theta) F_{C1} = \zeta R_1 + \zeta F_{M1} \implies (\theta_1 + \theta) F_{C1} - \zeta R_1 - \zeta F_{M1} = 0 \quad (7.3)$$

For number of jobs $1 < j \leq L - 1$

$$(\theta_1 + \theta) F_{Cj} = \zeta R_j + \zeta F_{Mj} \implies (\theta_1 + \theta) F_{Cj} - \zeta R_j - \zeta F_{Mj} = 0 \quad (7.4)$$

For number of jobs $j = L$

$$(\theta_1 + \theta) F_{C(L)} = \zeta R_{(L)} + \zeta F_{M(L)} \implies (\theta_1 + \theta) F_{C(L)} - \zeta R_{L} - \zeta F_{ML} = 0 \quad (7.5)$$

The next set of balance equations are for operating in node failed state ($FM$)

For number of jobs $j = 0$

$$(\eta + \zeta + \lambda) F_{MO} = \theta_1 F_{CO} + \xi S_{LP} \implies (\eta + \zeta + \lambda) F_{MO} - \theta_1 F_{CO} - \xi S_{LP} = 0 \quad (7.6)$$

For number of jobs $j = 1$

$$(\eta + \zeta + \lambda) F_{M1} = \theta_1 F_{C1} + \lambda F_{MO} + \xi R_1$$

$$\implies (\eta + \zeta + \lambda) F_{M1} - \theta_1 F_{C1} - \lambda F_{MO} - \xi R_1 = 0 \quad (7.7)$$

For number of jobs $1 < j \leq L - 1$

$$(\eta + \zeta + \lambda) F_{Mj} = \theta_1 F_{Cj} + \lambda F_{M(j-1)} + \xi R_j$$

$$\implies (\eta + \zeta + \lambda) F_{Mj} - \theta_1 F_{Cj} - \lambda F_{M(j-1)} - \xi R_j = 0 \quad (7.8)$$

For number of jobs $j = L$

$$(\eta + \zeta) F_{ML} = \theta_1 F_{CL} + \lambda F_{M(L-1)} + \xi R_L$$

$$\implies (\eta + \zeta) F_{ML} - \theta_1 F_{CL} - \lambda F_{M(L-1)} - \xi R_L = 0 \quad (7.9)$$

The next set of balance linear equations are developed during operations in active
phase $R$. In this phase, operation is divided into active and sleep modes as earlier indicated. The balance equations are given below;

For number of jobs $j = 0$

\[(\xi + \zeta + \lambda)S_{LP} = \theta F_{CO} + \eta F_{MO} + \mu R_1\]

\[\implies (\xi + \zeta + \lambda)S_{LP} - \theta F_{CO} - \eta F_{MO} - \mu R_1 = 0 \quad (7.10)\]

For number of jobs $j = 1$

\[(\xi + \zeta + \lambda + \mu)R_1 = \theta F_{C1} + \eta F_{M1} + \lambda S_{LP} + \mu R_2\]

\[\implies (\xi + \zeta + \lambda + \mu)R_1 - \theta F_{C1} - \eta F_{M1} - \lambda S_{LP} - \mu R_2 = 0 \quad (7.11)\]

For number of jobs $1 < j \leq L - 1$

\[(\xi + \zeta + \lambda + \mu)R_j = \theta F_{Cj} + \eta F_{Mj} + \lambda R_{(j-1)} + \mu R_{(j+1)}\]

\[\implies (\xi + \zeta + \lambda + \mu)R_j - \theta F_{Cj} - \eta F_{Mj} - \lambda R_{(j-1)} - \mu R_{(j+1)} = 0 \quad (7.12)\]

For number of jobs $j = L$

\[(\xi + \zeta + \mu)R_L = \theta F_{CL} + \eta F_{ML} + \lambda R_{(L-1)}\]

\[\implies (\xi + \zeta + \mu)R_L - \theta F_{CL} - \eta F_{ML} - \lambda R_{(L-1)} = 0 \quad (7.13)\]

In order to compute the state probabilities, a program was developed in MATLAB using inputs from the generator matrix ($G$) and the state probability vector $P$ together with the normalization equation.

From the state probabilities; a number of steady-state availability, reliability and performance measures can be computed using the following equations.

From Figure 7.2(b), when the queue is full, any arriving packets are blocked. The blocking probability ($P_B$), the effective arrival rate ($\lambda_{k,e}$) and the rate at which jobs are lost ($\lambda_{k,l}$) due to blocking can be computed using equations 5.1, 5.2 and 5.3 as discussed in section 5.3. Other performance measures including $MQL, R_T$ and $u$ are similarly computed using equations, 5.4, 5.7 and 5.6 respectively.
Given service is only possible when the system is in the active state with jobs in the system, $\gamma$ may be computed using equation \ref{eq:gamma}. 

$$
\gamma = \mu \sum_{j=1}^{L} P_{i,j}; \quad i = 2; \quad j = 1, 2, \ldots, L \quad (7.14)
$$

$$
P_{SLP} = 1 - \left( \sum_{j=1}^{L} \sum_{i=0}^{N} P_{i,j} + P_{FMi,0} + P_{FCi,0} \right); \quad i = 0, 1, \ldots, N; \quad j = 0, 1, 2, \ldots, L 
$$

\begin{equation}
(7.15)
\end{equation}

Since the CH may fail in any state during operations, the probabilities of system failing in the various states may be computed using equation \ref{eq:pslp}. 

Probability the CH fails during operation $P_{FM}$

$$
P_{FM} = \sum_{j=0}^{L} \sum_{i=1}^{N} P_{i,j}; \quad i = 1; \quad j = 0, 1, \ldots, L \quad (7.16)
$$

Probability CH fails with jobs in the system $P_{FMi}$

$$
P_{FMi} = \sum_{j=1}^{L} P_{i,j}; \quad i = 1; \quad j = 1, 2, \ldots, L \quad (7.17)
$$

Probability the CH fails when the system is empty $P_{FM0}$

$$
P_{FM0} = 1 - \left( \sum_{j=1}^{L} \sum_{i=0}^{N} P_{i,j} + P_{SLP,0} + P_{FC,0} \right); \quad i = 0, 1, \ldots, N; \quad j = 1, 2, \ldots, L 
$$

Likewise, the channel may fail at any time during operations. The probability of channel failures may be computed using equations \ref{eq:pslp}, \ref{eq:pfmi} and \ref{eq:pfm0}.
Probability of the channel failing during operation $P_{FC}$

$$P_{FC} = \sum_{j=0}^{L} P_{i,j} \quad i = 0; \quad j = 0, 1, \ldots, L$$  \hspace{1cm} (7.19)

where $i = 0$ is the probability of the channel being in failed state with or without any jobs in the system.

Probability channel fails when jobs are in the system $P_{FC1}$

$$P_{FC1} = \sum_{j=1}^{L} P_{i,j} \quad i = 0; \quad j = 1, 2, \ldots, L$$  \hspace{1cm} (7.20)

Probability the channel fails when the system is empty $P_{FC0}$

$$P_{FC0} = 1 - \left( \sum_{j=1}^{L} \sum_{i=0}^{N} P_{i,j} + P_{SLP,0} + P_{FM,0} \right) \quad i = 0, 1, \ldots, N; \quad j = 1, 2, \ldots, L$$  \hspace{1cm} (7.21)
Table 7.1: Transition Matrix G

\[
\begin{pmatrix}
F_{c0} & F_{m0} & R_0 & F_{c1} & F_{m1} & \ldots & F_{m(L-2)} & R_{(L-2)} & F_{c(L-2)} & F_{m(L-1)} & R_{(L-1)} & F_{cL} & F_{mL} & R_L \\
(\theta_1 + \theta) & -\zeta & -\zeta & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-r_1 & (\zeta + \eta + \lambda) & -\zeta & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-r_2 & -\theta & -\eta & (\eta + \zeta + \lambda) & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
r_7 & -\lambda & 0 & -\theta_1 & (\zeta + \eta + \lambda) & -\zeta & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r_8 & 0 & 0 & -\lambda & -\theta & -\eta & (\zeta + \lambda + \mu) & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r_9 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & (\theta_1 + \theta) & -\zeta & -\zeta & 0 & 0 & 0 \\
r_{10} & 0 & 0 & 0 & 0 & 0 & \ldots & -\lambda & 0 & -\theta_1 & (\zeta + \eta + \lambda) & -\zeta & 0 & 0 & 0 \\
r_{11} & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & -\lambda & -\theta & -\eta & (\zeta + \lambda + \mu) & 0 & 0 & -\mu \\
r_{12} & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & (\theta_1 + \theta) & -\zeta & -\zeta \\
r_{13} & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & -\lambda & 0 & -\theta_1 & (\zeta + \eta) & -\zeta \\
r_{14} & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & -\lambda & -\theta & -\eta & (\zeta + \lambda) \\
r_{15} & 1 & 1 & 1 & 1 & 1 & \ldots & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{pmatrix}
\]

\( P \) is an \(((L+1) \times 1)\) vector given by: \( P = (0, 0, 0, 0, 0, \ldots, 0, 0, 0, 0, 0, 1) \) and

Vector “\( X \)” for the unknowns is a \((3L \times 1)\) and is given by:

\[
X = (F_{co}, F_{Fm}, R_0, F_{c1}, F_{Fm1}, R_1, F_{c2}, F_{Fm2}, R_2, \ldots, F_{c(L-2)}, F_{Fm(L-2)}, R_{(L-2)}, F_{c(L-1)}, F_{Fm(L-1)}, R_{(L-1)}, F_{cL}, F_{FmL}, R_L)
\]
7.4 Numerical Results and Discussions

In order to show the effectiveness of the proposed model, experimental results are comparatively presented in this section. Two analytical solution approaches are used to obtain results; Spectral Expansion solution technique and Kolmogorov Forward Equations. The obtained results are finally validated using a dedicated simulation program developed using C++. The metrics used to evaluate the performance of the proposed model includes MQL, response time, throughput, and blocking probability. These are observed when the system is subjected to numerous operating conditions, including different finite queue lengths, varying node and channel failure rates, in addition to changing number of sources.

7.4.1 Parameter Choice

In this section, parameters used for the experiments are presented. This study utilized 25 to 35 nodes for optimal CH operation. Considering a generic system, use is made of parameters presented previously in section 5.5.1. Service rate $\mu = 300/hr$, channel restoration rate $\theta = 0.6/hr$ and CH repair rate of $\eta = 0.5/hr$ are kept the same throughout the study. Table 7.2 provides a summary of other parameters used.

Table 7.2: Used parameters and values

<table>
<thead>
<tr>
<th>Figure</th>
<th>$\lambda/hr$</th>
<th>$\xi/hr$</th>
<th>$\zeta/hr$</th>
<th>Queue size(L)-Packets</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3 &amp; 7.4</td>
<td>0-12</td>
<td>0.001</td>
<td>0.001</td>
<td>100</td>
</tr>
<tr>
<td>7.5 &amp; 7.6</td>
<td>0-9</td>
<td>0.001</td>
<td>0.001</td>
<td>10,30,50</td>
</tr>
<tr>
<td>7.7(a) &amp; 7.7(b)</td>
<td>8</td>
<td>0.001</td>
<td>0.0001-0.01</td>
<td>100</td>
</tr>
<tr>
<td>7.8(a) &amp; 7.8(b)</td>
<td>8</td>
<td>0.0001-0.01</td>
<td>0.001</td>
<td>100</td>
</tr>
</tbody>
</table>

7.4.2 Results and Discussions

In the first set of experiments, the effects of varying the arrival rate on the overall system performance are established. In this experiment a constant number of
sources, 25, 30 and 35 are maintained throughout each set of experiments as the arrival rate is varied from $\lambda = (1 - to - 9)\text{pkt/hr}$. The obtained system performance results are analysed and presented in terms of MQL, response time, and throughput using Figures 7.3(a), 7.3(b), and 7.4. In Figure 7.3(a) MQL is presented against varying arrival rates. The results clearly show that when relatively low arrival rates are considered, systems exhibit close MQL performance as a result of low server utilization hence shorter job queuing time. However as the arrival rate is increased, the systems with fewer sources perform better.

