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Abstract

This paper considers wireless multicast networks where network coding (NC) is applied to improve network throughput. A novel joint topology and cross-layer design is proposed to maximize the network throughput subject to various quality-of-service constraints, such as: wireless multicast rate, wireless link capacity, energy supply and network lifetime. Specifically, a heuristic NC-based link-controlled routing tree algorithm is developed to reduce the number of required intermediate nodes. The proposed algorithm facilitates the optimization of the wireless multicast rate, data flow of wireless links, energy supply and lifetime of nodes through a novel cross-layer design. The proposed joint topology and cross-layer design is evaluated and compared against other schemes from the literature. The results show that the proposed scheme can achieve up to 50% increase in the system throughput when compared to a classic approach.

Index Terms

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I. INTRODUCTION

In today’s environment, wireless multicast networks (WMNs) are facing important challenges as the network performance is highly affected by various dynamics of the wireless links (e.g. limited channel bandwidth, severe power constraint, unstable signals, interferences, etc.). For this reason, multicasting has attracted an increasing interest in wireless communications with extensive investigations [1]–[6]. With the advances of wireless communications, the demand of high-throughput multicasting is crucial, especially in services which require high-rate multicasting traffic, e.g. teleconferencing [5], [7] and multimedia streaming [8].

Network coding (NC) is regarded as an effective technique to improve the throughput of a communication system [9], [10]. As compared to the conventional store-and-forward manner at intermediate nodes in a multi-hop wireless network (e.g., a WMN), NC has been applied at the intermediate nodes to dramatically improve the throughput of wireless networks [11]–[13]. By performing algebraic linear/logic operations on received packets at the intermediate nodes with NC techniques, the bandwidth could be saved for a higher system throughput. Many NC-based protocols have been proposed and investigated for multicast channels (e.g. in [4], [6]).

In NC-based WMNs, topology design has significant impact on the system throughput [14]–[19]. In [14], an iterative cross-layer optimization was proposed to allocate physical and medium access layer resources for network planning. The scheduling for optimising the NC-based multicast was investigated in [15], [16]. In [17], the topology was shown to affect the efficiency of NC in improving the system throughput since destination nodes may not receive enough linearly independent NC-based data packets to recover the original data packets. Moreover, the use of all available nodes in WMNs to support the multicasting for a group of destination nodes
causes a large increase in energy consumption within the network [18]. In [19], the opportunity and applicability of NC in practical wireless networks have been shown to be dependent on the topology design. Therefore, the topology design is a challenging task for NC-based WMNs, especially for multimedia applications which require high multicasting traffic along with high quality-of-service (QoS).

In this paper, we consider a WMN consisting of multiple multicasting sets in which each source node multicasts data to a group of desired destination nodes with the assistance of multiple relay nodes. We jointly exploit the advantages of linear NC techniques and QoS-driven adaptive data streaming in designing a WMN topology that can maximize the system throughput. To the best of our knowledge, existent proposals in the literature (e.g. [14]–[19]) do not entirely tackle the topology design under the constraints of QoS including multicasting rate, capacity of wireless channels, energy supply, and node/network lifetime. The proposed multicast topology design is a novel cross-layer scheme that investigates wireless multicast characteristics at underlying layers while guaranteeing the end-to-end QoS required by multimedia users. Furthermore, the proposed design makes use of the medium access control (MAC) to allocate the transmission time, energy supply and data rate to a node (e.g. in [20]–[23]), while the routing is used to determine an effective path for different data flows. The work in this paper is extended from [24]. The main contributions of this paper can be summarized as follows:

- The WMN topology design is formulated as a cross-layer optimization problem. The objective is to maximize the system throughput over the design metrics including wireless multicasting rate of source nodes, amount of wireless data flows, energy supply at nodes, and lifetime of nodes subject to various QoS constraints on flow conservation, wireless link capacity, wireless multicast traffic rate, node energy, total energy, and network lifetime. This
NC topology design problem is shown to be NP-hard in the WMN.

- A heuristic NC-based link-controlled routing tree (NC-LCRT) algorithm is proposed associated to the analysed NP-hard problem to construct a multicasting tree. By referring to the LCRT algorithm [25], the NC-based LCRT algorithm employs a minimal number of nodes for NC operations, as contrast to the design using all nodes in the system.

- Based on the NC-based LCRT algorithm, the cross-layer design, targeting at optimizing the wireless multicast rate, data flow of wireless links, energy supply, and nodes’ lifetime, are studied. By solving the cross-layer optimization problem, it is demonstrated that, given fixed lifetime of nodes, the optimization problem is a linear programming problem and the network lifetime constraint can be relaxed as the lifetime of nodes approaches network lifetime.

The performance of the proposed cross-layer design is evaluated and compared against other solutions from the literature. The results show that the system throughput increases as either total energy available for network increases or network lifetime decreases. Additionally, with linear NC technique, a significantly improved throughput is achieved with the proposed protocol compared to the non-NC-based LCRT protocol, especially with a large wireless multicast set.

The rest of this paper is organised as follows: Section II describes the network model of a wireless multicast network and discusses various topology design aspects. The formulation of the cross-layer optimization problem is presented in Section III. Sections IV and V present the proposed heuristic NC-based LCRT algorithm and cross-layer design. Numerical results are presented and discussed in Section VI. Finally, Section VII draws the main conclusions from this paper.
II. NETWORK MODEL AND TOPOLOGY DESIGN ASPECTS

In this section, we first present the network model under investigation of the paper, and then introduce different aspects in designing multicast topology in WMNs.

A. Network Model

We consider the scenario in which the number of wireless multicast groups and the members (e.g., sources, destinations) in each group are known. Hence, the network conditions should be unchanged over the time period of the multicasts. As such, our study employs a quasi-static WMN with a set of $N$ nodes (denoted as $\mathcal{N}$) in which each node acts as either a source, or a relay, or a destination. Let the distance and the capacity of the wireless link between two adjacent nodes (say the $i$-th node and the $j$-th node) be $d_{i,j}$ and $C_{i,j}$, respectively, where $\{i, j\} \in \mathcal{N}$. Suppose there are $M$ wireless multicasting groups and $S_m = \{n_{m,0}, n_{m,1}, \ldots, n_{m,|S_m|-1}\}$, $m \in \mathcal{M} \triangleq \{1, 2, \ldots, M\}$, represents a subset of nodes in $\mathcal{N}$ that requires to join in the $m$-th group, where $n_{m,0}$ is the source of this group and $n_{m,l}$, $l \in \mathcal{L}_m \triangleq \{1, 2, \ldots, |S_m| - 1\}$, is a destination of this group. Also, let $K_m$ and $B_m$ denote the number of packets and the size of packets multicasted in the $m$-th group. For simplicity, let us assume $K_m = K$ and $B_m = B$, $\forall m \in \mathcal{M}$.

B. Topology Design Aspects

In NC-based WMNs, the topology design was shown to be crucial for implementing linear NC techniques [19]. In this paper, we consider the following topology design aspects:

1) Wireless Multicast Traffic Rate: If the transmission rate of wireless multicasting in the $m$-th group is $R_m$, the multicasting performance is acceptable if $R_m \geq \delta_m$ can always be guaranteed during the communication, where $\delta_m$ is the rate for the basic-layer performance rate.
2) **Wireless Flow Conservation:** For an intermediate node on a WMN multi-hop path, the amount of total outgoing wireless multicast traffic should be equal to the amount of total incoming wireless multicast traffic. However, in the case of source or destination nodes, the amounts of incoming traffic and outgoing traffic are different and the difference should be the amount of traffic generated at sources. Therefore, if we let \( f_{n,m,0}^{(n_m,0,n_m,l)} \) be the amount of wireless data flow from source node \( n_{m,0} \ (m \in M) \) to destination node \( n_{m,l} \ (l \in L_m) \) on link \( i \rightarrow j \ (\{i,j\} \in \mathcal{N}) \), we have

\[
\sum_{a \in \mathcal{N}} f_{a,n}^{(n_{m,0},n_{m,l})} - \sum_{b \in \mathcal{N}} f_{b,n}^{(n_{m,0},n_{m,l})} = \begin{cases} 
-R_m & \text{if } n = n_{m,l}, \\
R_m & \text{if } n = n_{m,0}, \\
0 & \text{otherwise}.
\end{cases}
\]  

(1)

3) **Wireless Link Capacity:** The total amount of wireless multicasting traffic through the wireless link from the \( i \)-th node to the \( j \)-th node (\( \{i,j\} \in \mathcal{N} \)) should not exceed the capacity limitation of the wireless link, i.e. \( C_{i,j} \). Here, for simplicity, let us consider additive white Gaussian noise (AWGN) channels\(^1\). Thus, \( C_{i,j} \) can be determined by \( C_{i,j} = W \log_2(1 + \gamma_{i,j}) \) [bits/s], where \( W \) and \( \gamma_{i,j} \) denote the channel bandwidth and the signal-to-noise ratio (SNR) of the wireless link \( i \rightarrow j \), respectively. With linear NC technique, the traffic amount of the wireless link \( i \rightarrow j \) is determined by \( \sum_{m=1}^{M} \max_{l \in L_m} f_{i,j}^{(n_{m,0},n_{m,l})} \). Therefore, we have the following constraint

\[
\sum_{m=1}^{M} \max_{l \in L_m} f_{i,j}^{(n_{m,0},n_{m,l})} \leq C_{i,j} = W \log_2(1 + \gamma_{i,j}).
\]  

(2)

4) **Node energy consumption:** The energy consumption at the \( n \)-th node is defined as the energy needed to transmit and receive data throughout the lifetime of this node. Let \( \alpha_n \) and \( \beta_n \ (n \in \mathcal{N}) \) be the energy consumed to transmit and receive a unit of data at the \( n \)-th node

\(^1\)It is noted that the communications may suffer from interference, which can be either avoided or exploited by using various interference cancellation and multi-user detection techniques [26], [27].
respectively, $T_n$ be the lifetime of the $n$-th node and $t_{i,j}$ ($\{i,j\} \in \{a,b,n\}$) be the transmission delay from the $i$-th node to the $j$-th node. With linear NC technique, the traffic amount of the wireless link $i \rightarrow j$ is determined by $\sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f^{(n_m,0,n_m,l)}_{i,j}$ where $\mathcal{L}_m = \{1, 2, \ldots, |S_m| - 1\}$. The constraint on the energy consumption at the $n$-th node is therefore formulated as follows:

$$\max_{a \in \mathcal{N}} [\alpha_n t_{n,a} \sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f^{(n_m,0,n_m,l)}_{n,n}] + \max_{b \in \mathcal{N}} [\beta_n t_{b,n} \sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f^{(n_m,0,n_m,l)}_{b,n}] T_n \leq E_n,\quad (3)$$

where $E_n$ denotes the energy supply at the $n$-th node.