In Figure 7.3(b), response time is presented against arrival rate. When light loaded systems are considered, close response time performances are observed. However, the difference become significant when the systems are heavily loaded. These findings can be used by engineers for planning WSN coverage during deployment.

Throughput analysis is presented in Figure 7.4. It is observed that throughput in this particular scenario is nearly same to the arriving packets. This is because the queue capacity is high (i.e. $L = 100$ packets). Almost all the packets arriving in the system are processed and forwarded to the sink. However, highly loaded systems are observed to give higher throughput.
In the next set of experiments, 30 sources are used to analyse the system performance when subjected to changing queue capacities. For every experimental run, a different queue length is chosen and fixed as the arrival rate is varied from $\lambda = 1, 2, \ldots, 9/hr$. The other parameters are kept constant as given in table 7.2. Performance results are analysed in terms of MQL, response time, throughput and blocking probability.

In Figure 7.5(a), MQL is gradually increased with the increase in arrival rate. After arrival rate of $\lambda = 5/hr$, operating with low queue size of $L = 10$ packets exhibit signs of performance degradation as the arrival rate is additionally increased. However, steep MQL gradients are observed with larger queue capacities as arrival rate is further increased. Depending on the queue sizes used, larger sizes enable the system to hold more packets and shows signs of performance degradation at higher arrival rates. In the case of response time, Figure 7.5(b), lower queue sizes ensure a faster response is achieved. On the contrary, larger queue sizes keep the packets for longer times in the queue hence the reason for delayed response time.

From Figure 7.6, throughput is observed to vary linearly with an increase in arrival rate. However, past $\lambda = 5/hr$, low queue size $L = 10$ packets start to exhibit increased discrepancies between solution approaches as a sign of system
instability. A similar behaviour is observed with $L = 30$ packets past arrival rate of $\lambda = 8/hr$. Therefore, this study confirms that larger queue capacity enables high throughput as a compromise to increased response time and delay in WSN systems hence may be used for performance optimisation. Finally in Figure 7.6(b), it is observed that when queue size of $L = 10$ packets is used; the system starts to block packets very early and the blocking probability increases faster and sharply as the arrival rate is increased further. The same trend is observed when $L = 30$ packets past arrival rate of $\lambda = 8/hr$. However, when $L = 50$ packets is used, the blocking probability is kept minimal nearly for the whole experiment period. In summary, the choice of queue capacity is a compromise of desired QoS in relation to throughput, response time and queue length, which may be determined by application demands.

The last set of experiment analyses effects of channel and node failures on system throughput and MQL. In both cases, 25 sources and arrival rate of $\lambda = 8/hr$ is maintained. When analysing channel failures, the node failure rate of $\xi = 0.001/hr$ is maintained as the channel failure rate is varied between $\zeta = 0.0001, 0.0005, \ldots, 0.1/hr$. The same principle and similar values are used alternately when analysing node failures. In both cases, the other parameters are...
(a) Throughput Vs Arrival rate  
(b) Blocking Probability Vs Arrival rate

Figure 7.6: Effects of Variable Buffer Capacity on Throughput and Blocking Probability

kept the same as given in table 7.2. Obtained results are presented comparatively in Figure 7.8.

(a) MQL Vs Channel Failure  
(b) Throughput Vs Channel Failure

Figure 7.7: Effects of Variable Channel Failure Rate on MQL and Throughput

Figures 7.7(a) and 7.8(a) illustrate the effects of variable channel and node failure
rates on the system MQL respectively. In both cases, failure rates of below $\xi = \zeta = 0.001/hr$ are preferred for better performance. Operating with higher failure rates destabilizes the system and results into degraded overall performance. For illustration purposes, Figures 7.7(b) and 7.8(b) show how these failures reduce system throughout when both failure rates are higher than $\zeta = \xi = 0.001/hr$ respectively.

Figure 7.8: Effects of Variable Node Failure Rate on MQL and Throughput

In the next category of experiments, presented and analysed are results for operating in the various states. The results shown here are a subset of those presented in Figure 7.3. The first experiment, Figure 7.9(a) illustrates system operation modes in phase $R$ of the active state. It is observed that in the initial stages when arrival rate is small, the CH spends more time in the sleep state in comparison to active state with jobs. However, time spent in the sleep state linearly reduces as active time with job increases linearly with the increase in arrival rate. As the arrival rate is increased further, active time with jobs surpasses sleep time when state probability is 0.5. The arrival rate at this point is dependent upon the number of sources used. Higher sources reach this mark at low arrival rate while lower sources reach it last at higher arrival rates. Beyond this point, more increases in arrival rate decrease sleep time even more as active time with jobs is increased further. This analysis is significant in the identification of appropriate
time to enter the sleep state for energy conservation purposes.

Figure 7.9: Effects of Variable Channel and Node Failure Rate on MQL and Throughput

Figure 7.9(b) shows a pattern of channel failure during operations. At low arrival rates, the probability of the channel failing without jobs is higher compared to periods when the arrival is higher. The opposite is the same for channel failures with jobs in the system. However, mean failure time during operation is nearly the same throughout the operation period. Figures 7.10(a) and 7.10(b), on the other hand, illustrates node failure when loaded with jobs and when the system is empty. Unlike channel failures, sharp increase and decrease are observed in the case of node failures with and without jobs respectively. As arrival rates are increased, the failures increase of decrease in a diminishing pattern before reaching a maximum and minimum after which further arrival rates have no effects in the respective cases.

Finally, Figure 7.11 shows the timing relations between active times, active times with jobs and sleep times when observed under different queue capacities. Like noted earlier in Figure 7.9(a), sleep time is higher at low arrival rates and decreases linearly as arrival rate is increased. This analyses can also be employed for...
Figure 7.10: Effects of Variable Channel and Node Failure Rate on MQL and Throughput establishing the appropriate times to enable sleep operations in order to conserve the limited CH energy.

Figure 7.11: Active operation States
7.5 Chapter Summary

In addition to the limited resources that constrain WSN performance, wireless communication links used are further affected by the prevailing environmental conditions which vary from one application habitat to another. For agricultural monitoring, terrain and changing weather conditions can be detrimental to the communication links to the point of disconnections. On the other hand, urban WSNs suffer from clear line of site and interference from other radio frequencies that may occasionally cause link failures. In smart industry applications, WSNs suffer from noise-related interferences that eventually degrade overall network performance. It is therefore, clear that the low-power radio links used in WSNs can be very unpredictable with side effects varying from one application environment to another.

In this study, the model of a clustered WSN with unreliable links in addition to the limited memory capacity and frequent node failures is presented. The model is then used to evaluate system performance when subjected to node and channel failures, repairs and restoration. We also incorporate, and analyze effects caused by the widely used sleeping schedules on overall network performance.

To solve the performability models, two analytical approaches; system of linear simultaneous equations and spectral expansion exact solution techniques were employed and validated using a simulation program. Results obtained from the three approaches closely match with discrepancy below 2%. The obtained results were then analysed in two categories as; system performance and operation state probabilities. From the results, a mechanism that can be used to control sleep scheduling based on network traffic is proposed. This study may also be used as a guide for developing a network deployment, and a system optimization tool for better performance.
Chapter 8

Modelling and Evaluation of Clustered Wireless Sensor Network Energy Consumption

8.1 Introduction

Limitation of power in WSN has remained critical hence optimizing the limited power is significant for prolonging network lifetime and providing good QoS. Several approaches are proposed in literature for maximizing WSN lifetime. These include use of energy efficient MAC layer protocols implementing Active/Sleep schedules [Valera et al., 2014] [Van Dam and Langendoen, 2003], energy efficient routing protocols [Tyagi and Kumar, 2013] [Abdulaleem and Ma, 2014] and implementation of charging schemes using on-board solar systems [Corke et al., 2007] and lately the introduction of a multi-node wireless energy transfer system based on magnetic resonant coupling [Xie et al., 2015]. In other areas, independent performance and availability/reliability studies have incorporated various power saving schemes in order to further evaluate and enhance WSN performance and availability [Chiasserini and Garetto, 2006]. Bearing in mind, the drawbacks of using pure performance and availability studies, energy saving prototypes developed from such models are likely not to provide desired results when subjected
to WSN systems with failures and repairs/replacement. In order to resolve these concerns, it is important to consider integrated performance and availability models for developing energy saving models for WSNs.

This chapter presents an analytical modelling approach for comparing mean energy consumption in the various sensor node operative states and uses the prototypes developed to evaluate energy consumption of the performability models developed in chapter 7. To the best of our knowledge, this is the first study to present an analytical modelling approach for comparing average energy consumption of various configurations in the presence of failures and repairs/replacement.

Over the years several approaches and models have been proposed for energy evaluation and performance of WSNs [Chiasserini and Garetto, 2006] [Zhang and Li, 2012]. In [Chiasserini and Garetto, 2006], authors classified energy costs in relation to the state of the system and routing dynamics. Transferring data packets between two nodes in the network involves both transmission and receiving energy. In addition to the transceiver electronics and processing energy spent when receiving, transmitting a packet requires energy amplification. This is assumed to be proportional to the square distance between the communicating nodes. In the same study energy consumed at each node due to the operational state of the sensor and energy spent during transition from sleep to active state are also considered. However, the energy consumed in idle mode and while switching from active to sleep mode has not been considered. Similar approaches were used by [Zhang and Li, 2012], [Odey and Li, 2012], [Zhang et al., 2011], [Jurdak et al., 2010]. In addition, authors in [Jurdak et al., 2010] present energy costs for listening, sensing, sleeping and switching between the operation states. Studies in both [Zhang and Li, 2012] and [Zhang et al., 2011], have extended the model used in [Chiasserini and Garetto, 2006]. However, in both cases energy spent in idle and sleep states of the model used is not given. In full active phase, the energy cost for receiving data packets is also not considered. In another study [Odey and Li, 2012], authors reiterate the significance of including energy costs for being in sleep and idle states.

In a more recent study [Chan et al., 2015], authors have proposed a novel framework enabling an adaptive duty cycling scheme for sensor networks that take into
account the operating duty cycle of the node, and application-level QoS requirements. Using the CTMC model developed, they derive key QoS metrics including loss probability, latency as well as power consumption, as functions of the duty cycle. Finally, they formulate and solve the optimal operating duty cycle as nonlinear optimization problem using latency and loss probability as the constraint. However, this study does not consider system performance effects resulting from node and channel failures.

8.2 Energy Consumption Model

In order to model energy consumption of a WSN node and network, the possible power consumption states are first identified and defined. Figures 7.2(a) and 7.2(b), are Markov models representing CH operating states. In these models, the system state at time \( t \) is described using a pair of integer valued random variables \( I(t) \) and \( J(t) \). The operative state \( I(t) \) represent node failed states, channel failed states, and the normal working period of the CH while \( J(t) \) represent the number of jobs in the system. At a time \( t \), the system state may therefore be described by \( Z = [I(t), J(t)], t \geq 0 \). \( Z \) therefore becomes an irreducible Markov process on a lattice strip (QBD) used to model the system whose state space is \( (0, 1, \ldots, N) \times (0, 1, \ldots, L) \). Here \( N \) and \( L \) represent the number of operative states, and system’s total job holding capacity respectively. Using the model, we identify possible energy transition events from the states. To compute energy measures steady state probabilities are considered together with transition energies into and out of the states. The following subsections present detailed discussions on various factors causing energy consumption.

8.2.1 Transmission Energy

Transmission energy is the energy spent when transmitting data packets from a given state \( Z(t) = [I(t), J(t)] \) at time \( t \) and can be calculated by the probability of being in state \((i, j) \times \) service rate (transmission) \( (\mu) \times \) energy required to
transmit one data packet. Letting \( Y_{i,j} \) = Energy spent during transition from state \((i, j)\) to state \(i, j - 1\) caused by the transmission of one packet from the buffer, then \( Y_{i,j} \) can be computed using equation 8.1.

\[
Y_{i,j} = P_{i,j} \times \mu \times e_{tx}; \quad i = 0, 1, \ldots, N; \quad j = 0, 1, \ldots, L \quad (8.1)
\]

Where:

- \( P_{i,j} \) - is the probability of being in state \((i, j)\) at time \(t\).
- \( \mu \) - is the variable data packet service rate
- \( e_{tx} \) - is the energy required to transmit one data packet (Joules/packet)

In the case of a single server system, the mean energy required to transmit data packets \( (E_{tx}) \), from a state may therefore be computed by summing up energy consumption for individuals transmissions.

\[
E_{tx} = \sum_{j=0}^{L} \sum_{i=0}^{N} P_{i,j} \times \mu \times e_{tx}; \quad i = 0, 1, \ldots, N; \quad j = 0, 1, \ldots, L \quad (8.2)
\]

where \( i \) and \( j \) represent operative state of the system and the number of jobs in the queue respectively.

Assuming a multi server system with \( W \) parallel servers, then only a maximum number of jobs equivalent to \( W \) servers can be serviced at one go if all the servers are operational. The formula for the mean energy given in equation 8.2 then changes to equation 8.3.

\[
E_{tx} = \min(j, w) \sum_{j=0}^{L} \sum_{i=0}^{N} P_{i,j} \times \mu \times e_{tx}; \quad i = 0, 1, \ldots, N; \quad j = 0, 1, \ldots, L \quad (8.3)
\]

where \( (w = 1, 2, \ldots, W) \) and \( j \) are the number of servers and jobs in the system respectively. Here the minimum of \( j \) and \( w \) becomes the number of jobs in service at any given time.
8.2.2 Receiving Energy

This is the energy consumed by the transceiver electronics when receiving a data packet. Since the node can only receive one data packet at a time, the energy required to receive a data packet while in a given operating state \((i, j)\) at time \(t\) may be given by multiplying the state probability by packet arrival rate and energy consumed to receive one data packet. Letting \(D_{i,j}\) be energy spent receiving data packets from state \((i, j)\) to state \((i, j + 1)\), caused by the arrival of a data packet in the system then \(D_{i,j}\) can be computed using equation 8.4.

\[
D_{i,j} = P_{i,j} \times \lambda \times e_{rx}; \quad i = 0, 1, \ldots, N; \quad j = 0, 1, \ldots, L \tag{8.4}
\]

where:

- \(P_{i,j}\) is the probability of being in state \((i, j)\) at time \(t\)
- \(\lambda\) is the variable packet arrival rate
- \(e_{rx}\) is the energy required to receive one data packet (Joules/packet)

The mean energy required to receive data packets \(E_{rx}\) in receiving state may therefore be computed using equation 8.5.

\[
E_{rx} = \sum_{j=0}^{L} \sum_{i=0}^{N} P_{i,j} \times \lambda \times e_{rx}; \quad i = 0, 1, \ldots, N; \quad j = 0, 1, \ldots, L \tag{8.5}
\]

8.2.3 Wake-Up Energy

This is the energy required to transit from sleep state into full active state \(E_{UP}\) at the arrival of a new job. This energy constitutes energy required to wake-up the node to be able to receive an incoming data packet. The energy required to wake-up the sensor node can therefore be computed using equation 8.6.

\[
E_{UP} = P_{i,j} \times \lambda \times e_{up}; \quad i = \text{Sleep state}; \quad j = 0 \tag{8.6}
\]
From [Jurdak et al., 2010], the power required to switch the radio from sleep back to active period is computed using equation 8.7.

\[ e_{up} = \frac{(I_{active} - I_{sleep}) \times \beta \times V}{2} \]  

(8.7)

where \( \beta \) accounts for the time required returning to active mode and the factor 2 accounting for switching back to sleep mode from active mode. \( V \) is the applied voltage and \( I_{active} \), and \( I_{sleep} \) are currents drawn in active and sleep modes respectively.

### 8.2.4 Sleep State Energy

While in sleep mode, several transceivers offer different energy levels [Odey and Li, 2012], the transceivers differ on the number of circuitry switched off and in the associated recovery times and start-up energy. An example is the case of a complete shut down of the transceiver where the starting energy has to include initialization and configuration of the radio as opposed to light sleep mode requiring restarting of a little circuitry since operations and configurations are maintained. The sleep energy may be computed using the equation 8.8.

\[ E_{sp} = P_{i,j} \times e_{sl} \times t_{sl}; \ i = \text{Sleep state}; \ j = 0 \]  

(8.8)

Where \( e_{sl} \) and \( t_{sl} \) are consumed power (Joules/sec) and time in (seconds) taken in sleep state respectively.

### 8.2.5 Idle State Energy

In some cases, the nodes may be idle waiting for packet arrival in situations where sleep mechanism is not implemented or where sleep scheduling is achieved through other MAC protocols, e.g. using adaptive sleep schedule MAC protocols [Zhou et al., 2011]. In such cases, the energy spent in the idle state is significant enough hence should be considered in the overall energy consumption evaluation.
Like the case with sleep state, the time taken in idle state contributes to the amount of energy consumed. The mean idling energy may be computed using equation 8.9.

\[ E_{ID} = \sum_{t=0}^{T} P_{i,j} \times e_{id} \times t_{id}; \quad i = \text{idle state}; \quad j = 0; \quad t = 0, 1, 2, \ldots, T \]  

(8.9)

Where \( e_{id} \) and \( t_{id} \) are consumed power and time taken in idling state respectively. \( t_{id} \) is an instance of time varied from \( t - t_0 - T \).

### 8.2.6 Node Failed State Energy

In the event of node failure, the node is assumed to continue receiving data packets as long as the buffer is not full hence incurring some energy costs. In addition, restarting a node may require energy. The resulting energy expended in this state therefore comprises receiving and rebooting energy and its mean may be computed using equation 8.10.

\[ E_{FM} = \sum_{j=0}^{L} P_{i,j}(\eta e_{nrb} + \lambda e_{Rx}); \quad i = FM; \quad j = 0, 1, 2, \ldots, L \]  

(8.10)

Where \( \eta \) and \( e_{nrb} \) are node repair rate and the subsequent energy spent rebooting the node after repair. \( FM \) is the node failed state.

### 8.2.7 Channel Failed State Energy

During operations, the channel may also fail hindering data packet transfer between the CH and the rest of the network. In such circumstances, the CH will not be able to transmit or receive any data packets. Instead, it is assumed that various nodes intending to transfer data to the CH will continue waiting for the restoration of the channel after which they will contend for a chance to transmit
their data packets. The CH on the other hand will continue to hold the data packets in its queue until the channel is restored after which it will forward the data packets to the sink. The mean energy spent for channel restoration may be computed using equation 8.11.

\[ E_{FC} = \sum_{j=0}^{L} P_{ij} \theta e_{crb} ; \quad i = FC; \quad j = 0.1.2.\ldots,L \]  \hspace{1cm} (8.11)

Where \( \theta \) and \( e_{crb} \) are channel restoration rate and the subsequent energy in (Joules/sec) consumed during channel restoration. \( FC \) is channel failed state.

### 8.3 Case Study Model

In this section, using energy computations given in section 8.2, specific mean energy spent in the various operative states of the proposed performability models given in Figures 7.2(a) and 7.2(b) are derived. The power consumption results obtained from the two models are eventually used to compare power saving differences between models with active/sleep implemented and those without.

The models above are similar with a slight difference in state \( R \) in which Figure 7.2(a) never gets to sleep mode, instead it stays idle for the periods when there are no jobs in the system. On the other hand Figure 7.2(b) presents a case in which the system always goes into sleep mode whenever there are no data packets in the system to be served. In order to compute the energy consumption for the models, first steady state probabilities for the various operating states are computed as detailed in section 8.2 above. The energy consumed in each state is then computed as detailed in the following subsections.

#### 8.3.1 Mean Energy Spent in the Active Phase \( R \)

The Energy spent in this phase is denoted \( E_{AR} \). In this phase, the CH is operating normally. Transitions between active/sleep modes are fully dependent
upon the availability of the jobs in the system as seen in the above sections. The composition of energy spent in this phase includes:

a.) Energy Spent while receiving data packets - $E_{rx}$
b.) Energy spent while transmitting data packets - $E_{tx}$
c.) Energy spent transiting from sleep to full operation mode - $E_{UP}$
d.) Energy spent restarting the CH after node and channel failures $E_{FM}$ and $E_{FC}$ respectively
e.) Energy spent in sleep state $E_{sp}$

Since the expected number of jobs in phase $R$ may vary over time, the energy spent also varies accordingly. Taking into account energy consumed rebooting the system after node failures, the mean energy spent during CH operations may be expressed as:

$$E_{AR} = \sum_{j=0}^{L} E_{tx} + E_{rx} + E_{sp} + E_{UP} + E_{FM} \text{ Joules; } \ i = 2; \ j = 0, 1, 2, \ldots , L$$

(8.12)

In the absence of failures this may reduce to:

$$E_{AR} = \sum_{j=0}^{L} E_{tx} + E_{rx} + E_{sp} + E_{UP} \text{ Joules; } \ i = 2; \ j = 0, 1, 2, \ldots , L$$

(8.13)

### 8.3.2 Mean Energy Spent in Sleep Mode

Depending on the application area, the CH may be configured to either go into deep or light sleep or even dynamically choose between the two depending upon the state of the traffic [Jurdak et al., 2010]. In such cases, a significant power consumption variation in the states is eminent. Most of the available radio
transceivers already have the variable power consumption for sleep states incorporated as deep and light sleep states. In order to evaluate the power levels in the proposed models, equation 8.8 is altered as given in equations 8.14 and 8.15 below for computation purposes.

\[ E_{sp-deep} = \sum_{t=0}^{T} P_{i,j} (t_{sl1}e_{sl1} + e_{up} + e_{rx}) \text{ Joules; } i = 2; j = 0; t = 0, 1, 2, \ldots, T \]  
(8.14)

\[ E_{sp-deep} = \sum_{t=0}^{T} P_{i,j} (t_{sl2}e_{sl2} + e_{up} + e_{rx}) \text{ Joules; } i = 2; j = 0; t = 0, 1, 2, \ldots, T \]  
(8.15)

Where \( e_{sl1} \) and \( e_{sl2} \) are energy spent in deep and light sleep states respectively. From the two equations, it is possible to derive the formula for computing a dynamically changing sleep state. For the model of Figure 7.2(a), it is important to note that the distribution of the sleep time \( t_{sl} \) is similar to that of idling time in the model of Figure 7.2(b).