5) Total available energy: It is observed that the equal energy allocation for all nodes in the network may not be optimal since the energy allocation at each node depends on many factors, such as the role of node (source and/or relay and/or destination), the location of node, and the connections of a node with the other nodes in the topology [28]. It is assumed that the energy supply of all nodes can be adaptively adjusted in a centralised manner to meet the total energy consumption constraint in the whole network. Let us denote $\xi$ as the total energy available for the whole network and assume that the energy of nodes is continuously distributed. We have the following constraint on the total energy consumption

$$\sum_{n=1}^{N} E_n \leq \xi.\quad (4)$$

6) Network lifetime: Let us denote $\tau$ as network lifetime, which is defined as the minimum duration for the survival of all nodes in network satisfying the wireless multicast rate constraints. This means $\tau = \min_{n \in \mathcal{N}} T_n$. In other words, the lifetime of the $n$-th node ($n \in \mathcal{N}$) is not less than the network lifetime, i.e.

$$T_n \geq \tau.\quad (5)$$
III. CROSS-LAYER OPTIMIZATION PROBLEM FORMULATION

Based on the above analysed conditions, we present the following optimization problem that proposes an NC-based multicasting topology for the maximum throughput.

Design network topology to maximize system throughput:

\[
\sum_{m=1}^{M} R_m
\]  
(6)

Over design variables:

1) Wireless multicast rate: \( \{R_m\} \quad (m \in \mathcal{M}) \)

2) Amount of flow on wireless links: \( \{f_{i,j}^{(n_m,0,n_m,l)}\} \), \( (m \in \mathcal{M}, l \in \mathcal{L}_m, \{i, j\} \in \mathcal{N}, i \neq j) \)

3) Energy supply at nodes: \( \{E_n\} \quad (n \in \mathcal{N}) \)

4) Lifetime of nodes: \( \{T_n\} \quad (n \in \mathcal{N}) \)

Subject to:

1) Flow conservation constraint:

\[
\sum_{a \in \mathcal{N}, a \neq n} f_{a,n}^{(n_m,0,n_m,l)} - \sum_{b \in \mathcal{N}, b \neq n} f_{b,n}^{(n_m,0,n_m,l)} = \begin{cases} 
-R_m & \text{if } n = n_{m,l}, \\
R_m & \text{if } n = n_{m,0}, \\
0 & \text{otherwise,}
\end{cases}
\]  
(7)

where \( n \in \mathcal{N}, m \in \mathcal{M}, l \in \mathcal{L}_m \).

2) Wireless link capacity constraint:

\[
\sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f_{i,j}^{(n_m,0,n_m,l)} \leq W \log_2(1 + \gamma_{i,j}), \quad \{i, j\} \in \mathcal{N}
\]  
(8)

3) Multicast traffic rate constraint:

\[
R_m \geq \delta_m, \quad m \in \mathcal{M}
\]  
(9)
4) Node energy constraint:

\[
\left[ \max_{a \in \mathcal{N}} \alpha_n t_{n,a} \sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f^{(n_m_0,n_m_1)}_{n,a} + \max_{b \in \mathcal{N}} \beta_n t_{b,n} \sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f^{(n_m_0,n_m_1)}_{b,n} \right] T_n \leq E_n \quad (10)
\]

5) Total energy constraint

\[
\sum_{n=1}^{N} E_n \leq \xi \quad (11)
\]

6) Network lifetime constraint:

\[
T_n \geq \tau, \quad n \in \mathcal{N} \quad (12)
\]

It can be observed that, if we consider a wired network with no constraints on the energy and network lifetime (i.e. with invariant link capacity and without constraints (10), (11) and (12)), the proposed WMN topology design problem can be regarded as the topology design problems in previous work for unicast and multicast wired networks, which were shown to be NP-hard [18]. Accordingly, the proposed topology design problem contains these problems as special cases, and thus is also NP-hard.

Since this is an NP-hard problem, in the next section, we first develop an heuristic algorithm, called the NC-based LCRT, to construct an NC supported wireless multicasting tree with the minimum number of intermediate nodes and wireless links while guaranteeing the maximal recovery capacity of original packets from the NC-based combined packets received at the destination nodes. We then optimise the wireless multicasting rate under different requirements, including the number of flows on wireless links, the energy supply at nodes, and the lifetime of nodes, so as to maximise the network throughput.

IV. HEURISTIC NC-BASED LCRT ALGORITHM

In the proposed NC-based LCRT algorithm, in order to be able to recover original packets from the linear NC-based combined packets, the destination nodes should try to receive at least
combined packets from the other nodes. Additionally, LCRT metric\(^2\) is utilised in the NC-based LCRT algorithm to evaluate the availability of nodes and measure the weight of nodes with respect to interference from the other nodes. The LCRT multicast tree employs a minimal number of on-tree forwarding nodes that can reliably cover a group of receiver nodes. Here, the availability of the \(n\)-th node \((n \in \mathcal{N})\) is defined as

\[
\theta_n = \frac{C_n}{\sum_{i \in \mathcal{N}} \sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f^{(n,m,n,m,l)}_{n,i}} \tag{13}
\]

where \(C_n\) denotes the total capacity available at the \(n\)-th node. Taking into account the benefit of randomised linear NC in exploiting various data streams [9], [29], the weight of the \(n\)-th node \((n \in \mathcal{N})\) is defined as

\[
\eta_n = \vartheta_n \theta_n \tag{14}
\]

where \(\vartheta_n\) denotes the number of flows coming from the \(n\)-th node.

Let us define the level of a node as the least number of wireless hops from a node to the source node. By using LCRT, source node \(n_{m,0}, m \in \mathcal{M}\), assigns a node level to a multicast node according to its hop distance to \(n_{m,0}\). Destination nodes \(n_{m,l}\) \((l \in \mathcal{L}_m)\) of the \(m\)-th wireless multicast are assigned the levels of \(Q_m\). Also, let \(u_{q,m}, c_{q,m}\) and \(v_{q,m}\) denote the number of uncovered nodes, covered nodes and fully covered nodes, respectively, at the \((q_m+1)\)-th level of the \(m\)-th wireless multicast. Here, node at the \((q_m+1)\)-th level is fully covered if it is covered by at least \(K\) nodes at the \(q_m\)-th level. This condition is helpful in assisting the recovery of original packets at destinations. The NC-based LCRT algorithm is summarised in Algorithm 1 and, for clarity, the flowchart of this algorithm is illustrated in Fig. 1. Protocols can be designed

\(^2\)LCRT metric is defined as in [25, Sect. 4.3.1, eq. (3)] to represent the relationship between the number of downstream nodes, the number of nodes and the availability of nodes.
to enable nodes in the network to cooperatively set up the NC-based multicast topology by using the proposed algorithm in a distributed fashion.

![Flowchart of NC-based LCRT algorithm](image)

Denote $N'$ as the total of nodes required in the proposed NC-based LCRT algorithm, $Q_{\text{max}}$ as the maximum node level for all $M$ wireless multicasts, i.e. $Q_{\text{max}} = \max_{m \in M} Q_m$, $\chi_q$ and $\psi_q$ ($q \in \{1, 2, \ldots, Q_{\text{max}}\}$) as the total number of nodes at the $q$-th level and the total number of nodes required at the $q$-th level in the NC-based LCRT algorithm, respectively. We have

$$M + \sum_{q=1}^{Q_{\text{max}}} \chi_q = N', \quad (15)$$

$$M + \sum_{q=1}^{Q_{\text{max}}} \psi_q = N'. \quad (16)$$
Algorithm 1 NC-based LCRT algorithm

for $m = 1$ to $M$ do

$q_m = Q_m - 1$

while $q_m > 1$ do

$c_{q,m} = 0$

while $u_{q,m} > 0$ do

* $n_{m,0}$ selects a maximum of $K$ $q_m$-th nodes based on 2 ordered criteria: i) covering a maximum number of the $(q_m + 1)$-th nodes and ii) having maximum weight values (see (14)) ⇒ Determine $v_{q,m}$

if $v_{q,m} > 0$ then

$u_{q,m} = u_{q,m} - v_{q,m}$

$c_{q,m} = c_{q,m} + v_{q,m}$

else

$u_{q,m} = u_{q,m} - 1$

$c_{q,m} = c_{q,m} + 1$

end if

end while

$q_m = q_m - 1$

end while

if $u_{0,m} \geq K$ then

* $n_{m,0}$ selects a minimum number of the 1st nodes ($\geq K$) to cover all the 2nd nodes ⇒ Determine $c_{0,m}$, $c_{1,m}$

else

* $n_{m,0}$ selects all the 1st nodes, i.e. $c_{0,m} = u_{0,m}$ ⇒ Determine $c_{1,m}$

end if

end for
Here, \( \chi_q \) can be given by

\[
\chi_q = \bigcup_{m=1}^{M} \chi_{q,m} = \bigcup_{m=1}^{M} (c_{q-1,m} + u_{q-1,m}),
\]

(17)

where \( \chi_{q,m} (m \in M) \) denotes the number of nodes at the \( q_m \)-th level of the \( m \)-th wireless multicast. From Algorithm 1, the number of nodes required at the \( q \)-th level in the NC-based LCRT algorithm can be expressed by

\[
\psi_q = \bigcup_{m=1}^{M} \psi_{q,m} = \bigcup_{m=1}^{M} c_{q-1,m},
\]

(18)

where \( \psi_{q,m} (m \in M) \) denotes the number of nodes required at the \( q_m \)-th level of the \( m \)-th multicast using Algorithm 1.