### 8.3.3 Mean Energy Spent in Idle Mode

In order to account for the energy expended while the system is operating in idling state, the computation formula is given considering energy spent while idling together with the energy used to receive the first data packet.

\[ E_{ID} = \sum_{t=0}^{T} P_{i,j} (e_{id}t_{id} + \lambda e_{rx}) \text{ Joules; } i = 2; j = 0; t = 0, 1, 2, \ldots, T \]  
(8.16)
8.3.4 Residual Energy

With all possible energy expenditure in all the states known, letting the initial energy be $E_{in}$, then the residual energy $E_{rsd}$ of the CH may be computed using the equation provided below;

$$E_{rsd} = E_{in} - E_{FR} \text{ Joules} \quad (8.17)$$

The residual energy may then be used to determine the levels necessary for CH operations.

8.4 Numerical Results and Discussions

Numerical results presented in this section show the effectiveness of the energy evaluation models developed for a typical wireless CH operations based on the radio sleep schedule mechanism. The parameters used in this numerical study are mainly taken from the data sheets for the existing wireless motes. In tables 8.1, 8.2 and 8.3, parameter specifications for radio transceivers, Micro-controllers and Sensors are presented. In this study Telos Mote radio transceiver (CC2420) and controller (MSP430F4794) specifications were used for the experiments [Texas, 2003] [Texas, 2011]. In order to evaluate the overall CH energy consumption, this study considers energy consumed when receiving and transmitting data packets in addition to the energy consumed while transiting between the operative states.
### Table 8.1: Transceiver Parameter Specifications

<table>
<thead>
<tr>
<th>Mote</th>
<th>Transceiver</th>
<th>Idling ($IDW$)</th>
<th>Deep Sleep ($ESDW$)</th>
<th>Transmitting ($ETxW$)</th>
<th>Receiving ($ETxW$)</th>
<th>Full to Reduced Mode ($FNW$)</th>
<th>Sleep to Full Operations ($EUFW$)</th>
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<td>Telos</td>
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<td>0.05742</td>
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<td>Mica2</td>
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<td>0.000003</td>
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<tr>
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<td>0.0041976</td>
</tr>
</tbody>
</table>

### Table 8.2: Micro-controller (Processor) Parameters Specifications

<table>
<thead>
<tr>
<th>Mote</th>
<th>Controller</th>
<th>Idling ($EIDW$)</th>
<th>Deep Sleep ($ESDW$)</th>
<th>Running (W)</th>
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</tr>
<tr>
<td>Imote2.0</td>
<td>Intel PXA271</td>
<td>0.1395</td>
<td>0.001755</td>
<td>0.198</td>
</tr>
</tbody>
</table>

### Table 8.3: Sensor Parameter Specifications

<table>
<thead>
<tr>
<th>Mote</th>
<th>Type</th>
<th>$E_{off}$</th>
<th>$E_{on}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telos</td>
<td>DS1820</td>
<td>0</td>
<td>0.003</td>
</tr>
<tr>
<td>Mica2</td>
<td>DS1820</td>
<td>0</td>
<td>0.003</td>
</tr>
<tr>
<td>Imote2.0</td>
<td>TMP175</td>
<td>0.000003</td>
<td>0.001</td>
</tr>
</tbody>
</table>
To begin with, the experiments are done using the same parameters presented in chapter 5. During each experiment, a constant number of sources, 25, 30 or 35 are maintained throughout the experiments. In all the experiments, use is made of a fixed queue length of $L = 100$ packets while arrival rate is varied from $\lambda = 1 - 20$ packets/hr.

From the models of Figure 7.2, transmission energy consumption is only possible while operating in state $R$. However, in addition to receiving packets in state $R$, the CH may also consume energy while receiving in state $FM$ as illustrated in Figure 8.5(b). The overall receiving energy is therefore the sum of energy consumed while receiving data packets in both states. During channel-failed state, the CH does not spend energy to either received or transmitted data packets. Figures 8.1(a) and 8.1(b) present energy consumed while transmitting and receiving data packets respectively. In both cases, energy increases linearly with increase in arrival rate from low values. As the arrival rates are increased further, energy consumption levels start to exhibit a reducing effect and finally remain constant irrespective of increments of arrival rates. The constant consumption level is an effect of the buffer size and implies that the buffer is full hence not able to admit more arriving data packets.

While in sleep state, the CH consumes the lowest power to maintain circuitry ready for wakeup in the event of data arrival. In addition, the CH spends energy during wakeup. Figures 8.2(a) and 8.2(b) illustrate sleep and wakeup energy consumptions respectively. In both figures, higher energy is consumed at low arrival rates. As the arrival rate is increased, the CH becomes more busier thereby reducing sleep time. Sleep time eventually becomes negligible at higher data arrivals rates. Consequently, both sleep and wakeup energies fall as illustrated in both figures. It is also observed that wakeup energy is much higher than sleep energy.

In order to evaluate the effectiveness of the energy saving model 7.2(b), we compare its energy consumption with that of the model implementing idle state 7.2(a). The differences in energy consumption are determined by comparing idle and sleep state energies as illustrated in Figures 8.3(a) and 8.3(b) respectively. The overall sleep state energy for the CH life time is computed by summing up energies con-
Figure 8.1: Energy Spent Transmitting and Receiving Data Packets

(a) Transmitting Energy  (b) Receiving Energy

Figure 8.2: Energy Spent Sleeping and Waking up from sleep state

(a) Sleeping Energy  (b) Wake-up Energy

sumed during wakeup and sleeping times. In both cases, the consumptions are higher at low arrival rate but reduce with the same gradient and finally becoming
almost the same at higher arrival rates. However, idling energy remains higher at all levels of arrival rate as illustrated in Figure 8.3(a). In this figure, we observe a similar trend where energy saving is higher at low arrival rate and reduces to negligible levels at higher arrival rates. In Figures 8.4(a) and 8.4(b), total energy consumed when operating with sleep and Idle states enabled are illustrated. In both figures, low energy is consumed during low arrival rates. This increases gradually with increasing arrival rates. Similar to transmission and receiving energies, total energy consumed in both states becomes constant after reaching some point determined by buffer size limitations. If the buffer sizes are known, this observation may be used by the designers to optimize energy consumption of sensor nodes and the overall WSN.

Finally, Figure 8.5(b) illustrates the energy consumption for receiving data packets in node failed state $FM$. Again, more energy is consumed at low arrival rates since the buffer is always empty while operating in this state hence more jobs can be stored awaiting repair completion. As arrival rates are increased, more data packets are stored during operations hence limiting the number of packets that can be accepted during node failure. The trend continues with increasing arrival rates.

Figure 8.3: Idling and Overall Sleep State Energy
(a) Total Energy Expended Operating with Idling

(b) Total Energy Expended Operating with Sleep

Figure 8.4: Total Energy Expended during Full operation with Sleep & Idle states

rate and finally becoming very small at higher arrival rates.

(a) Total Energy Saving in Sleep State

(b) Node Failed state energy

Figure 8.5: Total Energy Saving and Energy Expended in Node Failed states
To summarise, in all the cases observed, more sources lead the CH to consume higher amount of energy compared to fewer sources. Thus, the CH depletes its energy faster when many sources are involved. This becomes a trade off when configuring network coverage and may be significant in optimization studies.

In this model, we note that frequent CH wakeup may result into high-energy consumption negating the purpose of entering sleep mode. A mechanism of controlling when to enter sleep state is therefore necessary. In order to determine control levels, we consider operations in phase $R$ as given in Figure 7.11. During low arrival rates, sleep time is higher, this is a perfect time for the system to enter sleep state at the end of service for the last job and only resume when a new arrival occurs. As soon as mean active time with jobs become greater than mean sleep time due to increased traffic intensity, the system’s mean active time with jobs quickly gets high hence higher energy consumptions observed as arrival rates are further increased. In Figure 7.11, considering all the observations made with buffer capacities of $L = 10, 50$ and $100$ packets, if 30 nodes are deployed, then operations below arrival rates of $4 – 5 \text{ pck/hr}$ is preferable. Above arrival rates of $6 \text{ pck/hr}$, the overall saving is very small and at times negative hence the system should be kept in the idling state. In comparison, the observations made in Figure 8.5(a), indicate minimal saving hence keeping the system in idling state is preferable in order to eliminate frequent wakeup energy consumption. Table 8.4 gives a summary of proposed operation levels beyond which the CH should not enter sleep mode based on service of last data packet.

Based on the main objective of conserving WSN energy using On-demand sleep scheduling this study identifies the need for setting operation levels that minimize wastage of the limited energy. The proposed model can further be used by designers to developed planning, deployment, and optimization tools by considering performability findings outlined.
Table 8.4: Proposed limits for regulating sleep schedules

<table>
<thead>
<tr>
<th>Sources</th>
<th>Proposed Arrival Rate (λ)</th>
<th>Proposed MQL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Upper Limit</td>
</tr>
<tr>
<td>25</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>30</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>35</td>
<td>4.2</td>
<td>6</td>
</tr>
</tbody>
</table>

8.5 Chapter Summary

Energy conservation of WSNs continues to attract research work in various application environments due to the limitation of this resource. Despite numerous proposals made, integrated performance and availability studies of WSNs remain a green research area. Likewise, energy models incorporating integrated performance and availability studies in the presence of failures are lacking. In this chapter, models used for steady state analysis of performance and availability of WSNs presented in the preceding chapters are further considered for evaluating energy consumption of such systems. More precisely, consideration is given to energy consumption of the CH.

Initially, mathematical models for obtaining energy consumption from the various operative states are developed. To evaluate energy consumption in the various states, mean values obtained at steady state are used to compute consumed energy in the individual states. Summing up all the consumptions in the various states, mean overall CH consumption is obtained. Using the model, a case study of a clustered WSN is considered for energy evaluation under different conditions. Results obtained are further validated using simulation results. The two sets of results comparatively presented in all the figures closely follow each other with a discrepancy below 0.1 %.

Using the obtained results, we determine optimal sleep operation range suitable for On-demand sleep schedule technique that employs a second low power radio transceiver to regulate sleep based on packet arrivals. Above the optimal levels, we propose the system should be left to idle based on high traffic intensities. It is also possible to consider other sleep scheduling techniques if found appropriate.
for conserving energy at high traffic intensities.
Chapter 9

Modelling Arrival Distributions in Wireless Sensor Networks

9.1 Introduction

The emergence of WSNs has significantly facilitated human interaction with the physical environment. Depending on the area of application and data collection techniques employed, the distribution pattern of arriving data packets at the CH from the sensing nodes may vary considerably. In addition, various MAC layer protocols have been proposed, each influencing packet transmission differently, thereby altering arrival patterns at the CH.

In order to characterize arrival distribution of packets in WSNs, numerous studies compare Quantile-Quantile (Q-Q) plots with empirical data and draw conclusions based on similarities [Chiasserini and Garetto, 2006], [Wang et al., 2012]. However, simple eye checks can easily lead to drawing incorrect evaluation of results hence the need for more statistical analysis of the empirical data to establish the best theoretical distributions for packet arrival rates.