**Lemma 1.** WMN topology designed with the proposed NC-based LCRT algorithm allows a lower number of nodes involved than the total number of nodes in the network, especially when the number of data packets is small. It can be easily observed that \( c_{q,m} + u_{q,m} \geq c_{q,m} \geq v_{q,m} \) \( \forall q \in \{1, 2, \ldots, Q_{\text{max}}\} (m \in M) \), and thus, from (17) and (18), \( \psi_q \leq \chi_q \). Accordingly, from (15) and (16), we obtain \( N' \leq N \), which means the proposed NC-based LCRT algorithm reduces the number of required nodes. Furthermore, when \( K \) is small, as shown in Algorithm 1, we can easily find \( K \) \( q_m \)-th nodes to cover a \((q_m + 1)\)-th node. So, it can be approximated that \( c_{q,m} \approx v_{q,m} \ll c_{q,m} + u_{q,m} \). Consequently, a significantly reduced number of required nodes is achieved with the proposed NC-based LCRT algorithm.

**Remark 1.** System throughput of WMN increases as the number of nodes required for multicast decreases. Suppose that the wireless multicast requirements need a total of \( N' \) nodes \((N' < N)\). From the total energy constraint in (11), if the energy supply at each node is fixed and \( N \) decreases to \( N' \), then \( \sum_{n=1}^{N} E_n \) decreases to \( \sum_{n=1}^{N'} E_n \). This means that more energy can be allocated for \( N' \) nodes while still satisfying the total energy constraint. Let us denote \( E_{n'} \)
(n′ ∈ \{1, 2, \ldots, N\'}) as the energy supply at n'-th node in the new topology design. From (11), \(E_{n'}\) can be easily allocated satisfying \(E_{n'} > E_n\) and \(\sum_{n=1}^{N'} E_{n'} = \sum_{n=1}^{N} E_n\). Thus, given the node energy constraint in (10) with \(E_{n'}\), higher data flow can be allocated for wireless links. Accordingly, from flow conservation constraint in (7), the wireless multicast rates can be increased for a higher system throughput.

![Diagram](image)

Fig. 2. An example of NC-based LCRT algorithm.

In order to illustrate the steps in the proposed NC-based LCRT algorithm, let us consider an example as shown in Fig. 2. In this example, there are 19 nodes in total. For simplicity, 1 multicast requirement of 3 data packets (i.e. \(M = 1\) and \(K = 3\)) is considered\(^3\) with the multicast node set \(S_1 = \{1, 16, 17, 18, 19\}\), where node 1 is source node and nodes \(\{16, 17, 18, 19\}\) are destination nodes. Suppose that nodes \(\{2, 3, \ldots, 7\}\), \(\{8, 9, \ldots, 15\}\) and \(\{16, 17, 18, 19\}\) are in

\(^3\)The example is extendible to a general model with more than 1 multicast requirement by employing multicast requirements in parallel.
the first, the second and the third levels, respectively. All the nodes in the first and the second
levels in Fig. 2 are assumed to be available to support node 1 in the multicast. We assume that
nodes \{2, 3, 7, 8\} have the lowest weight compared to the other nodes in the same level. Since
\( K = 3 \), we only select a maximum of 3 nodes in the second level to cover each third-level node.
Specifically, the second-level node sets \{9, 10, 11\}, \{10, 11, 12\}, \{11, 12, 14\} and \{14, 15\} are
selected to cover destination nodes 16, 17, 18 and 19, respectively. These second-level node sets
are selected since they have higher weight and cover a maximum number of third-level nodes.
Here, node set \{11, 12, 14\} are selected to cover node 18 instead of \{11, 12, 13\} even though
node 13 has a higher weight, however it covers only node 18 while node 14 covers 2 nodes 18 and
19. For the first-level nodes, node set \{4, 5, 6\} is selected since these nodes have higher weight
and cover all the second-level nodes. Accordingly, our proposed NC-based LCRT algorithm has
saved 4 nodes in the network while guaranteeing the maximal recovery capability of original
packets at destination nodes. This means that an improved system throughput is achieved with
our proposed NC-based LCRT algorithm (see Remark 1).

V. CROSS-LAYER DESIGN

In the previous section, we have obtained the WMN topology tree using the proposed NC-
based LCRT algorithm for the required number of multicasted packets. Then, based on this
developed topology tree, we aim to find optimal design metrics that can maximize the system
throughput for transmitting a data packet subject to various design constraints. In this section,
we prove the tractability of the optimization problem over the design variables in the proposed
topology tree. Specifically, multicast rate for various wireless multicast requirements, flows on
wireless links, energy supply at nodes and lifetime of nodes are considered in the optimization
problem.
Let us assume that our topology design for NC-based multicast requires a total of \( N' \) nodes \((N' < N)\) in a node set \( \mathcal{N}' \). The optimization problem (6) is to maximize the system throughput over wireless multicast rate \( \{R_m\} \), flows across links \( f_{i,j}^{(n_m,0,n_m,l)} \), node energy supply \( E_n \) and node lifetime \( T_n \). Then, subject to constraints (7), (8), (9), (10), (11) and (12), we can reformulate (6) as

\[
\begin{align*}
\max_{\{R_m, \{f_{i,j}^{(n_m,0,n_m,l)}\}, \{E_n\}, \{T_n\}\}} & \sum_{m=1}^{M} R_m \\
\text{s.t.} & \sum_{a \in \mathcal{N}', a \neq n'} f_{n',a}^{(n_m,0,n_m,l)} - \sum_{b \in \mathcal{N}', b \neq n'} f_{b,n'}^{(n_m,0,n_m,l)} = \begin{cases} 
-R_m & \text{if } n' = n''_{m,l} \\
R_m & \text{if } n' = n''_{m,0} \\
0 & \text{otherwise}, 
\end{cases} \\
& \sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f_{i,j}^{(n_m,n_m,0,n_m,l)} \leq W \log_2 (1 + \gamma_{i,j}), \{i,j\} \in \mathcal{N}' \\
& \forall m \in \mathcal{M}, l \in \mathcal{L}_m
\end{align*}
\]

(19)

(20)

Lemma 2. Given a fixed topology tree and fixed lifetime of nodes, the optimization problem (19) subject to constraints (20), (21), (22), (23), (24) and (25) is a linear programming (LP) problem.

**Proof.** From (19), it can be seen that the system throughput function is linear over multicast rate variables of wireless multicast requirements \( R_m (m \in \mathcal{M}) \). With fixed topology tree and fixed
\( T_{n'} (n' \in \mathcal{N}') \), we can easily prove that the constraints (20), (21), (22), (23), (24) and (25) are linear over \( R_m, f_{n', a, n'_m, i}^{(n'_m, 0, n'_m, l)}, E_{n'} (n' \in \mathcal{N}', m \in \mathcal{M}, l \in \mathcal{L}_m) \). Therefore, the system throughput optimization problem is a linear programming problem. 

**Remark 2.** Given a fixed topology tree, the system throughput increases as lifetime of nodes approaches network lifetime. This can be observed from the flow constraint in (20) and node energy constraint in (23).

From Remark 2, in order to maximize the system throughput, the lifetime of nodes should be set as the network lifetime (i.e. \( T_{n'} \approx \tau \)), and thus the network lifetime constraint (25) can be relaxed. The optimization problem (19) subject to constraints (20), (21), (22), (23) and (24) can be now rewritten as

\[
\begin{align*}
& \max_{\{R_m\}, \{f_{n', a, n'_m, i}\}, \{E_{n'}\}} \sum_{m=1}^{M} R_m \\
\text{s.t.} \quad & \sum_{a \in \mathcal{N}', a \neq n'} f_{n', a}^{(n'_m, 0, n'_m, l)} - \sum_{b \in \mathcal{N}', b \neq n'} f_{b, n'}^{(n'_m, 0, n'_m, l)} = \begin{cases} 
-R_m & \text{if } n' = n'_m, l, \\
R_m & \text{if } n' = n'_m, 0, \\
0 & \text{otherwise},
\end{cases} \\
& \quad n' \in \mathcal{N}', m \in \mathcal{M}, l \in \mathcal{L}_m \\
& \sum_{m=1}^{M} \max_{l \in \mathcal{L}_m} f_{i, j}^{(n'_m, 0, n'_m, l)} \leq W \log_2 (1 + \gamma_{i, j}), \{i, j\} \in \mathcal{N}' \\
& \quad R_m \geq \delta_m, m \in \mathcal{M} \\
& \quad T_0 \leq E_{n'}, n' \in \mathcal{N}' \\
& \quad \sum_{n'=1}^{N'} E_{n'} \leq \xi
\end{align*}
\]
As proved in Lemma 2, the optimization problem \((26)\) subject to constraints \((27), (28), (29), (30)\) and \((31)\) is a LP problem, and thus this problem is tractable using various well-known solvers, such as interior-point, active-set and simplex algorithms.

**Remark 3.** System throughput increases as either total energy available for network increases or network lifetime decreases. This can be shown through the energy constraints in \((30)\) and \((31)\).

**Remark 4.** A significantly improved throughput is achieved with NC technique, especially with a large multicast node set. Let us consider the non-NC-based LCRT protocol in which linear NC is not applied at intermediate nodes. Without linear NC technique, the wireless link utilisation and node energy constraints are given by
\[
\sum_{m=1}^{M} \left| S_m \right| - 1 \sum_{l=1}^{f_{i,j}^{(n_m',m,n_m,i)}} \leq W \log_2(1 + \gamma_{i,j}), \{i,j\} \in \mathcal{N}',
\]
respectively. It can be observed from \((32)\) and \((33)\) that, subject to fixed SNR of the wireless link and limited node power, we cannot allocate high data flows for wireless links in the non-NC-based LCRT protocol. However, in the proposed NC-based LCRT protocol, the summation of flows in the wireless link capacity and node energy constraints is replaced by the maximum of flows (see \((28)\) and \((30)\)). This means that, as \(|S_m|\) is large, much higher data flows can be allocated for wireless links, and thus the system throughput is significantly improved.

**VI. Numerical Results**

The performance evaluation of the proposed solution is carried out through numerical results and compared against other solutions from the literature, namely non-NC-based LCRT protocol
and a classic protocol using all nodes (e.g. [30], [31]). The algorithms were implemented in MATLAB and compared in terms of the throughput. For simplicity, a multicast requirement is considered. The proposed algorithm can be applied for multiple multicast requirements by either separately treating each multicast or simultaneously processing all multicasts in a centralised manner.