In addition to the common practice of comparing empirical data with the theoretical exponential distributions of Q-Q plots based on simple eye checks, this chapter presents a detailed study of the possible distributions to prevent drawing
incorrect conclusions. In this study, the use of estimated maximum likelihood parameters of empirical distributions is made to generate theoretical distributions. By further conducting Kolmogorov-Smirnov Test for each generated data series, a possible corresponding theoretical distribution is obtained. In order to acquire realistic results, the study considers properties of the commonly used CSMA/CA and TMAC protocols that may result in different arrival distribution at the CH.

The remainder of the chapter is organised as follows: Section 9.2 presents a detailed description of the proposed model followed by mathematical models of a WSN arrival distribution processes in section 9.3. In section 9.4, the experimental test bed is presented followed by a detailed discussion of the obtained results in section 9.5. The chapter summary is finally presented in section 9.6.

### 9.2 Model Description

In this section, a generic model based on the IEEE/Zigbee 802.15.4 standards is proposed. The model assumes deployment of a WSN with homogeneous sensor nodes directly connected to the CH. The pattern of arriving data packets at the CH is similar in all clusters hence the study focuses on the analysis of the inter-arrival distribution of packets at a single CH. The resulting distribution pattern may therefore be useful for modelling performance and availability studies of WSNs. The reference network topology is presented in Figure 9.1. In this arrangement the CH near to the sink connect directly to the sink and also acts as a router to other CHs far away from the sink. The CH operation remains as discussed in section 2.5. In order to justify the use of specific distribution patterns, the general practice has been to compare WSN empirical data with theoretical exponential distribution in a Q-Q plot [Chiasserini and Garetto, 2006].

In [Wang et al., 2012], authors compared theoretical exponential results with empirical results obtained from an experiment test bed where all the nodes use TinyOS CSMA/CA MAC protocol. Sensor nodes forward their generated packets through intermediary nodes to the final node from where they record packet inter-arrival times. The empirical CDF of the inter arrival time are next plotted
against the exponential distribution model. The results reveal that exponential distribution closely models the inter arrival rates except for the low periodic traffic. However, in this work, only event based application scenarios are considered. Furthermore, relay traffics are not characterized and the effects of MAC protocols other than CSMA/CA are not analysed.

In this study, we investigate and establish the best-fitting inter arrival distribution both at the relay nodes and at the CH. Initially, we identify and characterize WSN applications in order to determine appropriate data delivery models that mainly depend on application requirements. Based on the delivery models, a simulation test bed using Castalia, running on OMNET++ platform is set and investigations performed using Kolmogorov-Smirnov test as detailed in the subsequent sections.

9.3 Modelling WSN Arrival Processes

Arrival processes are well covered in the literature. Bernoulli and Poisson processes have been used successfully to model arrivals in various WSN application environments. However, the diversity of application areas with varying requirements may imply that different application environments define dissimilar arrival processes. It is possible to characterise WSN arrival processes as either discrete-
time or continuous-time stochastic processes. As described in [Donald et al., 2013], a stochastic process is defined as a mathematical abstraction of an empirical process whose development is governed by probabilistic laws (examples are Poisson and Bernoulli processes). This may also be expressed from probability theory as a family of random variables \( \{X(t), t \in T\} \), defined over some index set or parameter space \( T \) representing a time range. \( X(t) \) denotes the state of the process at time \( t \). Depending on the nature of the time range, the process is classified as a discrete-parameter or continuous-parameter process as follows:

1. If \( T \) is a countable sequence, for example, \( T = \{0, 1, 2, 3, \ldots\} \), then the stochastic process \( \{X(t), t \in T\} \) is said to be a discrete parameter process defined on the index set \( T \). The geometric distribution is well known for modelling inter-arrival time distribution for events in this category.

2. If \( T \) is an interval or an algebraic combination of intervals, for instance, \( T = \{t : -\infty < t < +\infty\} \), then the stochastic process \( \{X(t), t \in T\} \) is called a continuous-parameter process defined on the index set \( T \). The exponential distribution is well known for modelling inter-arrival time distribution for events in this category.

From the previous sections, it is notable that arrival distribution in wireless sensor networks may fall in either of the two processes mentioned above.

9.3.1 The Bernoulli Process

Bernoulli processes consider a discrete time period such that the \( k \)th trial is associated with the arrival of at least one customer at the Service centre (CH) during the \( k \) period. The random variables \( X_n \) are i.i.d Bernoulli with common parameter \( p \in (0, 1) \). The natural sample space in this case is \( \Omega = \{0, 1\}^\infty \). Letting \( S_n = X_1 + \cdots + X_n \) (the numbers successful packet arrivals at the CH in \( n \) steps). The random variable \( S_n \) is binomial, with parameters \( n \) and \( p \), so that its
Probability Mass Function (PMF), is given by:

\[ p_{S_n}(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, ..., n \]  

(9.1)

The expected value of of the random variable \( S_n \) and the variance are given in equations 9.2 and 9.3 respectively.

\[ \mathbb{E}[S_n] = np \]  

(9.2)

\[ \text{var}(S_n) = np(1-p) \]  

(9.3)

where \( \mathbb{E}[S_n] \) denotes the expected operator.

The Bernoulli process is also associated with geometric distribution with parameter \( p \) representing the number \( T \) of trials up to and including the first success. Letting \( T_1 \) be the first successful arrival at the CH, then \( T_1 = \min \{ n | X_n = 1 \} \). Its PMF, mean and variance are given by equations 9.4, 9.5 and 9.6 respectively [Gamarnik et al., 2005].

\[ p_{T_1}(k) = (1-p)^{n-k}p, \quad k = 1, 2, ....; \]  

(9.4)

\[ \mathbb{E}[T_1] = \frac{1}{p} \]  

(9.5)

\[ \text{var}(T_1) = \frac{1-p}{p^2} \]  

(9.6)

Recalling that the time \( T \) until the first success is a geometric random variable, supposing a process is watched for successful arrival for \( n \) time step without recording any success, the remaining time until the first arrival may be expressed as \( T - n \), according to Bernoulli process, the future of the process after time \( n \) constitutes an independent fresh-start Bernoulli process whose future trials until
the first successful arrival is described by the same geometric PMF and may be presented as:

\[ P(T_n = t | T > n) = (1 - p)^{t-1} p = P(T = t) \quad t = 1, 2, \ldots, \quad (9.7) \]

This is the memoryless property of Bernoulli process showing that when observation is started at any point in time, the future is also modelled by a Bernoulli process, which is independent of the past.

An important random variable associated with the Bernoulli process is the time of \( k \)th arrival. For \( K \geq 1 \), let \( Y_k \) be the \( k^{th} \) arrival time. Formally, \( Y_k = \min \{ n | S_n = k \} \). For convenience, let \( Y_0 = 0 \). The \( k^{th} \) inter arrival time is then defined as \( T_1 = Y_1, T_k = Y_k - Y_{k-1}, \quad k = 2, 3, \ldots \) representing the number of trials following \( k-1 \)st arrival until the next arrival. Note that \( Y_k = T_1 + T_2 + \cdots + T_k \) and \( T_1, T_2, T_3, \ldots, T_k \) processes are all i.i.d geometric random variables with common parameter \( p \). The PMF of \( Y_k \) is given by equation 9.8 and is known as Pascal PMF of order \( k \).

\[ P(Y_k = t) = P(S_{t-1} = k - 1 \text{ and } X_t = 1) = P(S_{t-1} = k - 1).P(X_t = 1) \quad (9.8) \]

\[ = \binom{t - 1}{k - 1} p^{k-1} (1 - p)^{t-k} \cdot p \left( \binom{t - 1}{k - 1} p^k (1 - p)^{t-k} \right) \quad t = k, k + 1, \ldots \]

The Mean and variance of \( Y_k \) are given by equations 9.9 and 9.10.

\[ \mathbb{E}[Y_k] = \mathbb{E}[T_1] + \cdots + \mathbb{E}[T_k] = \frac{k}{p} \quad (9.9) \]
\[ \text{var}(Y_k) = \text{var}(T_1) + \cdots + \text{var}(T_k) = \frac{k(1 - p)}{p^2} \quad (9.10) \]

Finally, streams of i.i.d Bernoulli arrivals at the CH from the cluster nodes are considered. Suppose that \( \{X_n\} \) and \( \{Y_n\} \) are independent Bernoulli processes with parameters \( p \) and \( q \), respectively. If process \( Z_n \) records arrival at time \( n \) if and only if one or both processes record an arrival and it is formally given by
The merged random variables $Z_n$ are i.i.d, with parameter in equation 9.11 thus implying $Z_n$ is itself a Bernoulli process.

$$P(Z_n = 1) = 1 - P(X_n = 0, Y_n = 0) = 1 - (1 - p)(1 - q) = p + q - pq \quad (9.11)$$

On the other hand, splitting is a reverse if there is an arrival at time $n$ (i.e $X_n = 1$). Supposing the arriving packet finds the queue empty, then the packet enters the queue with probability $q$. However, if the queue is full, then the packet is dropped with probability $(1 - q)$. Noting that the decision to accept or discard a packet only depends on the state of the queue and therefore, independent for different arrivals, the accepted packets also follow Bernoulli process. Intuitively, in each time slot, there is a probability of $pq$ of accepted arrival independently of what happens in the other slots. In the same manner, the process of blocking arrivals is also a Bernoulli process with a probability of a blocked arrival at each time slot equal to $p(1 - q)$.

### 9.3.2 The Poisson Process

The Poisson process is a stochastic process that counts the number of events in a given time interval. The time between each pair of consecutive events has an exponential distribution with parameter $\lambda$ and each of these inter-arrival times is assumed to be independent of other inter-arrival times. It can be viewed as a continuous time analogue of the Bernoulli process applied to situations where there is no natural way of dividing time into discrete periods.

Consider a model for arrival distribution of packets at the CH. It is possible to characterize time into a one-minute period and record successful packet arrivals at the CH during every minute. If arrival rate is assumed to be constant over time, then the probability of arrivals should be the same during each period. Since it is assumed that different time periods are independent, the sequence of successes becomes a Bernoulli process. Taking the case of an event based and/or a hybrid WSN applications’ environment where nodes contend for channel availability and considering the back-off schemes configured in MAC protocols, it is possible to
have more than one arrival within a given time slot. However, the Bernoulli process does not keep track of the exact number of arrivals thereby hindering the calculation of expected packet arrivals within a specified period.

In order to deal with this drawback, a limiting situation with zero length time period may be considered and instead use made of a continuous time model. It is possible to consider an arrival process that evolves in continuous time, in the sense that any real number $t$ is a likely arrival time. Letting $P(k, \tau) = \mathbb{P}$ (there are exactly $k$ arrivals during an interval of length $\tau$), and assuming this probability is similar for all intervals of the same length $\tau$. Let us also introduce a positive arrival rate (intensity) $\lambda$. With these in mind, an arrival process is called a Poisson process with rate $\lambda$ if it possesses the following properties:

1. **Time-homogeneity:** The probability $P(k, \tau)$ of $k$ arrivals is the same for all intervals of the same length $\tau$.

2. **Independence:** The number of arrivals during a particular interval is independent of the history of arrivals outside this interval.