A. Maximum Achievable Throughput

Let us first investigate the maximum achievable throughput of different protocols for the topology design in the WMN. Fig. 3 plots the maximum throughput of various protocols as a function of network lifetime. The WMN consists of 19 nodes and the multicast node set is assumed to be $S_1 = \{1, 16, 17, 18, 19\}$. The nodes are located in a scale of $100 \times 100$, where

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*Note that a specific scenario is considered for brevity, though the simulation for various number of nodes in the multicast set is straightforward.*
their coordinates are uniformly distributed in \((0, 100)\). In Fig. 3, it is assumed that there are 3 packets (i.e. \(K = 3\)), each having 1000 bits (i.e. \(B = 1000\)), that need to be transmitted from node 1 to nodes \(\{16, 17, 18, 19\}\). The SNR of the adjacent wireless links is 10 dB and the channel bandwidth is 300 KHz. The minimum wireless multicast traffic requirement, i.e. \(\delta_1\), is 500 bits/s. The total energy available for the whole network, i.e. \(\xi\), is \(200 \times 10^6\) Joules. The energy to transmit and receive a bit is 1 and 0.1 Joule, respectively, i.e. \(\alpha = 1\) and \(\beta = 0.1\). First, it can be observed in Fig. 3 that the proposed NC-based LCRT protocol achieves an improved performance of up to 50% over the two compared protocols in terms of maximum throughput. This observation confirms the statements in Remarks 1 and 4 concerning the increased system throughput that can be achieved with the reduced number of nodes and linear NC technique in the proposed NC-based LCRT protocol. Secondly, in Fig. 3, the maximum throughput of all protocols is shown to decrease as the network lifetime increases. This performance reduction, as explained in Remarks 2 and 3, is caused by the limit of energy available for nodes in the WMN.

The impacts of total energy available for the WMN on the network throughput are shown in Fig. 4, where the maximum throughput of various protocols are plotted against the total energy available for the whole network (i.e. \(\xi\)). The number of multicast packets, packet size, SNR of wireless links, wireless multicast rate requirement, and energy to transmit and receive a bit are all set similar to those in Fig. 3. The network lifetime, i.e. \(\tau\), is assumed to be \(20 \times 10^3\) s. It can be seen in Fig. 4 that up to 50% increase in the throughput is achieved with the proposed NC-based LCRT protocol compared to the other protocols. This improved performance again confirms the statements in Remarks 1 and 4 regarding the increased system throughput achieved with the proposed NC-based LCRT protocol when the number of nodes required for the wireless
multicast decreases and linear NC technique is applied. Additionally, as shown in Fig. 4, the throughput of all protocols increases as the total available energy increases, which accordingly confirms the claim of the monotonically increasing throughput over total available energy in Remark 3.

B. Minimum Achievable Throughput

Investigating the minimum throughput that could be achieved with different protocols for WMN topology design, Figs. 5 and 6 show the minimum throughput as a function of network lifetime and total available energy with the same assumptions as in Figs. 3 and 4, respectively. It can be observed in Figs. 5 and 6 that the minimum throughput achieved with the proposed NC-based LCRT protocol is higher than that of both the all-node-based protocol and the non-NC-based LCRT protocol. For example, as shown in Figs. 5 and 6, when $\xi = 200 \times 10^6$ Joules and $\tau = 20 \times 10^3$ s, the proposed scheme improves the minimum throughput by 1600 and 3200.
Fig. 5. Minimum throughput versus network lifetime.

Fig. 6. Minimum throughput versus total energy.
bits/s compared to the all-noded-based protocol and non-NC-based LCRT protocol, respectively.

C. Impact of Transmission Delay

Taking into consideration transmission delay\(^5\) in WMN, Figs. 7 and 8 plot the transmission delay of various protocols versus network lifetime and total available energy for the WMN, respectively\(^6\). Similarly, we set the number of multicast packets, packet size, SNR of wireless links, wireless multicast rate requirement, and energy to transmit and receive a bit as those in Figs. 3 and 4. In Fig. 7, \(\xi\) is fixed at \(200 \times 10^6\) Joules and the transmission delay is shown as a function of \(\tau\), while \(\tau\) is fixed at \(20 \times 10^3\) s in Fig. 8 which plots the transmission delay over \(\xi\). As shown in Figs. 7 and 8, the transmission delay of the proposed protocol is 30\% less than that of the all-node-based protocol. This performance improvement reflects the above observations on the higher throughput achieved with the proposed NC-based LCRT protocol. Also, it can be observed that the transmission delay monotonically increases over the network lifetime, but decreases over the total available energy. Again, these observations are consistent with those of throughput in Figs. 3, 4, 5 and 6.

D. Impact of the Number of Multicast Data Packets

Considering the impacts of the number of multicast data packets on the system throughput of WMN, in Fig. 9, the maximum throughput of the proposed NC-based LCRT protocols is plotted as a function of total energy available in the network with respect to different number of packets,

\(^5\)Transmission delay is defined as the average of time to transmit a data packet from a source node to all destination nodes in a multicast node set.

\(^6\)As the performance of the non-NC-based LCRT protocol is clearly not as good as the other two protocols in terms of system throughput, in what follows, we only consider the all-node-based protocol and the proposed NC-based LCRT protocol for comparison.
Fig. 7. Transmission delay versus network lifetime.