3. **Small Interval probabilities:** The probabilities $P(k, \tau)$ satisfy

   $$P(0, \tau) = 1 - \lambda \tau + 0(\tau), \text{ and } P(1, \tau) = \lambda \tau + 0_1(\tau)$$

   Where $0(\tau)$ and $0_1(\tau)$ are function of $\tau$ that satisfy

   $$\lim_{\tau \to 0} \frac{0(\tau)}{\tau} = 0, \quad \lim_{\tau \to 0} \frac{0_1(\tau)}{\tau} = 0.$$ 

Suppose that the period $\tau$ is partitioned into smaller portions $\xi$ of length $\delta$ then the probability of receiving more than one arrival within the period becomes negligible as the length $\delta$ is made smaller. The probability of one successful arrival within each period may be given by $\lambda \delta$. Similarly, the probability of no arrival within each period is given by $1 - \lambda \delta$. The process may then be approximated using a Bernoulli process with the approximations becoming more accurate with smaller values of $\delta$. Thus the probability $P(k, \tau)$ of $k$ arrivals in time $\tau$ is approximately the same as the (binomial) probability of $k$ successes in $n = \tau/\delta$ independent Bernoulli trials with success probability $p = \lambda \delta$ at each trial. Keeping the length $\tau$ of the interval fixed and letting the period length $\delta$ decrease to zero, it can be noted that the number $n$ of periods tend to infinity, while the
product \( np \) remains constant and equal to \( \lambda \tau \). Under these circumstances, it can be proved that the binomial PMF converges to a Poisson PMF with parameter \( \lambda \delta \). This leads to the conclusion that

\[
P(k, \tau) = \frac{(\lambda \tau)^k e^{-\lambda \tau}}{k!}, \quad k = 0, 1, \ldots
\]  

(9.12)

It is possible to obtain the mean and variance using equations 9.13 and 9.14.

\[
E(N\tau) = \lambda \tau
\]  

(9.13)

\[
\text{var}(N\tau) = \lambda \tau
\]  

(9.14)

Where \( N\tau \) stands for the number of arrivals during a time interval of length \( \tau \). Assuming that the process starts at zero and there are no arrival during the interval \([0, t]\) and \( T > t \) then the probability law for the time \( T \) of the first arrival may be derived using equation 9.15.

\[
F_T(t) = P(T \leq t) = 1 - P(T > t) = 1 - P(0, t) = 1 - e^{-\lambda t}, \quad t \geq 0.
\]  

(9.15)

Differentiating the CDF \( F_T(t) \) of \( T \), and obtaining the Probability Density Function (PDF) formula, it can be shown that

\[
f_T(t) = \lambda e^{-\lambda t}, \quad t \geq 0,
\]  

(9.16)

This shows that the time until the first arrival is exponentially distributed with parameter \( \lambda \). The mean and variance are given by equations 9.17 and 9.18 respectively.

\[
E[T] = \frac{1}{\lambda}
\]  

(9.17)
\[ \text{var} [T] = \frac{1}{\lambda^2} \quad (9.18) \]

Like with Bernoulli process, Poisson process has several parallel properties:

1. **Independence and Memorylessness:** Consider two disjoint sets of times \( G \) and \( H \), such that \( G = [0, 1] \cup [4, \infty] \) and \( H = [1.6, 3.8] \), as an example. If \( W \) and \( Y \) are random variables determined by what happens in \( G \)(respectively, \( H \)), then \( G \) and \( H \) are independent.

2. **Fresh-Start Property:** A Poisson process starting at time \( t > 0 \) is a probabilistic replica of the Poisson process starting at time 0, and is independent of the portion of the process prior to time \( t \). This implies the Poisson process starts a fresh at each time instant.

3. **Memoryless Inter-Arrival Time Distribution:** The exponential PDF (Inter-arrival time in the Poisson process) is memoryless. Given time \( t \) and past history, the future is a fresh-starting Poisson process hence the remaining time until the next arrival has the same exponential distribution with same parameter \( \lambda \).

4. **The \( k^{th} \) Arrival Time:** The time \( Y_k \) of the \( k \)th arrival is the sum of the previous inter-arrival times until \( k \)th inter-arrival time as illustrated in equation 9.19. All the inter-arrival times are independent exponential random variables with common parameter \( \lambda \).

\[ Y_k = T_1 + T_2 + \cdots + T_k; \quad (9.19) \]

The PDF, mean and variance of \( Y_k \) are given using equations 9.20, 9.21 and 9.22 respectively. Equation 9.20 is the **Erlang PDF of order** \( k \).

\[ f_{Y_k}(y) = \frac{\lambda^k y^{k-1} e^{-\lambda y}}{(k-1)!} \quad (9.20) \]

\[ \mathbb{E}[Y_k] = \mathbb{E}[T_1] + \cdots + \mathbb{E}[T_k] = \frac{k}{\lambda} \quad (9.21) \]
\[ \text{var}(Y_k) = \text{var}(T_1) + \cdots + \text{var}(T_k) = \frac{k}{\lambda^2} \quad (9.22) \]

5. **Merging of Poisson Processes:** Consider a single WSN cluster as shown earlier in Figure 9.1 coordinated with a central CH. The traffic model is similar to that given in Figure 4.2. The CH receives traffic from cluster nodes with rates \( \lambda_n \) and from other CH’s with rate \( \lambda_r \). If the arrivals are merged with rate \( \lambda_k \) whenever an arrival occurs from both sources, it turns out that the merged process is also Poisson with rate \( \lambda_k = \lambda_n + \lambda_r \). The probability of packets arriving from within the cluster may be computed using equation 9.23. Consequently, the probability of packet arriving from other CHS may be computed using equation 9.24.

\[ P(\text{InternalArrival}) = \frac{\lambda_n}{\lambda_n + \lambda_r} \quad (9.23) \]

\[ P(\text{ExternalArrival}) = \frac{\lambda_r}{\lambda_n + \lambda_r} \quad (9.24) \]

Recalling that the length \( \delta \) is chosen very small to enable only one successful arrival within a slot, the probability \( p \) of successful arrival and probability \( (1 - p) \) of no packets arrival both remain constant. Since the arrival rates are independent and remain constant, this satisfy the time-homogeneity property. Further more, since different intervals in each of the two arrival processes are independent, the necessary conditions for a Poisson process are met.

6. **Splitting of a Poisson Process:** We now consider a CH receiving and separating packets from other CHs to be forwarded to the sink from local packets requiring further processing before they can be forwarded to the sink. Packets are separated with probability \( p \) and \( (1 - p) \) for local and other CHs respectively. Packets arrive at the CH according to a Poisson process with rate \( \lambda_k \) and each one is a local or transit packet independent of other packets and their respective arrival times. The process of the local
packet arrivals is therefore Poisson with rate $\lambda_k p$.

Time-homogeneity is satisfied by the fact that $\lambda$ and $p$ do not change with time (constant). Moreover, the fresh start property clearly holds since there is no dependence between events in disjoint time intervals. Again, focusing on the interval of small length $\delta$, the probability of a local arrival is approximately the probability of a successful packet arrival, and this turns out to be a local packet, implying $\lambda_n \delta p$. In addition, the probability of having two arrivals at the same time is negligible compared to $\delta$ hence the necessary properties are all met. This brings us to the conclusion that the local packet arrivals form a Poisson process and, in particular, it can be shown that the number $L_\tau$ of such arrivals during an interval of length $\tau$ has a Poisson PMF with parameter $\lambda_n p \tau$.

### 9.4 Simulation Test Bed

Many WSN simulators are proposed for use in various test environments [Musznicki and Zwierzykowski, 2012] [Sundani et al., 2011] [Xian et al., 2008]. In this study, the required simulator is to incorporate a wide range of platforms, including a realistic wireless channel and radio model based on measured data. Based on these, Castalia simulator using OMNET++ platform was preferred. Castalia is highly parametric and usable in evaluating different platform characteristics for specific applications [Sundani et al., 2011]. For this study, a part from the varied radio and channel platforms, Castalia also offers an opportunity to manipulate properties of the various MAC-Layer protocols.

The simulation was set up based on a WSN cluster topology with one central CH receiving packets from all other cluster nodes as illustrated in Figure 4.2. The main objective of the simulation runs is to record packet inter-arrival times at the CH. In order to obtain appropriate results for investigations and evaluation, the scope of the trials considered use of various parameter settings; First, a decision is made for a suitable number cluster nodes for each test run. During investigation clusters are set up using between 10 to 40 nodes. In addition, prop-
erties for various MAC protocols are considered. In this case, TMAC, CSMA and CSMA/CA were used. To provide a broad spectrum of results for effective investigation and evaluation, nodes were set to transmit at different packet rates to the CH. In order to represent different application scenarios, a range of packet rates were varied from 1 packet every 5 seconds to 1 packet every 10 minutes. During each simulation run, a record of packet inter-arrival time instances were obtained. Other parameters used in the test include a CC2420 radio transceiver that is compatible with IEEE 802.15.4/Zigbee standards [Chipcon Product]. The transceiver operates at 2.4 GHz with data rates of 250 Kbps. Similarly, internal MAC queue capacities of 32 packets, each having 105 bytes were also considered [Latré et al., 2005].

The data obtained from simulation runs was then processed using R-Studio to carry out Kolmogorov-Smirnov Test and statistically determine the inter-arrival distributions. R-Studio is a free open source Integrated Development Environment (IDE) for R programming language for statistical computing and graphic.

Kolmogorov-Smirnov Test (KS Test) is a tool used in statistics to confirm the hypothesis that a given empirical data follows known probability distributions. The KS Test statistic quantifies the distance between the empirical distribution function of the sample and the cdf of the reference distribution. Calculation of the null distribution of this statistic lies under the null hypothesis the sample drawn is from a continuous reference distribution. A detailed study on how K-S Test works is provided in [Jean Dickinson Gibbons, 2003] [Conover, 1999].

Consider a random sample \(X_1, X_2, X_3, \ldots, X_n\), drawn from some population, in this case \(X_i\) representing packet inter-arrival times at the CH from the wider WSN. From the random samples, empirical distribution function \(S(x)\) are determined. These are functions of \(X_i\) that are less than or equal to \(X\) for each \(X, -\infty < X < \infty\) and may be computed using equation 9.25.

\[
S(x) = \frac{1}{n} \sum_{i=1}^{n} I \{x_i \leq x\} \tag{9.25}
\]

where \(I\) is the indicator function equal to 1 if \(x_i \leq x\).
Equation 9.25 is used as an estimator of the unknown distribution function $F(x)$, of the random samples $(X_i)_s$. The empirical distribution function $F(x)$ are then compared with the hypothesised distribution function $F^*_x$ to establish any good agreement. For our case, several distributions were compared to establish possible distributions for various data delivery models discussed in section 4.2.1. A simple measure of the largest distance between the two functions $S(x)$, and $F^*_x$ was proposed by Kolmogorov in 1933.

Letting the test statistic $T$ be the greatest (denoted by “sup” for supremum) vertical distance between $S(x)$, and $F^*_x$, $T$ can be computed using equation 9.26

$$T = \sup_x |F^*_x - S(x)|$$

(9.26)

The testing conditions are given as:

$H_0$: $F(x) = F^*_x$ for all $X$ from $-\infty$ to $\infty$

$H_1$: $F(x) \neq F^*_x$ for at least one value of $X$

The quantile and $p$-values are usually given in the table. However, these are included as packages in the various statistical software programs. If $T$ exceeds the $1 - \alpha$ quantile, then $H_0$ is rejected at the level of significance $\alpha$. The approximate $p$-values are usually interpolated from the table. Nevertheless, in this study, R-Studio statistical program was employed to compute the Kolmogorov-Smirnov Tests.