Fig. 8. Transmission delay versus total energy.
i.e. $K$. Specifically, three scenarios $K = \{2, 3, 4\}$ are investigated under the same simulation settings of the other parameters as in Fig. 4. It can be observed in Fig. 9 that, the proposed scheme achieves a better performance of up to 50% for $K = 3$ compared to that for $K = 2$ and $K = 4$. In fact, when $K = 3$, we can take the most advantage of linear NC in the considered network (see Fig. 2) by exploiting a least number of wireless links and nodes to convey 3 data packets via NC to all destination nodes (as shown in Algorithm 1). When $K = 4$, more nodes and links are now required to convey 4 data packets, which causes higher energy consumption. Also, due to the availability and coverage of each node, some nodes may not have 4 wireless links with other nodes. Therefore, subject to constraints on energy consumption at node and in the whole network, linear NC is not efficiently exploited in the considered network model for the scenario $K = 4$. As $K = 2$, a lower number of nodes and links are used, and thus requires a lower energy supply. However, with a given energy and network lifetime to convey data to
4 destination nodes in the WMN model in Fig. 2, the transmission of only 2 packets does not take the most benefit of linear NC and thus also causes a decreased performance.

E. Impact of Data Packet Size

Let us now investigate the effects of the size of data packet on the system throughput of WMN. Fig. 10 shows the maximum throughput achieved with the proposed NC-based and all-node-based protocols against network lifetime with respect to different packet size, i.e. $B$. With the same network settings as those in Figs. 3 and 5, two cases $B = 1$ kbits and $B = 3$ kbits are considered in Fig. 10. The throughput of both the proposed NC-based LCRT and all-node-based protocols is shown to decrease as $B$ increases. In fact, the increase of packet size causes an increased transmission time and energy, but the energy supply is limited. This means lower data flows are allocated for wireless links, and thus causes decreased throughput. Additionally, it can be observed in Fig. 10 that, the proposed NC-based LCRT protocol achieves a better
performance of up to 50% than that of the all-node-based protocol for both cases of packet size. This again confirms the above observations as well as the statement in Remark 1 on the throughput improvement of the proposed protocol.

\section{F. Impact of Reception Energy of A Bit}

Another investigation is on the effects of energy for receiving and transmitting a bit\textsuperscript{7} on the system throughput. As shown in Fig. 11, the maximum throughput is plotted as a function of network lifetime for the proposed NC-based LCRT and all-node-based protocols with respect to different reception energy of a bit, i.e. $\beta$. Similarly, the WMN settings are assumed as those in

\textsuperscript{7}In practice, the transmission and reception energy of a data unit may vary depending on the hardware configuration, such as design of transmitting and receiving antennas.

\textsuperscript{8}The effects of transmission energy of a data unit, i.e. $\alpha_n$, on the throughput performance can be similarly observed, and thus is omitted for brevity.
Figs. 3 and 5. In Fig. 11, two scenarios $\beta = 0.08$ Joule/bit and $\beta = 0.1$ Joule/bit are considered. It can be observed that the throughput of the proposed NC-based LCRT protocol decreases as $\beta$ increases, while that of the all-node-based protocol is almost unchanged with varied $\beta$. In fact, the increased reception energy of a bit causes an increase of energy supplied at each node. Therefore, in order to guarantee the node energy and total energy constraints, the data flows allocated for wireless links should be lower, which accordingly results in a decreased system throughput. However, with all-node-based protocol, all available nodes can share their energy to assist the data multicast, and thus the increased reception energy of a data unit does not cause much effect on the throughput performance. Also, in Fig. 11, the proposed NC-based LCRT protocol is shown to achieve a higher throughput of up to 50% compared to the all-node-based protocol for both cases of reception energy of a bit, which again verifies the claim in Remark 1 in relation to the throughput improvement achieved with the proposed protocol.

VII. Conclusions

In this paper, we have proposed a joint topology and cross-layer design to maximize the system throughput of WMN. Given various constraints on QoS (e.g. wireless multicast rate, wireless link capacity, node energy, node lifetime, network lifetime, total energy consumption), we have developed an heuristic NC-based LCRT algorithm and optimized wireless multicast rate of source nodes, wireless data flows, energy supply at nodes and lifetime of nodes. It has been shown that the proposed design reduces the number of intermediate nodes for an increased system throughput and thus results in a reduced transmission delay. A significantly improved performance is also achieved with linear NC technique. The cross-layer optimization problem has been shown to be tractable as a linear programming problem with fixed node lifetime and relaxed network lifetime constraint. Furthermore, in the numerical results, we have analysed the
achievable throughput of the proposed NC-based LCRT protocol for the WMN with respect to
different number of transmitting packets, different packet size, and different values of energy to
receive a unit of data. The results showed that our proposed protocol can achieve an increase
in throughput of up to 50% as compared to that of the non-NC-based LCRT protocol and the
protocol using all nodes. Among our plans for the future is to further develop this protocol to
enable NC-based multicast routing to adapt to dynamic wireless link conditions as well as node
mobility. Also, our future work would be an investigation of the competition as well as the
fairness amongst various nodes in the cross-layer topology design for the WMN.

REFERENCES