### 9.5 Numerical Results and Discussions

This section presents results obtained using the Kolmogorov-Smirnov Test Statistic (KSTS). Table 9.1 provides a summary of the established distributions of packet inter-arrival times at the CH. Various experiments performed considered two main MAC-Protocols; TMAC and CSMA when subjected to different traffic loads. Figures 9.2 through 9.13 present the graphs obtained using different parameters as detailed in the table. In the Figures, $wta$, $wtf$ and $wts$ represent
waiting time of arrivals, waiting time of first part of arrivals and waiting time for the second part of arrivals respectively. These values are obtained as means of KSTS. In the table, $LCL$ and $UCL$ represent the Lower Confidence Level and Upper Confidence Level respectively.

From the obtained results, depending on the packet rate and MAC-protocols used, the distribution of packet inter-arrival times at the CH varies greatly. A number of theoretical distributions observed closely matched the empirical data sets recorded at the CH.
Table 9.1: A Summary of the Distribution of Packet Inter-Arrival Times

<table>
<thead>
<tr>
<th>Number of Nodes</th>
<th>MAC Protocol Used</th>
<th>Packet rate</th>
<th>Number of observations in the Simulated Series</th>
<th>ML Estimates of the parameters of empirical distribution</th>
<th>Kolmogorov Smirnov Test Statistic</th>
<th>P-Values</th>
<th>Corresponding theoretical distribution for the empirical data</th>
<th>Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>None</td>
<td>1 pck every 5 minutes</td>
<td>All: 246 Used: 214</td>
<td>Exponential rate: 16.11 LCL: 14.03 UCL: 18.03</td>
<td>Average of 87 runs (From LCL to UCL) 0.09</td>
<td>Average of 87 runs (From LCL to UCL) 0.09</td>
<td>Exponential</td>
<td>9.2(a, b, c, &amp; d)</td>
</tr>
<tr>
<td>10</td>
<td>TMAC</td>
<td>1 pck every 5 minutes</td>
<td>All: 279 Used: 249 First Part: 224 Sec. Part: 53</td>
<td>Mixed Log-Normal Mean1 = -5.14 Sdlog1 = 0.23 Meanlog 2 = - 0.52 Sdlog 2 = 0.02 Mixing proportion: 0.09</td>
<td>Average of 100 runs: 0.11</td>
<td>Average of 100 runs: 0.15</td>
<td>Mixed Log-Normal</td>
<td>9.3(a, b, c, d, e, &amp; f)</td>
</tr>
<tr>
<td>10</td>
<td>CSMA</td>
<td>1 pck every 10 minutes</td>
<td>All: 255 Used: 255</td>
<td>Exponential rate = 20.88 LCL: 18.38 UCL: 23.50</td>
<td>Average of 141 runs: 0.04</td>
<td>Average of 141 runs: 0.45</td>
<td>Exponential</td>
<td>9.4 (a, b, c, &amp; d)</td>
</tr>
<tr>
<td>20</td>
<td>None</td>
<td>1 pck every 5 minutes</td>
<td>All: 574 Used: 411</td>
<td>Mixed Log-Normal Mean1 = -5.18 Sdlog1 = 0.25 Meanlog 2 = - 0.58 Sdlog 2 = 0.04 Mixing proportion: 0.06</td>
<td>Average of 100 runs: 0.016</td>
<td>Average of 100 runs: 0.12</td>
<td>An unknown mixed distribution</td>
<td>9.7(a, b, c, d, e, f, g)</td>
</tr>
<tr>
<td>20</td>
<td>TMAC</td>
<td>1 pck every 5 minutes</td>
<td>All: 596 First part: 578 Sec. part: 18</td>
<td>Exponential rate= 16.91 LCL = 16.91 UCL = 17.69</td>
<td>Average of 153 runs (From LCL to UCL) 0.06</td>
<td>Average of 153 runs (From LCL to UCL) 4.60×e^-05</td>
<td>An unknown non-Exponential distribution</td>
<td>9.8(a, b, c, d)</td>
</tr>
<tr>
<td>20</td>
<td>CSMA</td>
<td>1 pck every 1 second</td>
<td>All: 1890 Used: 1889</td>
<td>Mixed Log-Normal Mean1 = -2.94 Sdlog1 = 0.05 Meanlog 2 = 0.10 Sdlog 2 = 0.2×e^-04 Mixing proportion 0.001</td>
<td>Average of 100 runs: 0.46</td>
<td>Average of 100 runs: 2.62×e^-03</td>
<td>An unknown mixed distribution</td>
<td>9.9 (a, b, c, d, e, f, g)</td>
</tr>
<tr>
<td>35</td>
<td>None</td>
<td>1 pck every 5 minutes</td>
<td>All: 644 Used: 612</td>
<td>Mixed Log-Normal Mean1 = -5.59 Sdlog1 = 0.93 Meanlog 2 = -3.66 UCL = -3.52</td>
<td>Average of 148 runs (From LCL to UCL) 0.08</td>
<td>Average of 148 runs (From LCL to UCL) 0.13</td>
<td>Log-Normal</td>
<td>9.10 (a, b, c, &amp; d)</td>
</tr>
<tr>
<td>35</td>
<td>TMAC</td>
<td>1 pck every 5 minutes</td>
<td>All: 998 Used: 958 First part: 914 Sec. part: 44</td>
<td>Mixed Log-Normal Mean1 = -5.25 Sdlog1 = 0.24 Meanlog 2 = -6.62 Sdlog 2 = 0.09 Mixing proportion: 0.05</td>
<td>Average of 100 runs: 0.19</td>
<td>Average of 100 runs: 3.18×e^-14</td>
<td>An unknown mixed distribution</td>
<td>9.11 (a, b, c, d, e, f, g)</td>
</tr>
<tr>
<td>40</td>
<td>None</td>
<td>1 pck every 5 minutes</td>
<td>All: 751 Used: 720</td>
<td>Mixed Log-Normal Mean1 = -5.77 Sdlog1 = 0.90 Meanlog 2 = -3.84 UCL = -3.71</td>
<td>Average of 100 runs (From LCL to UCL) 0.08</td>
<td>Average of 100 runs (From LCL to UCL) 0.07</td>
<td>Log-Normal at p-value 6% or less Not Log-Normal at traditional 10% significance level</td>
<td>9.12(a, b, c &amp; d)</td>
</tr>
<tr>
<td>40</td>
<td>TMAC</td>
<td>1 pck every 5 minutes</td>
<td>All: 1138 Used: 1106 First part: 1064 Sec. part: 38</td>
<td>Mixed Log-Normal Mean1 = -5.26 Sdlog1 = 0.22 Meanlog 2 = -6.62 Sdlog 2 = 0.09 Mixing proportion: 0.04</td>
<td>Average of 100 runs: 0.22</td>
<td>Average of 100 runs: 0.0</td>
<td>An unknown mixed distribution</td>
<td>9.13 (a, b, c, d, e, f)</td>
</tr>
</tbody>
</table>
In Figures 9.2 and 9.4, the empirical data follows known exponential distribution when a data rates of 1pck/5mins is used without consideration of MAC layer protocols. This translates to sending 12pck/hr. The same distribution is observed when CSMA/CA is used with 10 nodes transmitting packets at 1pck/10mins translating to 6pck/hr. The other distributions observed include Gamma distribution, Figure 9.6, when 20 nodes are considered to send packets at 1pck/5mins without the use of MAC protocols, log-normal distribution, Figure 9.10, when 35 nodes are used without considering MAC properties at data rates of 1pck every 5 minutes and Figure 9.12, log-normal at p-values below 6% observed when 40 nodes are used without MAC properties at data rates of 1pck every 5 minutes. However, Mixed log-normal distribution is observed in Figure 9.3 when using TMAC with 10 nodes transmitting 1pck every 5 minutes. In Figure 9.5, Mixed Log-Normal is also observed at p-values below 3.5%.

Figure 9.2: Comparison of Theoretical and Empirical Graphs for Exponential Distribution

Not Log-Normal is observed at traditional 10% significance level when 40 nodes each transmitting packets at 1pck/5mins are employed without MAC properties.
as illustrated in Figure 9.12. However Not mixed log-normal at traditional 5% or 10% is observed with 10 nodes transmitting at 1 packet every 5 seconds when CSMA is employed.

Figure 9.3: Comparison of Theoretical and Empirical Graphs for Mixed Log-Normal Distribution

The other category observed include an Unknown Non-Exponential distribution when using CSMA/CA and 20 nodes transmitting packets at 1pck/sec, Figure 9.8. The last group observed is an unknown mixed distribution illustrated in Figures 9.7, 9.9, 9.11 and 9.13. Their detailed parameters are given in table 9.1.
An important observation with MAC properties considered is the alteration of the inter-arrival distribution to an unknown mixed distribution because of the internal properties. TMAC presents a worst scenario (unknown mixed distribution) when the network has 20, 35 and 40 nodes each transmitting packets to the CH at $12\text{pck/hr}$. Mixed log-normal distribution is only observed when 10 nodes are sending packets at a constant rate. On the other hand, CSMA/CA presents various distributions influenced by the number of source nodes and packet rates. These include, exponential, an unknown non-exponential, mixed log-normal at some $p$-values and an unknown mixed distributions as summarized in table 9.1. However, the distributions become more distinct with no MAC properties employed. From the results, it is observed that there is no distinct packet arrival distribution pattern at the CH. Based on application scenario and operation parameters, different distributions may be obtained. The methodology and results obtained are therefore useful in scientific research for obtaining appropriate distributions for different WSN application scenarios.
9.6 Chapter Summary

In this chapter, a WSN clustered model coordinated by one central CH was proposed and inter-arrival time distribution for packets at the CH analysed in order to determine the resulting empirical distribution pattern. Based on application environment and criticality, data types are first characterised and classified into the various delivery models. In order to establish the actual inter-arrival distribution at the CH, use was made of Castalia simulator to run a number of simulations under varying conditions and inter-arrival times at the CH recorded. Using Kolmogorov-Smirnov Test Statistic on the empirical data sets, the real theoretical probability distributions were established.

Interesting observation noted is the fact that WSN packet inter-arrival time distribution at the CHs does not follow a specific probabilistic distribution pattern consistently. However, the distribution of arrivals is influenced by delivery models used, prevailing operation conditions and influences imposed by application protocols. It is therefore not proper to use a particular distribution for modelling packet inter-arrival times in all WSN system scenarios. For various studies, an evaluation of the appropriate distribution is necessary in order to provide realistic system models. The methods used are useful for determining appropriate distributions in any WSN application scenario for scientific research.

Identifying the appropriate arrival distribution ensures the resulting performance and energy saving models are a more realistic mimic of the actual system behaviour hence making related studies more relevant in the application environment. It is possible to extend this study to identify and classify relevant arrival distributions for various WSN application scenarios.
Figure 9.5: Comparison of Theoretical and Empirical Graphs for Mixed Log-Normal Distribution
Figure 9.6: Comparison of Theoretical and Empirical Graphs for Gamma Distribution
Figure 9.7: Comparison of Theoretical and Empirical Graphs for An Unknown Distribution
Figure 9.8: Comparison of Theoretical and Empirical Graphs for An Unknown Non-Exponential Distribution
Figure 9.9: Comparison of Theoretical and Empirical Graphs for Mixed Log-Normal Distribution
Figure 9.10: Comparison of Theoretical and Empirical Graphs for Log-Normal Distribution
(a) Histogram of Inter-Arrival Times

(b) Histogram of Inter-Arrival First Part

(c) Histogram of Inter-Arrival Second Part

(d) QQ-plot for Mixed Log-Normal Distribution

(e) Empirical & Theoretical Mixed Log-Normal Densities

(f) Empirical & Theoretical Mixed Log-Normal CDF

Figure 9.11: Comparison of Theoretical and Empirical Graphs for an Unknown Distribution
Figure 9.12: Comparison of Theoretical and Empirical Graphs for Log-Normal Distribution
Figure 9.13: Comparison of Theoretical and Empirical Graphs for an Unknown Distribution
Chapter 10

Conclusion

Today, smart grid, smart homes, smart industry, smart agriculture, intelligent transportation, green environment, are infrastructure systems that connect the world more than ever thought possible. The vision of such systems is associated with the single concept of IoT, where through the use of sensors, the entire physical infrastructure is closely coupled with information and communication technologies; where the intelligent monitoring and management can be achieved via the usage of networked embedded devices. In such dynamic systems, devices are interconnected to transmit useful measurement information and control instructions via distributed sensor networks. With the rapid technological development of sensors, WSN has become the key technology for IoT, and is regarded as a revolutionary information-gathering method to build the information and communication system which will greatly improve the reliability and efficiency of infrastructure systems.

Practically, the application of WSNs to the real world is unlimited. Most of the applications present varying QoS demands that complicate the design of WSNs due to several resource limitations hence the need for new paradigms. Lots of independent availability and performance studies are proposed in literature, but these do not address concerns of independent availability and performance studies. In order to address these concerns, this thesis presents research work on performability modelling of WSNs, which is a composite measure of performance and availability that provides the most realistic modelling approach. Two-dimensional
state space is used to represent the state of the system at a given time and spectral expansion method and system of linear simultaneous equations used extensively to solve the two-dimensional state space. In order to show the effectiveness of the models developed, numerical results are presented and validated using dedicated discrete event-based simulation programs developed in C++ and tested with known mathematical formulae. Some assumptions were considered to make analytical modelling tractable.

The approaches for the evaluation of the performability of WSNs presented in this thesis, provide powerful tools for making essential decisions for system design, optimising parameters to fit user requirements, and examining optimal design trade-offs. Furthermore, these models are useful for understanding the intricate interactions between systems’ operation states and subtle effects of various factors such as failures, repairs and delays.

Finally, sections 10.1 and 10.2 present thesis contributions and proposals of future work respectively.

10.1 Thesis Contributions

The following is a listing of the major contributions of this thesis:

1. Performance modelling and analysis continue to play a significant role in supporting research, system design, development and optimization of computer and communication systems and applications. The high demand for use of WSNs in various application environments also brings with it the need for performance and availability modelling for optimization and deployment of WSNs. Several pure performance and availability studies for WSNs exist but no record of composite studies of such exists [Chiasserini and Garetto, 2006], [Hashmi et al., 2010], [Qiu et al., 2011], [Chan et al., 2015]. The challenges resulting from pure performance and availability studies hence continue to affect QoS provision in WSNs. In chapter 5, the need to integrate performance and availability modelling for WSNs is justified. The
study presents a systematic modelling approach that considers performance and availability together in the presence of failures, repairs/replacement and restoration. First, a queuing model for the arriving data packets is developed. A performability model whose inputs are data packets derived from the queue model is developed to capture the desired system behaviour in the various operative states. The proposed model is then analysed and evaluated using Markov chain. Two analytical approaches; Spectral expansion solution techniques and Poisson approximation are employed to compute the models and results verified using a discrete event simulation program developed in C++.

From the results, it is confirmed that system failures degrade network performances hence require to be contained within some limit for better performance. In addition, setting appropriate arrival rates to adequate levels is important in maintaining acceptable queue lengths and system response times as per application requirements. Network coverage also plays a significant part on overall WSN performance. In order to cover a given area, the number of clusters required is determined by desired traffic intensity that provides optimum performance measures a part from use of maximum cluster node capacity. The numerical results and the methodology are therefore, significant to the system designers, users and scientists for development of planning and deployment tools, operation management and optimization studies.

2. In real-life situations, systems do not have infinite queues. This is worse in WSNs that are equipped with limited memory capacity that are dependent upon the physical size of the sensor nodes. As WSNs continue to find usage in diverse application environments, the demand for memory required for temporary data storage and information processing also vary considerably with high data intensive applications demanding more processing and storage space. The low memory capacities offered are therefore, not sufficient for use with all applications. Therefore, optimizing the available memory is significant for obtaining desired performance levels with better QoS. Additional studies are also necessary for establishment of optimal
queue capacities necessary for use with different applications. Chapter 5 presents an advancement of the model of chapter 4 that now incorporates bounded queues. Again, the developed performability model is resolved using Spectral Expansion Solution technique and validated using a discrete simulation program developed in C++. In addition to analysis of bounding effects alongside node failures’ effects on performance measures that include MQL, throughput, and response time, additional effect caused by limiting queue capacity is also analysed. Furthermore, the systems blocking probability is also analysed using various queue capacities and results used to propose possible queue capacities for use in various application environments. From the results, bounded queues limit the amount of data that can be stored as expected. These results into data packets being blocked and lost hence may negatively influence system performance. Considering queue capacity is useful, especially for optimisation of queue capacities at each node. The results and methodology are therefore, very significant to the designers and users in design production and system optimization.

3. Alternating sleep and active operation periods is widely used in WSNs to conserve the limited energy. However, use of sleep scheduling mechanisms may also affect WSNs performance negatively if not appropriately optimised. Chapter 6 critically analyses existing approaches for sleep scheduling and proposes appropriate models for evaluating performance and availability for clustered WSNs. The models developed improve the model in chapter 5 by incorporating the sleeping phase of the sensor node. QBD processes are then used for performability studies. Two analytical approaches; Spectral Expansion and System of simultaneous linear equation are employed to solve the models and the results further validated using results obtained using a discrete event simulation program developed in C++. Two sleep scheduling methods considered in this study are On-demand and Adaptive sleep scheduling schemes. In both cases, system performance is degraded at some levels of operations hence may require additional mechanisms for optimisation. System designers can use the results obtained to develop a more resilient system that regulates sleep schedules in line with traffic intensity.
4. In order to make the models more realistic to system behaviour, chapter 7 presents a critical analysis of WSN channel failures and its impact on performance and availability/reliability. First, the behaviour of WSN channel is analysed and incorporated into the system models developed in chapter 6. Specifically, use is made of On-demand sleep scheduling to develop the new model and QBD processes employed for performability studies using similar solution approaches. Using Spectral Expansion and System of simultaneous linear equations, the performability model is resolved and results further validated using solutions obtained from a discrete event simulation program developed in C++. The obtained results show how channel failure affects performance parameters and may result into system degradation if failure surpasses some levels. The results and methodology are therefore, useful to designers and user for deployment and performance optimisation.

5. Energy conservation remains a major challenge in WSNs despite many solutions proposed in various studies. Among the solutions offered, alternating sleep and active operations is widely used. In chapter 8, models developed in the preceding chapters are considered for evaluating energy consumption of WSN systems. First, analytical models used for obtaining energy consumed in various operative states are developed. Using the developed models in conjunction with steady-state probabilities obtained from performability models, mean energy consumptions in the various operative states are computed. As a case study, a comparison of energy consumption between two similar models, one implementing sleep state and the other implementing idle state is illustrated. Results show a good amount of energy saving while operating in the sleep state as expected. However, when traffic intensity increases to some level, wake-up energy becomes greater than sleep energy thereby negating the purpose for entering sleep state. Based on the results, appropriate level of traffic intensity that allows optimum energy saving in the sleep state is identified. The method employed is useful for designers and researchers for development of deployment tools and performance optimisation studies.

6. From the literature, the general practice used to determine arrival distribu-
tion of data packets in WSNs is to compare empirical data with the theoretical exponential distribution of Q-Q plots and draw conclusions based on simple eye checks. In addition, chapter 9, makes use of estimated maximum likelihood parameters of empirical distributions to generate theoretical distributions. For each data series generated, Kolmogorov-Smirnov Test is conducted to identify a possible corresponding theoretical distribution. Results obtained show a number of theoretical distributions, including; Gamma, Log-normal, Mixed-log-normal, and Exponential as potential candidates depending on the application set operation conditions. The choice of the right distribution is significant in ensuring realistic system models are obtained. The numerical results and methodology are therefore, useful for the designers, research scientist, and users of clustered WSNs for network planning, deployment, and optimization studies.

10.2 Proposals for Future studies

Below are suggestions for future studies

1. Today, WSNs play a significant role in the acquisition and distribution of information for IoT applications. Performance and availability/reliability of such systems remain critical in ensuring the provision of required QoS is achieved. In this study, a single queuing system implementing FCFS priority is used for performability studies. However, in a multi-application environment, different applications may require varying service priorities. Considering priority queuing in future performability studies is therefore, significant in ensuring provision of quality service to all applications.

2. Once deployed in the habitat, wireless sensors may relay their information to the Sink directly or through intermediary nodes. In a clustered network, all information is relayed to the sink through the CH, which may communicate directly or through other intermediary CHs towards the Sink. In both cases, the relaying nodes may have more than one next hope available to choose from, depending on the prevailing circumstances; appropriate choice of next
hope is determined. Multi-Server systems have been studied in related communication systems [Ever, 2007], [Gemikonakli, 2014]. It is possible to extend this study to consider a multi-server system for modelling available next hopes and channels for performability studies.

3. Several approaches are proposed in the literature for scheduling sleep periods widely used for conserving power for the sensor nodes [Yang and Vaidya, 2004], [Anastasi et al., 2009], [Ameen et al., 2010]. Based on these approaches, a number of pure performance and availability studies have been proposed [Chiasserini and Garetto, 2006], [Almazydeh et al., 2010], [Qiu et al., 2011]. Such models carry with them concerns emanating from pure performance and availability studies. In this study, a model for evaluating energy is proposed and used to study energy consumption for a performability model incorporating On-demand sleep scheduling scheme. Using a similar methodology it is possible to develop models for existing sleep scheduling algorithms and evaluate them for energy consumption. Furthermore, the obtained results are significant for improving system performance and optimisation studies.

4. Considering that WSNs have applications in diverse environments with varying QoS demands in addition to different deployment settings, the influence caused on arrival distribution of data packets at the intermediary nodes, and the CH may vary considerably. Moreover, the MAC protocols and other applications protocols running on top of MAC protocols like those for energy conservation [Heinzelman et al., 2000], [Ye et al., 2002], [Li, 2011] may also introduce delays that ultimately alter arrival distributions. In order to cover a wide range of applications and make the results more candid, the methodology employed in chapter 9 can be used for critical analysis of real-time WSN data collected using different data delivery models subjected to various application conditions.

5. In this chapter, only issues pertaining to a single CH are considered. For purposes of improving overall WSN QoS, it is possible to extend this research study to consider issues of network scalability.
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